Chapter 17: Magnetism

- What are northern lights?
- How does Earth protect us from the solar wind and damaging cosmic rays from Supernovae explosions?
- Do we really walk northward when following a compass?

Make sure you know how to
1. Explain the force between electrically charged objects (14.2).
2. Find the direction of the electric current in a circuit (16.2).
3. Apply Newton’s second law to a particle moving in a circle (4.4).

Figure 17.1 Aurora Borealis (northern lights)

[Chapter opening] They are called Auroras – Aurora Borealis (northern lights) in the northern hemisphere and Aurora Australis (southern lights) in the southern hemisphere. You can see them often in September-October or March-April if you travel to Alaska or to the southern parts of Australia. The view is magnificent – the whole sky lights up in a flickering greenish glow with red and sometimes blue or violet streaks. (Fig. 17.1) They are named after the Roman goddess of dawn, Aurora, as they often resemble the Sun rising. The word borealis comes from the Greek name for the north wind, ‘boreas.’ What makes the sky light up in a multitude of colors in particular locations at certain times? We know that the color of the auroras comes from the chemical composition of particles in the atmosphere, primarily oxygen and nitrogen. But why do we see the aurora mostly near Earth’s poles and only rarely anywhere else? Doesn’t Earth’s atmosphere surround the planet more or less uniformly? As we will learn in this chapter, it is not only the atmospheric gases but some other property of Earth that is responsible for the auroras.

[Lead] In the last three chapters we have been learning about electric phenomena. We learned that charged objects attract and repel each other. From our childhood experiences we know that magnets also attract and repel. However, in Chapter 14 we found that electrically charged objects do not exhibit magnetic properties. Are electricity and magnetism related in any way or are they...
totally different phenomena? Until 1820 a physicist chose the latter answer. However, today you use numerous devices whose work is based on the relationships between electricity and magnetism. In this chapter we will start learning what those connections are and how scientists discovered them.

17.1 The Magnetic Interaction

There are many legends describing the origin of the word magnet. The most popular involves Magnes, an old shepherd who lived about 4000 years ago in the part of Northern Greece called Magnesia. According to that legend Magnes was peacefully herding his sheep when suddenly the nails in his shoes and the metal tip of his staff got stuck to the large, black rock on which he was standing. To find the source of this attraction, he dug into the ground and found what became known as lodestones (the word ‘lode’ means lead or attract). Now we know that lodestones contain a natural magnetic material called iron oxide, Fe₃O₄, or magnetite.

We have all played with magnets. They stick to refrigerators, can pick up steel paper clips, and save cows from stomach punctures caused by accidentally swallowed nails. When brought near each other, magnets can both attract and repel. They can convert objects that were not previously magnets into magnets. In this section we investigate the behavior of magnets.

Magnetic poles

Some magnets have colors painted on them; often they are red and white. If you bring the same color sides of two magnets near each other, they repel. If you bring different color ends near each other, they attract (Fig. 17.2 a and b). Even if magnets do not have colors painted on them, you observe the same effect: one side of one magnet always attracts one side of another magnet and repels its other side. Additionally, both sides of a magnet attract other objects made of steel or iron even if those objects aren’t magnets themselves. However, they do not attract objects made of aluminum. Magnets do not attract all metal objects.

![Figure 17.2 Like magnet poles repel and unlike attract](image)

These two sides of a magnet are called poles—a north pole (sometimes marked red) and a south pole (white). It is interesting that these magnet poles can never be separated. If you break a magnet in two pieces, each piece interacts with other magnets as if it still has two poles—a north pole and a south pole (Fig. 17.3a). If you break one of those pieces again, each smaller piece has two poles! Unlike electrically charged objects that can have either negative or positive electric charge, a magnet with a single pole (a so-called monopole) has never been found.
The names ‘north’ and ‘south’ were motivated by experiments centuries before magnetism was understood. People noticed that if they put a tiny magnet on a low friction pivot (or let it float in a pale with water), one end always pointed in the direction of geographical north (that they could find using an independent method – stars) and the other end pointed toward geographical south (Fig. 17.4). This property of a magnet resulted in the names for the two ends: the north pole points toward geographical north; the south pole points toward geographical south. The device became known as a compass. The earliest clear evidence of navigational use of compasses was about 11th century China.

**Earth is a giant magnet**

To be used in navigation, the compass must dependably point in a particular direction no matter where you are located. Since the north pole of a compass needle is attracted to the south pole of another magnet, and since compasses that are not near other magnets always point toward Earth’s geographic north pole (approximately), it must mean that Earth is like a big magnet with its south pole at the geographic north pole and its north pole at the geographic south pole (Fig. 17.4).
Magnetic interaction depends on separation

Suppose you lay a compass on one side of a wooden table and place another magnet on the other side with one of its poles facing the compass. The compass needle may turn a little from its original orientation due to Earth’s magnetic pole, but not much (Fig. 17.5a). If you move the magnet closer toward the compass, the compass needle turns more (Fig. 17.5b). If you move the magnet next to the compass, the compass needle swings abruptly around so that one pole of the compass points towards the other-colored pole of the magnet (Fig. 17.5c). The interaction between the magnets increases in strength as their separation decreases.

The magnetic and electrical interactions are different

The behavior of magnets has similarities to the behavior of electrically charged objects – they attract and repel and the intensity of their interactions depend on their separation. However, there are significant differences. As mentioned earlier, magnets always have two poles. Also, we performed a testing experiment in Ch. 14 (Table 14.2) that disproved the hypothesis that the electric and magnetic interactions are the same. Electrically charged objects do not interact with magnets the same way the magnets interact with magnets. Charged objects such as a positively charge pith ball is attracted to both poles of a compass needle or of a magnet (Fig. 17.6a and b.) This attraction for both sides could be explained due to polarization of electric charge inside the metal magnet. We also found that a non-charged object (for example a packing peanut) is attracted to any charged object (Fig. 17.7a) but is not attracted to a compass or to a large magnet (Fig. 17.7b). Experiments with objects made of ceramic magnets (non-conducting materials) show that ceramic magnets do not interact with electrically charged objects at all.

The experiments described above disprove the idea that the attraction and repulsion of magnets is not simply an electric interaction. Magnetic poles are not electric charges. For many years, physicists explored these two phenomena completely independently because of the results of the experiments similar to just described.
Review Question 17.1
How do we know that magnetic poles are not electrically charged?

17.2 Magnetic field

We were able to better understand the mechanism for how electrically charged objects interact without contact by suggesting the existence of an electric field. Similarly, objects with mass could be thought to interact gravitationally without contact by the existence of a gravitational field. Since magnets can also interact without contact, it’s reasonable to suggest the existence of a magnetic field as the mechanism behind magnetic interactions.

Imitating our study of electric phenomena in Ch. 15, let’s suggest that a magnet produces a magnetic field with which other objects with magnetic properties (another magnet, anything made of iron, etc.) interact. The magnetic field is a magnetic disturbance produced by the magnet. The field affects other magnetic objects. How can we describe this field?

**Direction of magnetic field**

To describe a magnetic field we use a vector physical quantity called $\vec{B}$-field. In this section we will focus on its direction. One of the ways to determine the direction of the $\vec{B}$-field at a particular location is to place a compass at that location. The direction of the $\vec{B}$-field at that location is defined as the same direction that the north pole of the compass needle points when at that location. One does not need to have a compass at that location for the field to be there, the compass is just a detector. For example, when not too far from the magnet, the north pole of the compass needle points away from the north pole of the bar magnet and toward the magnet’s south pole, no matter where you place the compass (Fig. 17.8a). Figure 17.8a shows the direction of the $\vec{B}$-field at several other points near a bar magnet. Figure 17.8b shows the $\vec{B}$-field vectors near a horseshoe magnet. The compass in the above experiments is similar to a small charged object that we place in the electric field to determine the direction of $\vec{E}$ field vector.
Representing the magnetic field—field lines

Now, spread many tiny compasses on a table. They all point towards Earth’s geographical north. Now place a bar magnet in the middle of the compasses. The compasses are now oriented as shown in Fig. 17.9. The field direction at each position is in the direction that the north pole of the compass points when at that position.

If you follow the north-south axis for each compass as you move from compass to compass, you find that the compasses are tangent to lines surrounding the magnet—the dashed lines in Fig. 17.9. If instead of compasses, you use hundreds of tiny iron filings (which act like tiny compasses) sprinkled on a thin clear piece of plastic placed on top of the magnet, the filings form a pattern that looks identical to the lines formed by the compasses (Fig. 17.10). In addition, the filings that are directly on top of the magnet align with the north-south axis of the magnet. These so-called $\vec{B}$-field lines can be used to represent the $\vec{B}$-field produced by the magnet. Similar to $\vec{E}$-field lines, $\vec{B}$-field lines represent not only the direction of the $\vec{B}$-field but also its magnitude. We draw the lines closer together in the regions where $\vec{B}$-field magnitude is greater. If you use this method to construct multiple $\vec{B}$-field lines, the pattern looks as shown in (Fig. 17.11.)
Tip! Unlike $E$-field lines, which begin on positively charged objects and end on negatively charged objects, $B$-field lines do not have a beginning or an end. Notice in Fig. 17.11 that the lines form complete closed loops.

**The magnetic $B$-field and its representation by $B$-field lines** The direction of the magnetic $B$ field at a point is defined as the direction of a compass north pole when at that point. Magnetic field lines represent the $B$-field. The $B$-field vector at a point is tangent to the direction of the $B$-field line at that point. The density of lines in a region represents the magnitude of the $B$-field in that region—where the $B$-field is stronger, the lines are closer together.

Do other objects produce a magnetic field?

We can now explain the interaction between two magnets A and B in the following way. Magnet A creates a magnetic field around itself, which exerts a force or a torque on magnet B. Magnet B creates its own magnetic field that exerts a torque or a force on the magnet A. Do any other objects besides magnets create magnetic fields? We found earlier that stationary electrically charged objects do not. Maybe electric currents (moving electric charges) produce magnetic fields and are affected by magnetic fields?

**Observational Experiment Table 17.1** Do electric currents produce magnetic fields?

<table>
<thead>
<tr>
<th>Observational experiment</th>
<th>Analysis</th>
</tr>
</thead>
</table>
| Connect a battery, a switch, some wires, and a light bulb in a circuit as shown. The bulb indicates an electric current in the circuit. With the switch open, place compasses under, above, and at the sides of one of the wires. Notice the direction the needles point toward geographical north when there is no current. When there is current, the compasses point as shown. A compass on the right side. | }
The switch is then closed resulting in a current in the circuit. Notice the directions the compasses point now.

Reverse the direction of the current.

The compass needles reverse directions compared to the first experiment. The right side compass points down and the left side compass points up (not shown).

**Pattern**

Electric current affects the orientation of a compass needle, which means the current produces a magnetic field. The direction of the $\vec{B}$-field depends on the direction of the current. The magnetic field lines form closed circles around the current. Their direction changes when the direction of the current changes.

If we assume that the compass needle changes its orientation due to a magnetic field, then we conclude that an electric current does produce a magnetic field. As the current is made of electrically charged particles that are in collective motion with respect to the compass, this means that charged objects in motion produce a magnetic field and stationary charged objects do not. We know that motion is relative – an object seen as moving by one observer will be stationary for
some other. This reasoning leads us to the conclusion that magnetic effects are also relative – different observers will or will not detect a magnetic field from the same object.

The experiments in the above table allow us to deduce a pattern in the direction of the $\vec{B}$-field vectors and magnetic field lines around a current carrying wire. Examine the orientation of the compasses in the Observational Experiment Table 17.1. If you imagine grabbing the wire with your right hand with your thumb pointing in the direction of the current, your fingers wrap around the current in the direction of the $\vec{B}$-field (Fig. 17.12). This pattern for determining the direction of the magnetic field produced by the electric current in a wire is called the right hand rule for the $\vec{B}$-field.

![Figure 17.12](image)

**Right hand rule for the $\vec{B}$-field** To determine the direction of the $\vec{B}$-field line produced by a current, grab the wire with your right hand with your thumb pointing in the direction of the current. Your four fingers wrap around the current in the direction of the $\vec{B}$-field lines (Fig. 17.12). At each point on a line the $\vec{B}$-field vector is tangent to the line and points in the direction of the line.

If you use the right hand rule for the $\vec{B}$-field lines due to the current in a loop or coil, the pattern looks as shown in Fig. 17.13a. Notice that the $\vec{B}$-field produced by a current in a coil (Fig. 17.13a) and by a bar magnet (Fig. 17.13b) are very similar. The bar magnet has $\vec{B}$-field lines that leave the north pole, curl around, enter the south pole, then go through the magnet to the north pole again forming closed loops. Similarly, the current loop has $\vec{B}$-field lines that come out from inside the loop, curl around the outside of the loop, then go back inside the loop from the other side, again forming complete loops. The $\vec{B}$-field in the region to the right of the plane of the coil (or loop) looks just like the $\vec{B}$-field in the north pole region of the bar magnet. Likewise, the region to the left of the plane of the coil looks just like the $\vec{B}$-field in the south pole region of the bar magnet. Wire coils with current act magnetically in a very similar way to bar magnets and are known as electromagnets.
In 1820 Danish Professor Hans Oersted accidentally performed an experiment similar to that described in the Observational Experiment Table 17.1. He was using a long wire to demonstrate for students the heating effect of electric current in the wire. His assistant forgot to remove from the demonstration table a compass that Oersted used in his previous lecture on magnetism. One of the students noticed that when Oersted turned on the electric current in the wire, the compass needle abruptly changed direction. The student brought this to Oersted’s attention. History did not record the observant student’s name. Oersted however noted this observation and studied it in detail by placing the compass on different sides of the wire, above and below it. He was the first to observe the magnetic effect of electric current.

Conceptual Exercise 17.1 Draw the magnetic field lines for a solenoid when it is connected to a battery, as in Fig. 17.14a.

Sketch and Translate A solenoid is a wire wound with a large number of loops into a cylindrical shape. To draw the \( B \)-field lines produced by a current in the solenoid, we use the right hand rule for the \( B \)-field to draw lines produced by each loop, then combine them for a complete picture of the \( B \)-field produced by the solenoid.

Simplify and Diagram The lines inside each loop all point in the same direction (Fig. 17.14b). If the superposition principle applies to the \( B \)-field as it does for an \( E \) field, then the sum of all the \( B \)-field loops look like in Fig. 17.14c. The \( B \)-field produced by the current in a solenoid is nearly identical to the \( B \)-field produced by a bar magnet. The left side of this solenoid is a south pole, and the right side is a north pole. To test our answer we can take a compass needle and put it
close to the solenoid at different locations. We find that the $\vec{B}$-field produced by the current in the solenoid is indeed shaped like the $\vec{B}$-field produced by a bar magnet.

![Figure 17.14(b)(c) B-field of solenoid](image)

Try It Yourself: Use the right hand rule for the $\vec{B}$-field and the superposition principle to predict the direction of the magnetic field exactly in the middle between two straight wires oriented horizontally in the plane of the page. The current through the top wire is toward the right and the current through the bottom wire is toward the left.

Answer: The $\vec{B}$-field contribution of each wire at the point of interest points into the page. Therefore, the $\vec{B}$-field at that point points into the page.

Review Question 17.2

What is the direction of the $\vec{B}$-field at a point that is exactly in the middle between two straight wires with currents in the same direction?

17.3 Magnetic force exerted by magnetic field on a current-carrying wire

So far we learned that electric currents in wires produce a magnetic field just as a magnet produces a magnetic field. Thus it looks like a current carrying wire is similar to a magnet. If this is true, then a magnetic field should exert a magnetic force on a current carrying wire similar to the force it exerts on another magnet. To test this hypothesis we can place a bar magnet near a circuit with long connecting wires, so the effects of the magnet on the wires are visible. When we do that, we find that we need to vary the orientation and location of the magnet with respect to a wire to make it pull or push the wire. The effect is definitely there but the pattern is not clear. To investigate this phenomenon further we will collect more information.

A magnetic field exerts force on current carrying wire

To investigate how the orientation of the magnet and current carrying wire affect their interactions, we will conduct experiments described in Observational Experiment Table 17.2. They will involve a horseshoe magnet. This type of magnet is convenient because the magnetic field lines between the poles are almost parallel to each other and straight across from one pole to the other (see Fig. 17.15).
### Observational Experiment Table 17.2 Direction of magnetic force

<table>
<thead>
<tr>
<th>Observational experiment</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Hang a horizontal straight wire attached to a battery (switch open) between the poles a horseshoe magnet so that the wire is oriented parallel to the $\vec{B}$-field lines. Turn the current on by closing the switch. The wire does not move. If you reverse the current, the wire still does not move.</td>
<td>The $\vec{B}$-field lines for both experiments are parallel to the direction of the current. The magnet does not exert a force on the current carrying wire.</td>
</tr>
<tr>
<td>(b) Orient the wire perpendicular to the $\vec{B}$-field lines and turn the current on by closing the switch. The wire bends down.</td>
<td>The field lines are perpendicular to direction of the current in the wire. The force exerted on the wire by the magnetic field is downward and perpendicular to both the current and to the $\vec{B}$-field.</td>
</tr>
<tr>
<td>(c) Repeat the previous experiment but this time reverse the poles of the battery connected to the wire. When the switch is closed turning on the current in the opposite direction, the wire bends up.</td>
<td>The field lines are perpendicular to the orientation of the wire. The force exerted by the magnetic field on the current carrying wire is upward and perpendicular to both the wire and the $\vec{B}$-field.</td>
</tr>
</tbody>
</table>

### Pattern
- Magnetic field does not exert a force on the current carrying wire if the $\vec{B}$-field is parallel to the current.
- When the $\vec{B}$-field is perpendicular to the direction of the current, the magnetic field exerts a force on the current carrying wire that is both perpendicular to the direction of the current and to the $\vec{B}$-field. The direction of this force depends on the direction of the current and the $\vec{B}$-field.

A magnetic field does not exert a force on a current carrying wire when the current carrying wire is oriented perpendicular to the $\vec{B}$-field of the magnet. Other experiments besides those in
Table 17.2 indicate that even if the current-carrying wire is not perpendicular to the \( \vec{B} \)-field, provided that they aren’t exactly parallel, the \( \vec{B} \)-field still exerts a force on the current carrying wire that is perpendicular to both the direction of the current and the direction of the \( \vec{B} \)-field. The direction of the magnetic force that a magnetic field exerts on a current carrying wire is illustrated in Fig. 17.16 and described below.

**Right-hand rule for the magnetic force** Hold your right hand flat with your thumb extended from the four fingers. Orient your hand so that your right thumb points along the direction of the current, and your fingers point in the direction of the \( \vec{B} \)-field. The direction of the magnetic force exerted by the magnetic field on the current is the direction your palm faces—perpendicular to both the direction of the current and the direction of the \( \vec{B} \)-field. (Fig. 17.16)

![Figure 17.16 Right-hand rule for magnetic force](image)

The magnetic force has several important features:

1. Although we developed the right hand rule for the magnetic force for situations involving electric currents, we find later that the rule applies for individual moving positively charged objects. The force that the magnetic field exerts on a moving negatively charged object (such as an electron) is opposite the direction of the force on a moving positive charge.

2. If the current is parallel to the \( \vec{B} \)-field, the magnetic force exerted on it is zero. If the current is not parallel to the \( \vec{B} \)-field, the larger the angle between the current and the \( \vec{B} \)-field, the larger the magnetic force exerted on it (for \( 90^\circ \) angles or less).

3. We now have two right hand rules: 1) the right hand rule for the \( \vec{B} \)-field, and 2) the right hand rule for magnetic force. Right hand rule 1 determines the direction of the \( \vec{B} \)-field produced by an electric current. Right hand rule 2 determines the direction of the magnetic force exerted by a magnetic field on moving charged objects.

**Forces between current carrying wires**
If a current carrying straight wire produces a magnetic field, this field should exert a force on a second current carrying straight wire placed nearby. Similarly, the magnetic field produced by the second wire’s current should exert a force on the first wire’s current. According to Newton’s third law, these forces that wires exert on each other should point in opposite directions and have the same magnitudes. Let us test this reasoning with a simple experiment described in Conceptual Exercise 17.2.

**Conceptual Exercise 17.2 Testing experiment - interaction of two current carrying wires**

Imagine that you have two strips of aluminum foil each connected at their ends to the terminals of their own battery. The strips are positioned vertically next to each other. Predict what will happen (a) when the currents in the strips are in the same direction, and (b) the currents are in the opposite direction.

*Sketch and Translate* The situation is sketched in Fig.17.17a. The arrows beside the strips indicate the directions of the currents. We label the left strip A and the right strip B.

![Figure 17.17(a)](image)

*Simplify and Diagram* (a) Choose strip B as the system of interest. Consider the direction of the \( \vec{B} \)-field produced in the vicinity of B by strip A. Using the right hand rule for the \( \vec{B} \)-field, we find that \( \vec{B}_A \) points into the page where strip B is located (a cross represents a \( \vec{B} \)-field pointing into the page). Now we use right hand rule for the force to determine the direction of the force that the strip A magnetic field exerts on strip B (\( \vec{F}_{A \text{ on } B} \)). The force should point to the left (Fig. 17.17b). We repeat the same procedure for strip A as the system and find that the force exerted on it by the strip B magnetic field. The force that this field exerts on the strip A current \( \vec{F}_{B \text{ on } A} \) should point to the right (Fig. 17.17c). The strips should attract each other. (b) When the current in the B flows in the downward direction instead of up, the same analysis shows that the strips should repel. Both results are consistent with Newton’s third law – the forces that the strips exert on each other point in opposite directions. If we do the experiment, we see that the strips bow towards each other with the currents in the same direction and bow away from each other when the currents are in the opposite directions (Fig. 17.17d).
Try It Yourself: Explain why the two current carrying light coils of wire next to each other attract when the current is as shown in Fig. 17.18a.

Answer: Each coil is like a bar magnet with their poles in the same direction. The N pole on left coil attracts the S pole on right coil, and vice versa (see Fig. 17.18b).

Tip! Notice that in the conceptual exercise 17.2 the wires attracted and repelled each other via magnetic forces as if they were magnets. The wires do not interact with each other via electric forces as the net electric charge of each is zero.

The Conceptual Exercise 17.2 repeats the experiment that Andre Mari Ampere performed in 1820 soon after he learned about Oersted’s experiments. Ampere conducted numerous experiments with the goal of finding a mathematical expression for the force that current carrying wires exert on each other. He found that the mathematical equation describing the magnitude of these forces was similar to the force that electrically charged particles exerted on each other; it was directly proportional to the magnitude of each current and inversely proportional to the distance between the wires squared. His famous experiment established this as the basis for the unit of the electric current in the SI system – the ampere (Fig. 17.19).
**Definition of the ampere** Suppose two 1.0-m long parallel wires are separated by 1.0 m (see Fig. 17.19). You run equal magnitude electric currents through the wires so that they exert a force of $1.0 \times 10^{-7}$ N on each other. The current $I$ in each wire is then defined to have a magnitude of 1.0 A.

From this definition it is apparent that magnetic forces are rather weak. Unless the currents are very large, the magnetic forces they exert on each other will be small and difficult to detect. We can think of every current carrying object as a small magnet whose magnetic field lines can be determined using the right hand rule for the fields.

**Expression for magnetic force that magnetic field exerts on a current carrying wire**

To determine a mathematical expression for the magnetic force that a magnetic field exerts on a current-carrying wire, we hang a horizontal wire at its ends from conducting springs. The springs are connected to a battery so the current through the wire is in the direction shown in (Fig.17.20a). We place the wire between the poles of an electromagnet, which produces an approximately uniform variable $\vec{B}$-field in the region surrounding the wire (and no field in the region where the springs are). The right hand rule for magnetic force indicates that the magnetic field exerts a downward magnetic force $\vec{F}_B$ on the current in the wire. Earth exerts a downward gravitational force on the wire $\vec{F}_E$. These two forces are balanced by the net upward force of the two springs on the wire $\vec{F}_S$. (See the force diagram in Fig. 17.20b.) Knowing the spring constant of the springs and the mass of the wire, we can use the stretch of the springs to deduce the magnetic force exerted on different length wires when different currents pass through them. The collected data are shown in Table 17.3.
Notice that the magnitude of the magnetic force exerted on the wire is proportional to the current $I$ through the wire (the first three rows), to the length $L$ of the wire (the second three rows), and to the sine of the angle $\theta$ between the direction of the current and the direction of the $B$-field (the last four rows). Mathematically, we get for the force (the subscript $B$ on $W$ indicates that the magnetic field exerts the force on the wire):

$$F_{B \text{ on } W} \propto IL \sin \theta$$

(17.1)

This relation means that doubling the current in the wire or its length leads to a doubling of the force that the same magnetic field exerts on the wire. It also means that the magnitude of the force depends on the orientation of the magnetic field and the current carrying wire. When the wire and the $B$-field vector are perpendicular to each other, the magnetic field exerts the maximum magnitude force; when they are parallel, the same field exerts no force at all. This is consistent with our previous observations.

When one quantity is proportional to another quantity, their ratio is constant (for a car moving at constant velocity, the displacement is proportional to time elapsed, and the ratio is a constant value – the velocity of the car). We can rewrite the above relation as:

$$\frac{F_{B \text{ on } W}}{IL \sin \theta} = \text{const}.$$
This expression indicates that when you place a current carrying wire in a magnetic field, the ratio of force that the field exerts on the wire and the magnitude of the current, the length of wire and the sin of the angle between the wire and the direction of the $B$-field remains constant. Could it be that this ratio tells us something about the magnetic field itself? Or, to be more precise, does it provide information about the magnitude of the $B$-field? If this is true, and we change the magnet to a different magnet in the experiment, then the ratio should change. For example, intuitively we know that a bigger magnet should exert a bigger force on the same length wire with the same the current. When we do the experiment with a bigger magnet, we observe that it exerts a bigger force on the same current carrying wire. But it also matches the prediction based on the equation – the ratio $\frac{F_B \text{ on } W_{\text{max}}}{IL}$ increases.

We can use the above mathematical relation to define the magnitude of the $B$-field in a particular region as the ratio of the magnitude of the maximum force that the field exerts on a current carrying wire of length $L$ with current $I$ placed at that region. This maximum force is exerted when the wire is perpendicular to the direction of the $B$-field.

$$B = \frac{F_B \text{ on } W_{\text{max}}}{IL}$$  \hspace{1cm} (17.2)

*Tip!* The definition $B = \frac{F_B \text{ on } W_{\text{max}}}{IL}$ is an operational definition for the magnitude of the $B$-field. It does not explain why the magnitude has a particular value, but describes a method of determining this value. It is similar to the definition of the $E$ field as $E = \frac{F_E \text{ on system}}{q_{\text{system}}}$ that we had in Chapter 15. For the magnetic situations the current carrying wire is the system. Three properties – the magnitude of current, its orientation and the length of the wire are the important properties of the system.

Equation (17.2) allows us to define a unit of the $B$-field, known as the tesla $T$. A $B$-field of one tesla in a particular region means that if you take a 1 m long wire and let 1 A of current pass through it when it is oriented perpendicular to the $B$-field in that region (assuming the uniform field), the magnetic field will exert a force of 1 N on it: $1 \ T = 1 \ \text{N/A} \cdot \text{m}$. The unit is named in honor of the Serbian inventor Nikola Tesla (1857-1943). A 1-T $B$-field is very strong. By comparison, the $B$-field produced by Earth has an average value at the surface of $5 \times 10^{-5} \ T$. Good quality bar magnets produce a $B$-field near their poles of about 0.04 T. We can now use the definition of the magnitude of the $B$-field to rewrite the expression for the force as an equality:
Magnetic force exerted on a current. The magnitude of the magnetic force $F_{\text{on W}}$ that a magnetic field $\vec{B}$ exerts on a current $I$ passing through a wire of length $L$ is

$$F_{\text{on W}} = ILB \sin \theta,$$

where $\theta$ is the angle between the directions of the $\vec{B}$-field and the current $I$. The direction of this magnetic force is given by the right hand rule for the magnetic force.

Example 17.3 Magnetic field supports a clothesline You wonder if instead of supporting your clothesline with two poles you could replace it with a wire and then support it magnetically by running an electric current through it and using Earth’s $\vec{B}$-field, which near the surface has magnitude $5 \times 10^{-5}$ T and points north. Assume that your house is located on the island of Dominica near the equator where the $\vec{B}$-field produced by Earth is approximately parallel to the earth’s surface. The clothesline is 10 m long and with the hanging clothes has a 2.0 kg mass. What direction should you orient the clothesline and what electric current is needed to support it? Finally, decide if this seems like a promising way to support the clothesline—no poles needed!

Sketch and Translate Draw a sketch representing the situation and decide which way to orient the line and which way to run the current in it (Fig. 17.21a). The $\vec{B}$-field points northward (into the page) and you want the magnetic force exerted on the clothesline to point upward to balance the gravitational force exerted by the Earth on it. Using the right hand for the magnetic force, you point your fingers north in the direction of the magnetic field. You orient your hand so that your palm faces up (corresponding to an upward magnetic force exerted on the clothesline.) Your thumb now points toward the east, the direction the current needs to flow (see insert in Fig. 17.21a).

Figure 17.21(a)

Simplify and Diagram Assume the $\vec{B}$-field produced by Earth is uniform in the region of the clothesline. Draw a force diagram for the clothesline (Fig. 17.21b). Earth exerts a downward
gravitational force ($\mathbf{F}_{\text{E on C}}$) on the clothesline+clothes system. Earth’s magnetic field exerts an upward magnetic force on that system ($\mathbf{F}_{\text{B on C}}$). We choose the upward direction as positive.

Represent Mathematically If we want the system to remain at rest (zero acceleration), the $y$-components of the forces exerted on it must add to zero:

$$\sum F_y = F_{\text{E on C}_y} + F_{\text{B on C}_y} = (-F_{\text{E on C}}) + F_{\text{B on C}} = 0$$

or

$$-m_c g + ILB \sin \theta = 0$$

Solve and Evaluate

We can solve the above for the current $I$ and insert the known quantities to get:

$$I = \frac{m_c g}{LB \sin \theta} = \frac{(2.0 \text{ kg})(9.8 \text{ N/kg})}{(10.0 \text{ m})(5 \times 10^{-5} \text{ T})(\sin 90^\circ)} = 3.9 \times 10^4 \text{ A}$$

This is a serious problem. The wires in homes will only carry currents around 20 A before circuit breakers start to trigger for safety reasons. There’s just no way to safely run a current of 39,000 A through the electrical system of a residential home.

Try It Yourself: A 2.0-m long wire has a 10-A current through it. The wire is oriented south to north and located near the equator. Earth’s $\tilde{B}$-field has a $4.0 \times 10^{-5}$ T magnitude in the vicinity off the wire. What is the magnetic force exerted on the wire?

Answer: The wire is parallel to the $\tilde{B}$-field. Thus, $\sin \theta = \sin (0^\circ) = 0$ and the magnetic force exerted on the wire is zero.

There are significant differences between the properties of magnetic force and the electric and gravitational forces with which we are already familiar. What follows is a summary of these differences. While reading (a), (b) and (c) of this summary, answer the question “How do I know this?” Asking such question is called metacognition—thinking about our own thinking. Actively engaging in metacognition helps you remember things better and helps you use your knowledge to understand new phenomena.

Summary: The force caused by a magnetic field differs in a number of ways from the forces caused by gravitational and electric fields.
(a) The electric field exerts a force on objects with electric charge. The gravitational field exerts a force on objects with mass (mass can be thought of as a gravitational "charge"). However, every “magnetic object” that has ever been found has both a north pole and a south pole, but never just one pole in isolation.
(b) The gravitational and electric forces exerted on objects do not depend on the direction of motion of those objects, whereas the magnetic force exerted does. If the direction of the electric current is parallel or anti-parallel to the $\vec{B}$-field, no magnetic force is exerted on it.
(c) Finally, while the forces exerted by the gravitational and the electric fields are always in the direction of the $\vec{g}$ or $\vec{E}$ field (or opposite that direction in the case of a negatively charged object), the force exerted by the magnetic field on a current carrying wire is perpendicular to both the $\vec{B}$-field and the direction of the electric current (Fig. 17.22).

The direct current electric motor

The electric motor was one of the first applications of physicists’ understanding of magnetic forces. A motor is a device that transforms electric energy into mechanical energy, specifically kinetic energy (rotational or translational). It is used every day in devices such as refrigerators, kitchen fans, hair dryers, electric toothbrushes, drills, table saws, and many others. A simple motor consists of a rectangular current carrying coil placed between the poles of a large electromagnet (Fig. 17.23a). The coil is free to rotate around the axle. How can this device transform the energy stored in the battery into the rotational energy of the coil?

Let us start with the situation when the rectangular coil is oriented so that the plane of the coil is parallel to the $\vec{B}$-field. The currents through sides 1 and 3 of the coil are perpendicular to
the $\vec{B}$-field, which means the field exerts a force on them. The currents through sides 2 and 4 are parallel to the $\vec{B}$-field, which means the force exerted on them is zero.

According to the right hand rule for magnetic force, the magnetic field exerts an upward force on wire 1 of the coil and a downward force on wire 3 of the coil. (Fig. 17.23b) These forces each produce a torque around the axle that causes the coil to start rotating clockwise.

![Figure 17.23(b)](image)

As the coil turns, the orientations of the currents relative to the $\vec{B}$-field change, and as a result so do the magnetic forces exerted by the field on these sides. As the coil reaches an orientation with its surface perpendicular to the $\vec{B}$-field, the magnetic field exerts forces on each side of the coil that tend to stretch it but that do not have any turning ability (Fig. 17.23c). The coil turns past this orientation reaching the one shown in Fig 17.23d. Using the right hand rule for magnetic forces again, we find that the torques produced by the magnetic forces exerted on sides 1 and 3 cause the coil to accelerate in the counterclockwise direction, slowing down and reversing the direction of its rotation. This is a serious problem. If the current were reversed when the plane of the coil is perpendicular to the $\vec{B}$-field (Fig. 17.23c), the net torque would remain in the clockwise direction. Consequently, for the torque produced by the magnetic force exerted on the coil to remain clockwise the current through the coil must change direction each time the coil passes the vertical orientation.

![Figure 17.23(c)(d)](image)
This current reversal is made possible using a device known as a *commutator* (Fig. 17.24). A commutator consists of two semicircular rings that are attached to the rotating coil. Sliding contacts connect a battery to the commutator rings. The current direction is reversed in the middle of each rotation as the sliding contacts pass from one commutator ring to the next.

![Figure 17.24 Motor’s commutator ring](image)

**Torque on a current carrying loop**

We see that the torque produced by the magnetic forces exerted on a loop depends on the orientation of the loop relative to the \( \vec{B} \)-field. What is the magnitude of the torque that the \( \vec{B} \)-field causes on the loop? The magnitude of the torque that a force exerts on an object depends on how far from this axis the force is exerted. The torque that the magnetic force exerted on side 1 and on side 3 of the loop in Fig. 17.23b is each directly proportional to the distance \( \frac{D}{2} \) from the force to the axis of rotation—shown in Fig. 17.23a. As there are two equal magnitude torques causing the loop to turn in the same direction, the total torque is proportional to \( D \). In addition, the magnitude of the magnetic force on wire 1 and on wire 3 depends on the length \( L \) of that side of the loop (Fig. 17.23a). Therefore the total torque should be proportional to the product of \( D \) and \( L \), which is the area of the loop. In addition, if we have a coil with \( N \) loops, the torques exerted on each loop add.

Summarizing the above reasoning, we arrive at an expression for the magnitude of the torque that magnetic forces exert on a current carrying coil as

\[
\tau_{\text{on Coil}} = N B A I \sin \theta.
\]  

(17.4)

where \( N \) is the number of turns in the coil, \( I \) is the electric current in the coil, \( B \) is the magnitude of the \( \vec{B} \)-field, \( A \) is the area of the coil, and \( \theta \) is the angle between a vector perpendicular to the coil’s surface (the so-called normal vector) and the direction of the \( \vec{B} \)-field.

**Using a coil in a magnetic field to measure current—an ammeter**

We can use Eq. (17.4) as the basis for a method to measure the current through a wire. Take a coil of wire and place it between the poles of a horseshoe magnet whose \( \vec{B} \)-field is known. Remember that the \( \vec{B} \)-field in this region is approximately uniform. Orient the coil so that its normal vector is perpendicular to the \( \vec{B} \)-field. Connect this coil in series with the wire
you want to measure the current through. The $B$-field will exert forces on the current through the coil that produce a magnetic torque on the coil. If we attach springs to the turning sides of the coil, the springs will exert forces that produce a torque that opposes the torque produced by the magnetic forces. The coil then turns until the spring torques balance the magnetic torque (Fig. 17.25). The current through the coil can then be determined by Eq. (17.4):

$$I = \frac{F_{\text{on Coil}}}{NAB \sin \theta} = \frac{F_{\text{Springs on Coil}}}{NAB \sin \theta}.$$

The torque produced by the springs will be proportional to their stretch distance; so all quantities on the right hand side of the equation are measurable. This allows the measurement of the current $I$ through the coil. This method is what is used in analog ammeters used to measure the electric current in a circuit. Used in a different configuration, it can be turned into an analog voltmeter.

![Figure 17.25 Device for measuring electric current](image)

**Michael Faraday**

The motor about which you learned in this section is known as a *direct current electric motor*. It was invented by Michael Faraday in 1821. Remember from Chapter 15 that Faraday also brought the concept of a field to physics. Before him all magnetic effects were considered to happen without any mediator for the interactions. He imagined a current carrying wire as being surrounded by concentric circles (now called $B$-field lines) that affect another current carrying wire or a nearby magnet. It is interesting that although Faraday invented the concept of the magnetic field, the electric motor, the electric generator, and was the first to measure the magnitude of elementary charge, he did not use complex mathematics to describe his ideas. Faraday did not finish high school or college, as he had to work from age 12 in a bookbinder shop. However, he read the books he bound and attended public science lectures. His interest in science and his hard work eventually made him one of the most famous physicists of the 19th century. His strongest attribute was his ability to visualize, imagine, and represent physical processes in different ways, a very important ability for all people involved in science.
Review Question 17.3
A 0.7-m wire carrying a 0.1-A current is oriented parallel to the direction that a nearby compass points. Determine the magnetic force that Earth’s magnetic field exerts on the wire? Assume the magnitude of Earth’s $\vec{B}$-field at this location is about $10^{-5}$ T.

17.4 Magnetic force exerted on a single moving charged particle

We learned that a magnetic field exerts a force on current carrying wires. The current in a wire is the result of collective motion a huge number of electrically charged particles – electrons. Thus, it is reasonable that the magnetic field also exerts a force on each individual moving charged particle. It turns out that the understanding of this mechanism is crucial for our existence on Earth.

In everyday life charged particles zoom past and through us ever minute. They are called cosmic rays. They come at us from all directions in the Universe and bombard Earth and its inhabitants. Every minute about twenty of these fast moving charged particles pass through a person’s head (usually they are electrons, protons and other elementary particles produced by various astrophysical processes including those occurring in our Sun). These particles can cause genetic mutations, cancer, and other unpleasant effects. Fortunately, our bodies have multiple repair mechanisms and most of the damage gets repaired. But without the protection by Earth itself, there would be many more particles passing through our heads each minute (thousands). We will learn how Earth protects us from the damaging effects of these cosmic rays later in this section, but first we need to investigate the force that a magnetic field exerts on individual charged particles.

![Figure 17.26 Magnet distorts on old television set](image)

**Direction of the force that a magnetic field exerts on a moving charged particle**

Older non-LCD TVs had a beam of electrons that rapidly “painted” the glowing image seen on the screen. You can do a simple experiment with one of these old TVs, if you can find one. Bring a magnet close to the screen and move it around. You see the image being distorted by the magnet (Fig. 17.26). We do not advise that you perform this experiment as it might
permanently damage your TV. Instead, use a device called an oscilloscope (Fig.17.27a).
Electrons are emitted by a hot wire (called the cathode) at the back of the device. A potential
difference is maintained between the hot cathode and the hollow anode causing the electrons to
accelerate toward the screen. The screen is treated with a material known as a scintillator, which
glows green when hit with electrons. The location of this dot on the screen where electrons are
hitting the screen can be used to infer the path the electrons take while in flight inside the
oscilloscope.

If a magnetic field exerts a force on individual electrons in a way similar to the way it
exerts a force on the electric current in a wire, then we should be able to use the right hand rule
for magnetic force to predict the direction the electrons will be deflected. We need to be careful in
applying the rule since the rule was formulated in terms of electric current, which by convention
is the direction of in which positively charged particles move. The magnetic force exerted on
negative electrons moving towards the screen will be opposite the direction given by the right
hand rule for magnetic force. Orient the magnet so that the magnetic field it produces points into
the page (Fig. 17.27b). If the right hand rule for the magnetic force applies for this situation, it
predicts the electrons should deflect downwards. If we reverse the direction of the magnetic field,
the electrons should deflect upwards (Fig. 17.27c). When the experiment is performed, the
outcome is consistent with the predictions. This supports the idea that magnetic field exerts a
similar magnetic force on individual charged objects as it does on currents. Let’s construct a
quantitative relationship for the magnitude of the magnetic force exerted by a magnetic field on
an individual charged particle.

![Figure 17.27 An oscilloscope](image)

**Magnitude of the force that magnetic field exerts on a moving charged particle**

We know that magnetic field exerts a force on a current carrying wire of magnitude:

$$ F_{\text{on wire}} = ILB \sin \theta. $$

We will use this to develop an expression for the magnitude of the force that the magnetic field
exerts on a single charged object with charge \( q \) moving at speed \( v \). To do this, think of the
current $I$ in the wire as consisting a large number of positively charged particles each with charge $q$ (Fig. 17.28).

Imagine that between the two dashed lines there are $N$ moving charged particles. In a time interval $\Delta t$, all of them pass through the dashed line on the right. Thus, the electric current in this wire is:

$$I = \frac{Nq}{\Delta t}$$

The speed of the charged particles is $v = L/\Delta t$ since an object at the left dashed line takes a time interval $\Delta t$ to reach the right dashed line. Rearrange this for $\Delta t$ and substitute in the above to get:

$$I = \frac{Nqv}{L}$$

Inserting this into $F_{\text{on } W} = ILB \sin \theta$, we get:

$$F_{\text{on } W} = \left( \frac{Nqv}{L} \right) LB \sin \theta = N(qvB \sin \theta)$$

This is the magnitude of the force exerted by the magnetic field on all $N$ moving charged particles in the wire. The magnitude of the force exerted by the field on a single charged particle is then:

$$F_{\text{on } q} = |q|vB \sin \theta$$

**Magnetic force exerted by magnetic field on a charged particle** The magnitude of the magnetic force that a magnetic field exerts on a particle with electric charge $q$ moving at speed $v$ is:

$$F_{\text{on } q} = |q|vB \sin \theta$$  \hspace{1cm} (17.5)

where $\theta$ is the angle between the direction of the velocity of the particle and the direction of the $\vec{B}$-field. The direction of this force is determined by the right hand rule for the magnetic force. If the particle is negatively charged, then the force points in the opposite the direction.
Notice that the presence of $\sin \theta$ in the above formula indicates that the force exerted by the magnetic field on a single moving charged particle depends on the direction of the charged object’s velocity relative to the direction of the $\vec{B}$-field. If the velocity and the $\vec{B}$ field vector are parallel, the force is zero. The force is maximum when the object’s velocity and the $\vec{B}$-field are perpendicular.

**Right hand rule for the direction of the magnetic force exerted on a moving charged particle**

Hold your right hand flat with your thumb extended from your fingers and pointing in the direction of the object’s velocity. Point your fingers in the direction of the $\vec{B}$-field. The direction of the magnetic force exerted by the $\vec{B}$-field on the particle is in the direction your palm faces—perpendicular to both the velocity and the $\vec{B}$-field (Fig. 17.29). The force exerted by $\vec{B}$-field on a negatively charged particle is in the opposite direction.

**Tip!** Remember the magnetic field exerts a force on a moving charged particle only if there is a component of the particle’s velocity perpendicular to the direction of the $\vec{B}$-field.

![Figure 17.29 Right hand rule for magnetic force on charged particle](image)

**Quantitative Exercise 17.4 Particles in a magnetic field**

Each of the lettered dots shown in Fig. 17.30 represents a small object with electric charge of $+2.0 \times 10^{-6}$ C moving at the speed of $3.0 \times 10^7$ m/s in the directions shown. Determine the magnetic force (magnitude and direction) that a 0.10-T $\vec{B}$-field exerts on each object. The $\vec{B}$-field points in the positive $y$ direction. 

**Represent Mathematically**

First, use the right hand rule for the magnetic force to determine the directions of the magnetic force exerted on each object. (a) For object A, point the fingers of your right hand toward the top of the page in the direction of $\vec{B}$. Then, orient your hand so that your thumb points to the left in the direction of $\vec{v}$. With this hand orientation, your palm points into the paper, the direction of the magnetic force exerted on object A. (b) Object B moves in a direction opposite to $\vec{B}$ ($\theta = 180^0$); thus the magnetic force is zero. (c) For object C, your thumb needs to point out of the paper. Your palm then faces left, so the magnetic force exerted on...
object C points in the negative \( x \) direction. (d) For object D, point your fingers toward the top of the page and your thumb parallel to the paper pointing \( 37^\circ \) above straight right. Your palm faces out of the page in the direction of the magnetic force on D. Use \( F_{B \text{ on } q} = |q|vB \sin \theta \) to determine the magnitude of each force.

![Figure 17.30 Four charged particles moving in B Field](image)

**Solve and Evaluate** Use the Eq. (17.5) to determine the magnitude of each force:

\[
F_{B \text{ on } A} = (2.0 \times 10^{-6} \text{C})(3.0 \times 10^7 \text{ m/s})(0.10 \text{ T}) \sin (90^\circ) = 6.0 \text{ N}
\]
\[
F_{B \text{ on } B} = (2.0 \times 10^{-6} \text{C})(3.0 \times 10^7 \text{ m/s})(0.10 \text{ T}) \sin (180^\circ) = 0
\]
\[
F_{B \text{ on } C} = (2.0 \times 10^{-6} \text{C})(3.0 \times 10^7 \text{ m/s})(0.10 \text{ T}) \sin (90^\circ) = 6.0 \text{ N}
\]
\[
F_{B \text{ on } D} = (2.0 \times 10^{-6} \text{C})(3.0 \times 10^7 \text{ m/s})(0.10 \text{ T}) \sin (53^\circ) = 4.8 \text{ N}
\]

**Try It Yourself:** The equation below represents the solution to a problem. Devise a possible problem that is consistent with the mathematics:

\[
(1.6 \times 10^{-19} \text{C})v(0.50 \times 10^{-5} \text{T}) \sin (30^\circ) = 1.0 \times 10^{-18} \text{ N}
\]

**Answer:** A proton enters Earth’s magnetic field far above the Earth’s surface. The proton’s velocity makes a \( 30^\circ \) angle with the direction of the \( \vec{B} \)-field. The field exerts a \( 1.0 \times 10^{-18} \text{ N} \) force on the proton at the point of entry. What is the proton’s speed?

**Circular motion in a magnetic field**

Earlier we mentioned that Earth’s magnetic field protects us from the dangerous cosmic rays. Now we can explain how this happens. Imagine that a positively charged particle moves to the left across the top of your open textbook (position a in Fig. 17.31) when it enters a uniform magnetic field that points into the page. What is the path of the particle?

Using the right hand rule for the magnetic force, we find that the field exerts a magnetic force on that particle that points downward. As a result the direction of the particle’s velocity changes and now points slightly downward. The force exerted by the magnetic field on the charged particle always points perpendicular to its velocity. This deflects the particle further. Once it has made a quarter turn and reaches position b, it is now moving toward the bottom of the
page and the magnetic field exerts a force toward the right. This pattern persists with the magnetic force exerted on the particle always pointing towards the center of the particle’s circular path. Thus, in a uniform $B\text{-field}$ a charged particle that initially moves perpendicular to the $B\text{-field}$ field lines will move along a circular path in the plane perpendicular to the field. This helps us understand why Earth’s magnetic field serves as a shield against harmful cosmic rays causing them to deflect from their original trajectory toward Earth.

**Figure 17.31 Circular motion of charge in magnetic field**

**Example 17.5 Motion of protons in Earth’s magnetic field** What happens to a cosmic ray proton flying into Earth’s atmosphere above the equator at a speed of about $7\times10^7$ m/s? The average magnitude of Earth’s $B\text{-field}$ in this region is approximately $5\times10^{-5}$ T. The mass $m$ of a proton is approximately $10^{-27}$ kg.

**Sketch and Translate** Since we do not have information about the proton’s direction of motion relative to Earth’s $B\text{-field}$, we will consider two cases of motion: (i) perpendicular to Earth’s $B\text{-field}$ and (ii) at an arbitrary angle $\theta$ relative to it (Fig. 17.32a).

**Figure 17.32(a) Proton Earth’s magnetic field**

**Simplify and Diagram** Consider a short distance that the proton travels and assume that in this region the $B\text{-field}$ lines are parallel to Earth’s surface and have a constant magnitude of
5 \times 10^{-5} \text{T}. We neglect the gravitational force that Earth exerts on the proton since it is extremely small in comparison to the magnetic force exerted on the proton. Force diagrams have been drawn for the proton for the two cases (Fig. 17.32b).

Represent Mathematically

(i) **Proton moving perpendicular to the $\vec{B}$-field lines:** When the velocity of a charged particle is perpendicular to the $\vec{B}$-field, it will move in a circular path at constant speed. We can use the radial $r$ component form of Newton’s 2nd law to relate the magnetic force exerted on the proton to its resulting motion. The force exerted by the magnetic field points toward the center of the proton’s circular path (Fig. 17.32c):

$$a_r = \frac{v^2}{r} = \frac{1}{m} \sum F_r = \frac{1}{m} (F_{B \text{ on } P}) = \frac{1}{m} (|q|vB \sin \theta) = \frac{|q|vB \sin (90^\circ)}{m} = \frac{|q|vB}{m}$$

This equation can be used to determine the radius $r$ of the proton’s circular path and to determine the period $T$ of its motion noting that:

$$v = \frac{2\pi r}{T}.$$

Do not confuse the period $T$ with the unit for the magnetic field, the tesla T.

(ii) **Proton moving at an angle $\theta$ relative to the $\vec{B}$-field:** In this case the proton’s velocity has a component perpendicular to the $\vec{B}$-field (which will result in uniform circular motion as in (i)) and a component parallel to the $\vec{B}$-field (which causes zero magnetic force and hence constant velocity parallel to the $\vec{B}$-field). The combination of these two motions will be a helix (Fig. 17.32d). To determine the radius of the helix, use the component of the proton’s velocity tangent to the helix and perpendicular to the cylinder’s axis:

$$v_t = v \sin \theta$$

Now, determine the radius of the helix similar to how we determined the radius of the proton’s circular path in (i):

$$a_r = \frac{v_t^2}{r} = \left(\frac{v \sin \theta}{r}\right)^2 = \frac{1}{m} \sum F_r = \frac{1}{m} (F_{B \text{ on } P}) = \frac{1}{m} (|q|vB \sin \theta) = \frac{|q|vB \sin (\theta)}{m}.$$

The period $T$ of this motion can be determined using:
\[ v_t = v \sin \theta = \frac{2\pi r}{T}. \]

We can also determine the step \( d \) (labeled in Fig 17.32d) of the helix using the component of the proton’s velocity along the axis of the cylinder (parallel to the \( \vec{B} \)-field) and kinematics (the component of the proton’s velocity in this direction is constant):

\[ d = v_t T = (v \cos \theta) T \]

**Solve and Evaluate**

(i) **Proton moving perpendicular to the \( \vec{B} \)-field lines**: Above, we used Newton’s second law to develop an expression for the proton’s circular motion:

\[ \frac{v^2}{r} = \frac{|q| v B}{m} \]

Multiply both sides by the product of \( r \cdot m \) and then rearrange to get an expression for \( r \):

\[ \frac{v^2 m r}{r} = \frac{|q| v B m r}{m} \]

\[ \Rightarrow m v^2 = |q| v B r \]

\[ \Rightarrow r = \frac{m v}{|q| B} \approx \frac{(10^{-27} \text{kg}) (10^7 \text{m/s})}{1.6 \times 10^{-19} \text{C} (5 \times 10^{-5} \text{T})} \approx 10^3 \text{m}. \]

The period \( T \) of the proton’s motion is then:

\[ T = \frac{2\pi r}{v} \approx \frac{2\pi (10^3 \text{m})}{10^7 \text{m/s}} \approx 10^{-3} \text{s} \]

Check the units for the radius:

\[ \frac{\text{kg} \cdot (\text{m/s})}{\text{C} \cdot \text{T}} = \frac{\text{kg} \cdot (\text{m/s})}{\text{C} \cdot \left( \frac{\text{N}}{\text{A} \cdot \text{m}} \right)} = \frac{\text{kg} \cdot (\text{m/s})}{\text{A} \cdot \left( \frac{\text{m}}{\text{s}} \right)} = \frac{\text{m/s} \cdot \text{m}}{\text{C} \cdot \text{A} \cdot \text{s}} = \text{m} \]

We get the correct units.

(ii) **Proton moving at an angle \( \theta \) relative to the \( \vec{B} \)-field lines**: Since a specific angle is not mentioned, we will use \( 30^0 \). The Newton’s second law application to this process was:
Using the same procedure as in part (i), we can solve for the radius:

\[
\frac{(v \sin 30^\circ)^2}{r} = \frac{|q|vB \sin 30^\circ}{m}
\]

The period of the proton’s motion is:

\[
T = \frac{2\pi r}{v \sin \theta} \approx \frac{2\pi (10^3 \text{ m})}{(10^7 \text{ m/s}) \sin (30^\circ)} \approx 2 \times 10^{-3} \text{ s}
\]

The step of the helix is:

\[
d = (v \cos \theta)T \approx \left[(10^7 \text{ m/s}) \cos (30^\circ)\right](2 \times 10^{-3} \text{ s}) \approx 2 \times 10^4 \text{ m}
\]

Something interesting happens if we combine the equations for \( r \) and \( T \):

\[
T = \frac{2\pi r}{v \sin \theta} = \frac{2\pi}{v \sin \theta} \left(\frac{mv \sin \theta}{|q|B}\right) = \frac{2\pi m}{|q|B}
\]

The period of the proton’s motion does not depend on its speed or its direction of motion relative to the \( \vec{B} \)-field. It only depends on the magnitude of the \( \vec{B} \)-field, and the proton’s own properties (its charge \( q \) and mass \( m \)).

**Try It Yourself:** What happens to the motion of the proton that enters Earth’s magnetic field parallel to the field lines?

**Answer:** The motion of the proton will not be affected by magnetic field.

![Figure 17.33 Paths of particles entering Earth’s magnetic field from space](image)

**The auroras**

We learned in the previous example that charged particles moving in Earth’s magnetic field follow helical paths around the \( \vec{B} \)-field lines. If this model of the motion of electrically charged particles in a magnetic field is correct and Earth has a magnetic field similar to that of a
bar magnet, then charged particles entering the field should travel in a helical path that follows the \( \vec{B} \)-field lines and enters the Earth’s atmosphere near the poles (Fig. 17.33). When they enter the atmosphere moving at such high speeds, they collide with the molecules in the atmosphere. Such collisions lead to the molecules in the atmosphere becoming ionized. When the electrons recombine with the ionized molecules, the excess energy is radiated as light. We should see light in the upper atmosphere in the region of the magnetic poles. This is a testable prediction. This is in fact what people have been observing for hundreds of years (Fig. 17.34). These are the auroras mentioned at the beginning of the chapter. Often these charged particles moving toward the Earth come from solar flares that occur due to the interactions of the Sun’s hot ionized gas with its magnetic field. Therefore, when magnetic activity on the Sun is high, people on Earth see more intense auroras. On occasion, the auroras are visible in the locations far from the magnetic pole regions, sometimes even quite close to the equator.

\[ r = \frac{mv \sin \theta}{|q|\vec{B}}. \]

The largest this expression can be is when \( \theta = 90^\circ \), or:

\[ r = \frac{mv}{|q|\vec{B}}. \]

The mass of the proton is \( 1.67 \times 10^{-27} \text{ kg} \), and the \( \vec{B} \)-field has an average magnitude of \( 5 \times 10^{-5} \text{ T} \). This means the radius of this proton’s helical path is at most:

\[ r = \frac{mv}{|q|\vec{B}} = \frac{1.67 \times 10^{-27} \text{ kg}}{|1.6 \times 10^{-19} \text{ C}|} \left(\frac{10^8 \text{ m/s}}{5 \times 10^{-5} \text{ T}}\right) \approx 2 \times 10^4 \text{ m}. \]

This is 20 km or about 12 miles. Earth’s magnetic field extends several tens of thousands of miles above the surface so life is well protected. Even if the proton was traveling extremely close to
light speed, we are still protected. However, during the Apollo missions of the 1960’s and 70’s to the moon, humans were for the first time outside the protection of Earth’s magnetic field. Because of this, they were exposed to much higher levels of radiation than is present on Earth’s surface. Since magnetic solar activity was not very well understood then (and is understood only marginally better today) these missions were quite risky.

During intense magnetic solar activity, the exposure of astronauts in low Earth orbit (on the space shuttle, or on the International Space Station) can be dangerous as well, even though they are within Earth’s magnetic field. Early warning systems are in place so that astronauts have time to move to more sheltered areas reducing the danger. Research is ongoing to improve these safety measures as we consider plans for manned missions to Mars and permanent settlements on the Moon.

**Review Question 17.4**

If the magnetic force is always perpendicular to the velocity of a charged particle, does it do any work on it? Explain your answer.

### 17.5 Magnetic fields produced by electric currents

In the previous sections we learned how to calculate the force that a magnetic field exerts on current carrying wires and on moving charged objects, and the torque that magnetic forces exert on current loops. In all situations that we studied so far, we investigated the effects of the magnetic field for which the magnitude of the $\vec{B}$-field and its direction were known or could be inferred from the field’s effects on other objects (current carrying straight wires or wire loops).

To build electromagnets that will produce desirable $\vec{B}$ fields we need to know how to predict the magnitude and direction of a $\vec{B}$-field produced by a particular current configuration. To determine qualitatively the general shape of the $\vec{B}$-field produced by currents we can use the right hand rule for the $\vec{B}$-field. In this section we learn how to determine quantitatively the magnitude of a $\vec{B}$-field produced by simple electric currents.

**The $\vec{B}$-field caused by an electric current in a long straight wire**

The experiments with magnets and an electric current in a long straight wire in Table 17.1 indicated that the current produces $\vec{B}$-field lines that circle around the wire. This is supported by an experiment where we pass a long current carrying wire perpendicular through a sheet of paper with iron filings on it (Fig. 17.35a) or surrounded by magnets (Fig. 17.35b). The magnetic field lines form closed circles around the wire (Fig. 17.35c). This is consistent with the right hand rule for magnetic field; the lines in Fig. 17.35c circle the wire in the correct way.
What is the magnitude of the \( B \)-field at various points in the region surrounding the wire? In Section 17.3 we learned that the \( B \)-field at a specific location will produce a torque on a current carrying coil \( F_{\text{on } C} = NBAI \sin \theta \) (in this case the coil is the detector of the field). We can use that torque as a measure of the magnitude of the \( B \)-field at that location, that is, if we know the area of the coil and the number of loops. If we always measure the maximum torque exerted on the coil, we do not need to worry about its orientation relative to the \( B \)-field. We can use this method to investigate the magnitude of the \( B \)-field in the region around a long straight current. See Observation Experiment Table 17.4.

![Magnetic field due to long current carrying wire](image)

**Observational Experiment Table 17.4 \( B \)-field around a straight current carrying wire**

<table>
<thead>
<tr>
<th>Observational experiment</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Take a small light coil connected to a battery that can rotate (detector). Place it at different locations ( r ) from a straight current carrying wire (source) Use the maximum torque exerted on the detector by source’s magnetic field to determine the magnitude of the ( B )-field surrounding a along straight wire with the current ( I ) and at different distances ( r ) from the wire The current ( I ) in the source wire is varied as well.</td>
<td>Look for a pattern in how ( B ) depends first on ( I ) and then on ( r ).</td>
</tr>
<tr>
<td>( I ) (in the wire)</td>
<td>( r ) (distance between wire and detector)</td>
</tr>
<tr>
<td>( I )</td>
<td>( r )</td>
</tr>
<tr>
<td>( 2I )</td>
<td>( r )</td>
</tr>
<tr>
<td>( 3I )</td>
<td>( r )</td>
</tr>
<tr>
<td>( I )</td>
<td>( 2r )</td>
</tr>
<tr>
<td>( I )</td>
<td>( 3r )</td>
</tr>
<tr>
<td>( I )</td>
<td>( r/2 )</td>
</tr>
</tbody>
</table>

**Pattern**

The magnitude of the \( B \)-field created by a long straight current carrying wire is directly proportional to the magnitude of the current \( I \) and inversely proportional to the distance \( r \) between the wire and the location where the field is measured.

We can express mathematically the pattern identified in Table 17.4 as follows:

\[
B_{\text{straight wire}} \propto \frac{I}{r}
\]
Limiting case analysis can be used to see if this result makes sense. Intuitively we know that a wire with zero current does not produce a magnetic field. This agrees with the above equation:

$$B_{\text{straight wire}} \propto \frac{I}{r} \bigg|_{r=0} = 0.$$  

We also intuitively know that if we are infinitely far from the current-carrying wire, the $\vec{B}$-field should be zero. This also agrees with the above equation:

$$B_{\text{straight wire}} \propto \frac{I}{r} \bigg|_{r=\infty} = 0.$$  

Thus extreme case analysis is consistent with the expression for the magnitude of the $\vec{B}$-field.

Since we can experimentally measure all three of the quantities appearing in this relationship, it’s possible to determine the constant of proportionality that will turn it into an equation. Traditionally this constant is written as $\frac{\mu_0}{2\pi}$ where $\mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A}$. We now have an expression for the magnitude of the $\vec{B}$-field at a perpendicular distance $r$ from a long straight current carrying wire:

$$B_{\text{straight wire}} = \frac{\mu_0 I}{2\pi r} \quad (17.6)$$

Note that the farther you move from the current carrying wire (larger $r$), the smaller the magnitude of the $\vec{B}$ field; the greater the current (larger $I$), the larger the magnitude of the $\vec{B}$ field.

**Tip!** The above expression represents a cause effect relationship – the magnitude of the $\vec{B}$-field at a particular location depends both on the magnitude of the current in the wire creating the field (source) and the location where we measure the field. Compare it to the Eq. (17.2) $B = \frac{F_{\text{on W}}}{IL \sin \theta}$. The latter equation is the operational definition; the value of $B$ found using it does not depend on the current in the wire creating the field or its length (the $I$ and $L$ in the equation are both properties of the detector wire and not of the wire creating the field).

**Magnetic permeability**

With this value of $\mu_0$, the $\vec{B}$-field magnitude produced 0.10 m (10 cm) from a long straight wire with a current of 10.0 A is $2.0 \times 10^{-4} \text{T}$. The constant $\mu_0$ is known as the *vacuum permeability*; Eq. (17.6) assumes the region where we are measuring $\vec{B}$ field is in a vacuum (as opposed to some medium such as water, oil, etc.). The magnetic permeability of air is approximately equal to $\mu_0$ as well. However, for example, the magnetic permeability of iron is
about 1000 times greater than \( \mu_0 \). In other words, the \( \vec{B} \)-field within iron is 1000 times greater than if only air were there. In general, Eq. (17.6) is written as:

\[
B_{\text{straight wire}} = \frac{\mu I}{2\pi r}
\]

where \( \mu \) is the magnetic permeability of the substance present at the location of interest.

The magnetic permeability of materials is a complex subject. Even for a particular type of material, such as iron, its magnetic permeability depends on the history of the \( \vec{B} \)-field within the material. Meaning, the material has a ‘memory’ of the \( \vec{B} \)-field that was present at past times, and the current value of its permeability depends on that history.

**Magnetic fields produced by a circular electric current**

Equation (17.6) helps us find the magnitude of the \( \vec{B} \)-field at different locations near a straight current carrying wire. The \( \vec{B} \)-field lines for such a source are shown in Fig. 17.36a. Another common configuration is a circular loop or coil of current carrying wire (Figure 17.36b), such as found in an electric heater and in the circular coil of a motor. The \( \vec{B} \)-field due to current in a solenoid is shown in Fig. 17.36c.

![Figure 17.36](image)

**Figure 17.36** Expressions for magnetic field produced by (a) straight wire (b) loop or coil (c) solenoid

**\( \vec{B} \)-field due to electron motion in an atom**

Interestingly an electron in an early 20\(^{th}\) century model of the hydrogen atom can also be seen as a circular current (similar to Fig. 17.36b). In this model the electron is thought to move rapidly in a tiny circular path around the nucleus of the atom. The electron motion is like a circular electric current that produces a \( \vec{B} \)-field --see the next example. If these electron fields are significant, they can potentially explain magnetic properties of materials. Thus, it is important to be able to estimate their magnitude. We will use the equation for the magnitude of the \( \vec{B} \)-field at the center of the loop of radius \( r \) without derivation:

\[
B = \frac{\mu_0 I}{2r}
\]
Let us again examine it using extreme case analysis – the magnitude of the $B$-field at the center of the loop should be zero when there is no current and it should be zero when the loop is infinitely large – both of these cases are predicted by the equation.

**Example 17.6 Magnetic field produced by electron in a hydrogen atom** In the above mentioned early 20th century model of the hydrogen atom the electron was thought to move in a circle of radius $0.53 \times 10^{-10}$ m orbiting once around the nucleus every $1.5 \times 10^{-16}$ s. Determine the magnitude of the $B$-field produced by the electron at the center of its circular orbit.

**Sketch and Translate** A sketch of the moving electron is shown in Fig. 17.37 along with the known information.

![Figure 17.37 Electron in circular orbit produces magnetic field](image)

**Simplify and Diagram** The electron’s motion corresponds to a clockwise current (opposite the direction of travel of the negatively charged electrons). This is similar to a single loop current. The direction of the $B$-field at the center can be determined by the right hand rule for the $B$-field.

**Represent Mathematically** The magnitude of the $B$-field at the center of a circular current $I$ is:

$$B = \frac{\mu_0 I}{2r}.$$  

To determine $B$ we have to determine the electric current due to the electron’s motion. Electric current is $I = \frac{q}{\Delta t}$ where $q$ is the magnitude of the total electric charge that passes a cross section of a wire in time $\Delta t$. The cross section in this case is a single point along the electron’s orbit. The single electron passes that point once every $1.5 \times 10^{-16}$ s.

**Solve and Evaluate** Using these ideas, we can calculate the magnitude of the current:

$$I = \frac{q}{\Delta t} = \frac{1.6 \times 10^{-19} \text{C}}{1.5 \times 10^{-16} \text{s}} = 1.1 \times 10^{-3} \text{A}$$

The magnitude of the $B$-field at the center of the loop is then:
This is a huge $\vec{B}$-field, especially compared to the $10^{-5}$ T $\vec{B}$-field of Earth. We will learn in Chapter 27 that the proton nucleus of the hydrogen atom itself acts like a tiny magnet that interacts with the $\vec{B}$-field produced by the electron. This has a small but measurable effect on the light emitted by hydrogen gas.

**Try It Yourself:** We found in the example above that a moving electron in an atom produces a very strong magnetic field. Suggest an explanation for why all materials aren’t strong magnets.

**Answer:** One reason can be that the electron orbits of different atoms are oriented randomly. So the $\vec{B}$-field contributions from each add to zero. Another reason can be that most atoms have more than one electron and the $\vec{B}$-field contributions produced by each of these electrons add to zero so that the $\vec{B}$-field produced by each individual atom is zero.

**Review Question 17.5**

The current in a wire on the left goes into the page and the current in a wire on the right goes out of the page. Determine the direction of the $\vec{B}$-field at the wire on the right produced by the current in the wire on the left. Then determine the direction of the force that this $\vec{B}$-field exerts on the current in the wire on the right.

**17.6 Skills for magnetism problems**

Problems involving magnetic interactions are often of two main types: 1) determine the magnetic force exerted on a current or individual moving charged object by a magnetic field whose origin you do not know, and 2) determine the $\vec{B}$-field produced by a source such as an electric current. In this section we will develop skills needed to solve such problems. The general procedure is described on the left side of the following example and illustrated in the solution of the example.

**Example 17.7 Magnetic force problem** A metal wire of mass 5.0 g and length of 0.20 m is supported at its ends by two very light conducting threads. The wire hangs in a 49-mT magnetic field, which points perpendicular to the wire and out of the page. The maximum tension force each thread can exert on the wire before breaking is 39 mN. What minimum current through the wire causes the threads to break?

**Sketch and Translate**

- Sketch the process described in the problem. Show the direction of the $\vec{B}$-field and the wire is the system of interest. What downward
direction of the electric current (or the velocity of a charged particle) if known.

- Decide whether the problem asks to find a \( B \)-field produced by an electric current, or to find a magnetic force exerted by the field on a moving charged particle or on a wire with electric current.

<table>
<thead>
<tr>
<th>Simplify and Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Decide whether the ( B )-field can be considered uniform in the region of interest.</td>
</tr>
<tr>
<td>- Draw a force diagram for the system if necessary. Use the right hand rule for the magnetic force to find an unknown force, current, velocity, or field direction if needed.</td>
</tr>
<tr>
<td>- Use the right hand rule for the ( B )-field if needed.</td>
</tr>
</tbody>
</table>

- Nothing specific is mentioned about the \( B \)-field, so we consider it uniform in the vicinity of the wire. |
- Construct a force diagram for the wire (U 17.9). The wire interacts with the threads (\( \vec{F}_{\text{on W}} \)), Earth (\( \vec{F}_{\text{E on W}} \)), and the magnetic field (\( \vec{F}_{\text{B on W}} \)). With the \( B \)-field pointing out of the page, the magnetic force points down in the desired direction if the wire current flows toward the right. We choose the y-axis as pointing down.

<table>
<thead>
<tr>
<th>Represent Mathematically</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Describe the situation mathematically using the expressions for magnetic force exerted on a current or charged particle and the expressions for the ( B )-field produced by currents.</td>
</tr>
<tr>
<td>- If necessary, use Newton’s second law in component form and kinematics.</td>
</tr>
</tbody>
</table>

The wire is in equilibrium (\( \Sigma F_y = 0 \)). In component form
\[
F_{E_{\text{on W}y}} + 2F_{T_{\text{on W}y}} + F_{B_{\text{on W}y}} = 0
\]
\[
\Rightarrow mg + (-2F_{\text{on W}}) + ILB = 0
\]
Move the parts that do not contain \( I \) to the right side of the equation: \( ILB = 2F_{\text{on W}} - mg \). Then divide both sides by \( LB \):
\[
I = \frac{2F_{\text{on W}} - mg}{LB}
\]

Inserting the appropriate values:
\[
I = \frac{2(39 \times 10^2 \text{ N}) - (5.0 \times 10^{-3} \text{ kg})(9.8 \text{ N/kg})}{(0.2 \text{ m})(49 \times 10^{-3} \text{ T})} = 3.0 \text{ A}
\]
This is a large current for a thin wire but is not completely unreasonable.

Looking at a limiting case: if the wire mass is such that \( mg = 2F_{\text{on W}} \), then the required current is zero—the gravitational force that Earth exerts would be enough to break the wire.

**Try It Yourself:** An electron enters a \( 1.0 \times 10^2 \text{ T} \) \( B \)-field perpendicular to the \( B \)-field lines. It then completes a \( 1.0 \times 10^3 \text{ m} \) semicircular path and leaves the \( B \)-field region traveling in the opposite direction. What is the speed of the electron (its mass is \( 9.11 \times 10^{-31} \text{ kg} \))? 

\[
\text{Answer: } v = \frac{qrB}{m} = \frac{(1.6 \times 10^{-19} \text{ C})(1.0 \times 10^{-3} \text{ m})(1.0 \times 10^{-2} \text{ T})}{(9.11 \times 10^{-31} \text{ kg})} = 1.7 \times 10^6 \text{ m/s}
\]
Example 17.8 Determine the \( B \)-field Determine the \( B \)-field 5.0 cm from a long straight wire connected in series to a 5.0 \( \Omega \) resistor and a 9.0 V battery.

Sketch and Translate Make a sketch of the situation (see Fig. 17.38a). The sketch includes the electric circuit connected to the wire. The current in the circuit is clockwise.

\[ \text{Figure 17.38(a) Magnetic field due to current} \]

Simplify and Diagram Assume that the only contribution to the \( B \)-field at the point of interest comes from the long wire. The other three wires contribute as well, but since they are much further away by comparison, we will neglect their contributions. Assume also that the other connecting wires and the battery have zero resistance. Using the right hand rule for the \( B \)-field, we find that below the wire the field points into the paper and above the wire it points out of the paper (Fig. 17.38b.)

\[ \text{Figure 17.38(b)} \]

Represent Mathematically The magnitude of the \( B \)-field produced by a long straight current is given by Eq. (17.6):

\[ B_{\text{straight wire}} = \frac{\mu_0 I}{2\pi r} \]

Using the Ohm’s law and the loop rule we can determine the current through the wire (the current is in the positive direction):

\[ \Delta V_{\text{batt}} + \Delta V_{\text{w}} = 0 \]
\[ \Rightarrow \varepsilon + (-IR) = 0 \]

Solve and Evaluate Combining these two equations gives:

\[ B_{\text{straight wire}} = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{2\pi} \frac{\varepsilon}{r} = \frac{\mu_0}{2\pi} \frac{\varepsilon}{rR} = \frac{4\pi \times 10^{-7}}{2\pi} \frac{9.0 \text{ V}}{0.50 \text{ m}(5.0 \Omega)} = 3.6 \times 10^{-6} \text{T}. \]
A limiting case analysis can be done for the case of a discharged battery. If $\varepsilon = 0$, then there should be no current and the $\vec{B}$-field should be zero, consistent with the equation. Notice that the $\vec{B}$-field due to the current is an order of magnitude smaller than the Earth’s magnetic field. This will make it somewhat challenging to measure the wire’s magnetic field.

*Try It Yourself:* Estimate the magnitude of the $\vec{B}$-field produced by an electron beam in an older non-LCD TV at a point 1.0 m to the side of the beam. Assume that $10^{10}$ electrons hit the screen every second and that they move at a speed of $10^7$ m/s.

*Answer:* $3.2 \times 10^{-16}$ T.

Earlier in the chapter we learned that the unit of electric current, the ampere, is related to the magnetic force that the current in one wire exerts on the current in a second wire. Now we can analyze this situation quantitatively.

**Example 17.9 How the ampere is defined** Determine the magnetic force that one 1.0 m long wire exerts on a second 1.0 m long wire. The two wires are parallel, separated by 1.0 m, and have 1.0 A currents in the same direction.

*Sketch and Translate* This example emphasizes the importance of identifying the system of interest. Choose wire 2 in Fig. 17.39 as the system of interest. The current through wire 1 produces a magnetic field (called $\vec{B}_1$) that exerts a magnetic force on the system (wire 2). We could just as easily have chosen wire 1 as the system. In that case wire 2 would produce a magnetic field (called $\vec{B}_2$) that exerts a magnetic force on wire 1. We use the first choice.

![Figure 17.39 Experiment defining the ampere unit](image)

*Simplify and Diagram* In Conceptual Exercise 17.2 we learned how to determine the direction of the force that one wire exerts on the other. Before working though this example, review the reasoning used in that exercise to conclude that those two wires exert attractive forces on each other.
Represent Mathematically Use Eq. (17.6) to determine the magnitude of the $\vec{B}$-field produced by current $I_1$ in the vicinity of wire 2:

$$B_i = \frac{\mu_0 I_1}{2\pi r}$$

The magnitude of the magnetic force exerted by the wire 1 magnetic field on wire 2 is determined using Eq. (17.2):

$$F_{\vec{B}_1 \text{ on } W_2} = I_2 L B_i \sin \theta$$

Solve and Evaluate Combining these two equations we get (note that the current $I_2$ is perpendicular to the direction of $\vec{B}_1$, so $\sin 90^0 = 1$):

$$F_{\vec{B}_1 \text{ on } W_2} = I_2 L \left(\frac{\mu_0 I_1}{2\pi r}\right) \sin \theta = \frac{\mu_0 I_1 L}{2\pi r} L \sin \theta = \left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(1.0 \text{ A})(1.0 \text{ A})(1.0 \text{ m})(\sin 90^0) = 1.0 \times 10^{-7} \text{ N}.$$ 

This is just the result used to define the ampere current unit. It is the force that two 1.0 m long parallel wires carrying 1.0 A currents exert on each other when they are separated by 1.0 m.

Try It Yourself: Suppose that the electric power cord for an appliance has two 2.0 m long wires separated by 1.0 mm. On wire has a 1.0 A current flowing into the appliance and the other has 1.0 A current flowing out of the appliance. Determine the magnitude of the magnetic force that one wire exerts on the other, and whether the wires attract, repel, or do something else.

Answer: $1.0 \times 10^{-4} \text{ N}$. They repel.

Intensity Modulated Radiation Therapy (I.M.R.T.)

Intensity Modulated Radiation Therapy (IMRT) is a powerful cancer-fighting technology. A computerized tomography (CT) scans a cancerous area in the body and produces a three dimensional image of the tumor. This information is provided to a physics team and to the IMRT machine (Fig. 17.40), which has a linear accelerator for electrons. After getting the electrons to the desired kinetic energy, the electrons are bent by a magnetic field in a $90^0$ angle and hit a tungsten alloy target that produces x-rays (we will learn the mechanism behind this production in chapter 26). The x-ray beam is then shaped by 120 movable metal leaves that shape the beam to match the shape of the tumor. The accelerator and radiation device rotates around the patient with the leaves continually changing to match the 3-D shape of the tumor, as seen from that orientation. The tumor receives a high dose of x-rays but surrounding tissue receives little. IMRT works well for prostate cancer as well as for tumors of the head and neck, and other organs that lie near important body parts such as the eyes, optic nerves, brain, brain stem, salivary glands, bladder, rectum, small bowel, kidneys, liver, lung and spinal cord. In the next example, we consider the magnetic field that bends the electron beam.
Example 17.10 Magnetic field that bends electrons in IMRT device

Electrons in an IMRT are accelerated to high energy (relativistic calculations are needed for accurate analysis) when they reach a bending magnet that causes them to make a $90^\circ$ turn. Estimate the magnitude of the magnetic field needed for the machine. For the estimate, we assume the electrons are moving at $2 \times 10^8$ m/s, mass of the electrons is $9 \times 10^{-31}$ kg, and the radius of the turn is $5 \, \text{cm} = 0.05 \, \text{m}$.

Estimate the magnitude of the $\vec{B}$-field.

*Sketch and Translate* The process is sketched in Fig. 17.41a.

![Figure 17.41(a) Magnet bends electron beam](image)
Simplify and Diagram This is an estimate and we assume that the standard classic physics principles apply without modification. A force diagram for the electron part way around the 90° curved are is shown in Fig. 17.41b. The gravitational force that Earth exerts on the electron is assumed to be small compared to the force the magnetic field exerts on the electron. The latter force points perpendicular to the electron’s velocity at each point along its path – in the radial direction.

![Figure 17.41(b)](image)

Represent Mathematically Apply Newton’s second law to the radial direction for circular motion. In the radial direction the magnitude of acceleration is determined by the magnitude of sum of the forces exerted on the object in the radial direction and the mass of the object:

\[ m_{ei}a_r = m_{ei} \frac{v^2}{r} = \Sigma F_{\text{radial}} \]

The only force in the radial direction is the magnetic force. Insert the expression for the magnitude of the magnetic force on the electron: 

\[ F_{\text{B on El}} = evB \]

in the equation above. There \( e \) is the magnitude of the electron electric charge. We get:

\[ evB = m_{ei} \frac{v^2}{r} \]

Solve and Evaluate Divide each side of the equation by \( v \) and then rearrange the equation to determine the magnitude of the magnetic field \( B \):

\[ B = \frac{m_{ei}v}{evr} = \frac{(9 \times 10^{-31} \text{ kg})(2 \times 10^8 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C})(0.05 \text{ m})} = 0.024 \text{ T} \approx 0.02 \text{ T} . \]

This is an easily attained magnetic field. Thus, there should be no difficulty bending the electron beam in a 90° turn.

Try It Yourself: Suppose it was a beam of protons. How would this affect the required magnetic field?

Answer: Protons are about 2000 times more massive than electrons. Thus, the magnetic field would need to be about 2000 times greater (see the last equation).

Review Question 17.6

What is the difference between the right hand rule for the magnetic force and the right hand rule for the \( \vec{B} \)-field?
17.7 Putting it all together

So far in this chapter, we only considered applications involving the magnetic field, magnetic forces, and the torques they can produce. However, other practical applications involve a combination of magnetic and electric phenomena, including the magnetohydrodynamic generation of electric power and the measurement of the speed at which blood flows. Both of these applications involve electrically charged objects moving in regions that have both a non-zero $\vec{B}$-field and a non-zero $\vec{E}$-field. Additionally, the fields are perpendicular to each other. Let’s analyze this general situation first.

**Ions moving through perpendicular $\vec{B}$-field and $\vec{E}$-field**

Imagine that you have a $\vec{B}$-field in the region between two conducting plates that points into the page. At the beginning the plates are not charged. There is a mechanism that injects electrically charged particles between the plates perpendicular to the direction of the $\vec{B}$-field. Both positively and negatively charged particles are initially moving downward (see Fig. 17.42a). When the positively charged particles enter the $\vec{B}$-field region, according to the right hand rule for the magnetic force the field exerts a magnetic force on the positively charged particles toward the right (Fig. 17.42b). The trajectory of these particles becomes circular and they collide with and are collected by the plate on the right. The same thing happens to the negatively charged particles, except that the magnetic force exerted by the $\vec{B}$-field on them points to the left; so they collect on the left plate (Fig. 17.42c.) This device separates positively and negatively charged particles.

![Figure 17.42(a)(b)(c) Charge separation of moving charged particles](image)

As the plates become oppositely charged, they produce an $\vec{E}$-field, which points to the left in the region between the plates (Fig. 17.42d). This field now exerts an electric force on the positively charged particles that points to the left, and a force on the negatively charged particles that points to the right. In both cases, this electric force points in the opposite direction to the
magnetic force. The increasing $E$-field due to the accumulation of electric charge on the plates quickly becomes large enough so that it opposes further accumulation of charged objects on the walls. When the electric and magnetic forces exerted on the moving charged particles balance, the particles travel with constant velocity downward despite the presence of both a $B$-field and an $E$-field (Fig. 17.42c). Mathematically, for a positively charged particle:

$$F_{B_{on q}} + F_{E_{on q}} = 0$$

$$\Rightarrow (+F_{B_{on q}}) + (-F_{E_{on q}}) = 0$$

$$\Rightarrow qvB \sin(90^\circ) - q|E| = 0$$

$$\Rightarrow vB = |E| \quad \text{or} \quad E = vB$$

In Ch. 15, we learned that the $E$-field can be expressed in terms of the magnitude of the potential difference $\Delta V$ across the plates ($E = \Delta V/d$), where $d$ is the plate separation. Thus, the potential difference across the plates will be:

$$\Delta V = Ed = vBd$$

There have been multiple practical applications for such a process.

![Magnetohydrodynamic generator](image)

**Figure 17.43 Magnetohydrodynamic generator**

**Magnetohydrodynamic generator**

Magnetohydrodynamic (MHD) generation is one application the showed promise. A generator is a device that transforms some form of mechanical energy into electric potential energy. A magnetohydrodynamic generator transforms the random kinetic energy of high temperature charged particles into electric potential energy. This is how it happens. In a MHD generator, gaseous fuel such as pulverized coal enters a combustion chamber and burns at high temperature (1000-2000 °C) and pressure. If alkali metals, such as potassium, are injected into the burning gas, free electrons and positively charged ions are formed. The electrons and ions pass from the combustion chamber through a nozzle and into a magnetic field region such as pictured in Fig. 17.43. The negative electrons and positive ions accumulate on opposite plates at the sides of the leftward moving gas, producing a potential difference between the plates. The plates, like the terminals of a battery, serve as a supplemental power source for a load (depicted
Magnetic flow meters

This magnetic flow method with a $\vec{B}$-field perpendicular to flowing charged particles was used for several decades to measure blood flow speed through an artery during open-heart surgery. It has now been replaced by a Doppler method described in Chapter 20. However, magnetic flow meters are used widely in industrial applications in place of mechanical turbine or propeller flow meters. The magnetic flow meter has no rotor to stop turning or bearings to wear out and is virtually maintenance-free, especially in applications where debris or sand would foul the bearings. These meters are used for applications such as measuring municipal and industrial water and wastewater flow, cooling tower flow for power plants, and monitoring water usage from wells.

The magnetic flow meter works only for fluids with moving ions, which includes most fluids. A magnetic field is oriented perpendicular to the vessel. Oppositely charged ions in the fluid are forced by the magnetic field to opposite walls of the vessel, thus producing a potential difference $\Delta V$ across the walls of the vessel. By measuring $\Delta V$, the magnitude of the $\vec{B}$-field, and the diameter $d$ of the vessel, we can substitute into $v = \Delta V / Bd$ to determine the fluid's speed $v$. The fluid's volume flow rate can then be determined (see Chapter 11):

$$Q = Av = \left(\pi r^2\right) v = \frac{\pi d^2}{4} v$$

Commercial flow meters can measure flow rates from 12-3600 gallons/minute.

**Quantitative Exercise 17.11 Blood speed meter** Imagine that you are an inventor, the first to come up with the idea for a magnetic blood speed meter. You wonder if the general magnetic flow meter idea described above that led to the equation $\Delta V = vBd$ is feasible for measuring blood speed. Estimate the potential difference you would expect to measure as blood in an artery passes through a 0.10 T $\vec{B}$-field. The heart pumps about 80 cm$^3$ of blood each second (the approximate volume for each heart beat) and the diameter of an artery is about 1.0 cm.

Represent Mathematically Use $\Delta V = vBd$ to estimate the potential difference $\Delta V$ that you can expect across opposite walls of the artery; then decide if this is large enough to measure. We need first to estimate the speed of the blood using:

$$Q = Av = \left(\pi r^2\right) v = \frac{\pi d^2}{4} v,$$
where $Q$ is the volume blood flow rate.

**Solve and Evaluate** The speed of the blood in this artery is:

$$v = \frac{Q}{\pi d^2/4} = \frac{4Q}{\pi d^2} = \frac{4(80\text{ cm}^3/\text{s})}{\pi (1.0 \text{ cm})^2} = \frac{1 \text{ m}}{100 \text{ cm}}^3 = 1 \text{ m/s}$$

This speed will result in a potential difference across the walls of the artery of:

$$\Delta V = vBd = (1 \text{ m/s})(0.10 \text{ T})(1.0 \text{ cm})\left(\frac{1 \text{ m}}{100 \text{ cm}}\right) = 1 \times 10^{-3} \text{ V}$$

The potential difference is easily measured, so this is in fact a practical way to measure blood flow in major arteries.

**Try It Yourself:** The flow rate of water from a 5.0-cm diameter water pipe used to irrigate a field is 200 gallons/minute (1 gallon = $3.79 \times 10^{-3} \text{ m}^3$). The water passes through a 0.1 T magnetic field. Determine the average speed of the water in the pipe and the potential difference across the flow meter in that pipe.

*Answer:* 6.4 m/s and 0.016 V.

**Sunspots**

Aristotle, a Greek philosopher, said that the Sun and the heavens were ideal, an embodiment of unblemished perfection. Greek philosophers from the fourth century B.C. made references to spots on the Sun but could not explain what they were seeing. Astronomers in China in 28 B.C. recorded what looked like small, changing dark patches on the surface of the Sun. In 1611 using their newly invented telescopes, Johann Goldsmid of Holland, Galileo Galilei of Italy, Christopher Scheiner of Germany, and Thomas Herriot of England, conclusively observed and recorded what have become known as sunspots (see Fig. 17.44).

These scientists could not agree on what they were seeing. Some, like Galileo, believed that sunspots were part of the Sun itself, perhaps dark spots on its surface, or some sort of cloudlike formation. But other scientists, especially Scheiner, who was a Jesuit priest, believed the Catholic Church's doctrine that the heavens were the divine perfection of God, an extension of Aristotle's views. To admit that the Sun had spots or blemishes that moved and changed undermined that perfection. So Scheiner argued that the spots he and Galileo were seeing must be other planets or moons orbiting the Sun.
Galileo observed the spots moving and returning to the same location after about one month. He correctly explained this visible motion as the monthly rotation of the Sun on its axis. This explanation implied that the Sun was a regular physical object, not a perfect heavenly fire as Scheiner and many others had thought. The observations of the sunspots contributed to the building of the heliocentric (Sun-centered) model of the Solar system, and simultaneously got Galileo into trouble with Catholic Church.

What are sunspots? They look dark because they are cooler (4500 K) than other regions on the Sun’s surface (5600 K). In addition, observations reveal that they always come in pairs (see those pairs in Fig. 17.44). Finally, measurements of the Sun’s $\vec{B}$-field in the dark regions and in the hotter regions of the Sun’s surface indicate that the $\vec{B}$-field is much stronger in regions of sunspots than in other parts and that the $\vec{B}$-field lines are perpendicular to the Sun’s surface. The measurements are based on the effects of the magnetic field on light emitted by the Sun, studied in Chapter 27. The magnitude of the $\vec{B}$ field in sunspots (measured using spectral analysis techniques) is about 0.25 T—thousands of times stronger than the Sun’s overall magnetic field. In each pair of sunspots, physicists find that the magnetic field lines go in opposite directions: if they are coming out of one spot, they are going into the other. Is there any connection between the darkness of the sunspots and the presence of magnetic field in them?

Before trying to explain what makes sunspots cooler than other parts, we first need to ask why the Sun’s surface is so hot? We learn later that near the Sun’s center, nuclear processes cause the temperature to be extremely hot compared to the surface—around 10 million K. The Sun’s temperature decreases from the center to the outside layers. This temperature difference leads to the transfer of thermal energy outward from the hot center towards the cooler surface. Closer to the surface, the energy is transported by convection. Hot material rises to the surface, cools there and sinks back where it subsequently warms again (Fig. 17.45). This same mechanism warms water in a teapot by heating it at the bottom. The warm water at the bottom expands, its density decreases, and it rises due to the fact that buoyant force exerted on it by the surrounding water is greater than the gravitational force pulling it downward.

![Figure 17.45 Convention currents on Sun](image)

The Sun is comprised primarily of hydrogen. At the Sun’s high temperature under its surface nearly all hydrogen atoms are ionized with each atom producing a negative free electron.
and a positive proton nucleus. Why is the ionized gas at the sunspots cooler than the surrounding surface? Recall that a charged particle moving perpendicular to a $\vec{B}$-field travels in a circular path around the field lines (the cause of auroras on the Earth). The magnetic field ‘confines’ charged objects so that they can’t easily move in straight paths perpendicular to the field lines.

It is possible that the Sun’s magnetic field also confines the cooler matter near the surface preventing hotter matter from moving up in a convection current. Imagine that the field lines near the Sun’s surface are as shown in Fig. 17.46a. Hot charged particles from below the Sun’s surface rise and cool in the convection loop described earlier. When the cooler matter condenses and tries to return back down, it crosses one of these intense $\vec{B}$-field lines that is part of the magnetic loop shown in Fig. 17.46a. The cooler particles are in effect trapped as they circle around the intense $\vec{B}$-field line. This prevents hotter gas from rising to the surface. This produces a cooler spot on the surface that looks to us like a dark spot. The shown configuration of the field also explains why the $\vec{B}$-field lines point in the opposite directions in two neighboring spots. It is not fully understood why the Sun’s magnetic field is what it is, but the observed pattern does give some qualitative understanding of the darkness of sunspots.

This reasoning also explains another phenomenon – solar prominences (Fig. 17.46b). These are jets of solar material shooting at extremely high speeds (hundreds of miles per hour) into the Solar atmosphere. They appear as arks about 30,000 km high that can exist for days usually above sunspots. One can explain such arks as the streams of Sun’s matter that move in a helical path along the $\vec{B}$-field lines in a sunspot, much like particle streams that lead to the Earth’s auroras.

Many other phenomena on the Sun’s surface are now explained by the magnetic properties of the Sun. Sudden changes in the configuration and strength of the magnetic field lead to solar flares (see Fig. 17.47) in which charged particles (similar to the prominences) follow closed oblong magnetic field lines that erupt from the Sun’s surface. The similarity of the shapes of prominences and flares is due to the same underlying reason – the motion of the particles along
magnetic field lines. The major difference is that flares are much more violent. The flare emits into space huge amounts of energy (as much as $6 \times 10^{25}$ J) in several minutes (the entire Earth population uses about $6 \times 10^{20}$ J of energy in one year). The flare also emits high-energy charged particles in all directions. Some of them reach Earth and disrupt its magnetic field. This disruption leads to “magnetic storms” which in old times when navigation depended on compasses, resulted in many shipwrecks. The records of these shipwrecks allowed scientists to reconstruct the timeline of magnetic disruptions on the Sun and determine their periodic nature.

![Solar flares](image)

**Figure 17.47** Solar flares

**Mass spectrometer**

In Section 17.5 we learned that when a charged particle’s velocity is perpendicular to the direction of the $\vec{B}$-field, the particle moves in a circular path. The radius $r$ of the circle depends on the particle’s mass:

$$r = \frac{mv}{|q|B}$$

This leads to an important application—measuring the mass of ions, molecules, even elementary particles such as protons and electrons. This provides a method for determining the relative concentration of atoms of the same chemical element but that have slightly different masses (known as isotopes, which we learn about in Chapter 28). For example, the two most abundant isotopes of oxygen are oxygen-16 (8 protons and 8 neutrons in its nucleus) and oxygen-18 (8 protons and 10 neutrons in its nucleus). The only difference between the two isotopes is the numbers of neutrons in their nuclei. The isotopes behave very similarly in terms of atomic processes but have different masses.

Oxygen of course is one of the atoms found in water (H$_2$O). Under normal conditions, about 99.8 percent of water molecules contain oxygen-16 atoms and only 0.2 percent of water molecules contain oxygen-18 atoms. The water molecules containing oxygen-16 are lighter and evaporate from the oceans slightly faster than those containing oxygen-18. Thus, water vapor in the atmosphere has a slightly greater concentration of oxygen-16 leaving the oceans with a
slightly lower concentration of oxygen-16. Because atmospheric water vapor eventually returns to the ocean as rainfall and runoff, a stable ratio of oxygen-16 and oxygen-18 is maintained. However, if the evaporated water vapor instead falls as snow and becomes glacial ice, it does not return to the sea. This means that glacial ice has a slightly greater oxygen-16 concentration. The oxygen-16/18 isotope ratio changes over geologic time due to global climate conditions, and therefore can be used as a way to date plant and animal remains found in the ice. Therefore, being able to make accurate measurements of this isotope ratio is of practical importance.

Another familiar example is radioactive carbon dating. This is a method to determine the age of bones from formerly living animals by determining the ratio of the carbon-14 isotope (which is slightly radioactive) in the bones to carbon-12 (which is stable). How can one separate these isotopes to determine the ratio, which in turn helps determine the age of the bone? Since both are carbon, they behave nearly identically from a chemical point of view. Is there a non-chemical method for distinguishing them?

This is exactly what a mass spectrometer is designed to do. In a mass spectrometer, atoms or molecules are ionized by removing one of their electrons. An \( E \)-field is used to accelerate the ion which then passes through a device called a velocity selector that allows only those ions moving at a predetermined speed to pass through it (Fig. 17.48). The ions then enter a region with a uniform \( B \)-field, traveling perpendicular to the field lines. As a result, the ions move in circular paths. It is possible to measure the radius of the circles by observing the place where the ions strike a detector after moving half way around the circle. How do we determine the ion’s mass if we know its speed, electric charge, the \( B \)-field magnitude, and the radius of the ion’s path from where it strikes the detector?

**Quantitative Exercise 17.12 Mass spectrometer** Since the atom or molecule has had a single electron removed, its electric charge is \(+1.6 \times 10^{-19} \text{ C}\). Its speed is \(1.0 \times 10^6 \text{ m/s}\) when it enters a mass spectrometer’s \(0.50 \text{ T} \ B \)-field region. It moves in a circle of radius \(0.20 \text{ m}\) until it hits
the detector (Fig. 17.49). Determine (a) the magnitude of the magnetic force that the $\vec{B}$-field exerts on the ion and (b) the mass of the ion.

![Figure 17.49]  

Represent Mathematically The magnitude of the force that the magnetic field exerts on the moving ion is determined using Eq. (17.5):

$$ F_{B \text{on}q} = |q|vB \sin \theta $$

This force perpendicular to the ion’s velocity causes its radial (centripetal) acceleration:

$$ ma_r = |q|vB $$

The radial acceleration is $a_r = \frac{v^2}{r}$; thus $ma_r = m\frac{v^2}{r} = |q|vB$ and we can determine the mass of the ion:

$$ m = \frac{|q|rB}{v} $$

Solve and Evaluate Now insert the appropriate values to determine the two unknowns:

$$ F_{B \text{on}1} = |q|vB \sin \theta = (+1.6 \times 10^{-19} \text{C})(1.0 \times 10^6 \text{ m/s})(0.50 \text{ T})\sin(90^\circ) = 8.0 \times 10^{-16} \text{ N} $$

and

$$ m = \frac{|q|Br}{v} = \frac{(+1.6 \times 10^{-19} \text{C})(0.50 \text{ T})(0.20 \text{ m})}{1.0 \times 10^6 \text{ m/s}} = 1.6 \times 10^{-26} \text{ kg}. $$

While the magnitude of the force exerted on the ion is very small, it’s actually reasonable since it does represent the force exerted on a single ion. The mass of the ion is also reasonable.

Try It Yourself: What would be the radius of the circular path a particle of mass $1.7 \times 10^{-26} \text{ kg}$ would follow in this mass spectrometer?

Answer: 0.21 m.

Review Question 17.7

How does a MHD generator produce constant potential difference?
17.8 Magnetic materials

At the beginning of the chapter we learned about the properties of magnets. For example magnets strongly attract objects made from iron such as paper clips, but do not exert a magnetic force on objects made from aluminum such as soda cans. Also iron has the ability to greatly amplify the $\vec{B}$-field surrounding it and most magnets are themselves made from iron. Finally, Earth is a giant magnet. Thus, it seems that materials, even amongst metals, can have widely varying magnetic properties. How can we explain this?

**Earth as a magnet**

As you know, Earth has distinct north and south magnetic poles as if it had a giant bar magnet inside. The south magnetic pole is in close proximity to geographic North Pole while the north magnetic pole is near the geographic South Pole. The $\vec{B}$-field lines for Earth’s magnetic field look something like shown in Fig. 17.50a. The lines go almost vertically into Earth’s surface at the poles and at an angle at places where most of us live. The first person to propose that Earth was similar to a giant magnet was William Gilbert, a courtier of Queen Elizabeth I, in 1600. Andre Mari Ampere provided an explanation several hundred years later. He suggested that Earth’s magnetism is due to electric currents in its core (Fig. 17.50b). Now geologists know that Earth’s core has different layers. The outer core is mostly liquid nickel-iron (region 2 in the figure), both strongly magnetic materials. Earth’s rotation causes convection to occur in the outer core, and this circulation of the liquid nickel-iron generates and maintains Earth’s magnetic field. Sometimes, the circulation is disrupted causing the poles of Earth’s magnetic field to reverse (called a geomagnetic reversal.) The reasons for such disruptions are not well understood. These field reversals happen irregularly, with intervals of 50,000 years to 780,000 years. It has been 700,000 years since the last reversal.

![Diagram of Earth's magnetic field](image)

**Figure 17.50** Earth as a magnet

**Magnetic properties of materials**

Let’s see if we can understand why some materials interact magnetically much more strongly than others. We start with observational experiments.
Observational Experiment Table 17.5 Magnetic behavior of different materials

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>A piece of pyrolytic carbon is placed above a strong magnet; the carbon levitates above the magnet.</td>
<td>A force diagram for the carbon shows that the magnetic field produced by the magnet exerts an upward magnetic force on the carbon. It seems the carbon has become a strong magnet with its poles oppositely aligned to the magnet below it (its north pole facing the north pole of the magnet).</td>
</tr>
<tr>
<td>A piece of aluminum is placed above the same magnet; it falls downward with the acceleration slightly greater than $9.8 , \text{m/s}^2$.</td>
<td>A force diagram for the aluminum shows that the magnetic field produced by the magnet exerts a small downward magnetic force on the aluminum. It seems the aluminum has become a weak magnet with its poles in the same orientation as the magnet below it (its south pole facing the north pole of the magnet).</td>
</tr>
<tr>
<td>A piece of iron is placed above the same magnet; it falls downward with the acceleration significantly greater than $9.8 , \text{m/s}^2$.</td>
<td>A force diagram for the iron shows that the magnetic field produced by the magnet exerts a large downward magnetic force on the iron. It seems the iron has become a strong magnet with its poles in the same orientation as the magnet below it (its south pole facing the north pole of the magnet).</td>
</tr>
</tbody>
</table>

**Pattern**

All of the materials have magnetic properties. Some become ‘magnetized’ opposite the direction of the other magnet and some in the same direction as the external magnet. The amount of this ‘magnetization’ also varies in strength.

It turns out that all materials belong to one of the three groups encountered in the table: the materials that are repelled by magnets are called **diamagnetic** (like the pyrolytic carbon or water); those that are weakly attracted are called **paramagnetic** (like the aluminum); and those that are strongly attracted are called **ferromagnetic** (like the iron.) Ferromagnetic materials have the additional property that they retain their magnetization to a certain degree even after the magnetic field that magnetized them is removed. Our next task is to explain the mechanisms behind diamagnetism, paramagnetism, and ferromagnetism.
Magnetic properties of atoms

To explain these three magnetic properties of materials we have to understand the magnetic behavior of individual atoms. The model of an atom that we use has a point-like positively charged nucleus at its center and point-like negatively charged electrons moving at high speeds in circular orbits around the nucleus. The motion of each electron is equivalent to a tiny circular current loop. Each tiny current loop produces a magnetic field that closely resembles the field of a tiny bar magnet (Fig. 17.51a). Thus, each electron acts like a tiny bar magnet (Fig. 17.51b) creating its own tiny magnetic field. In addition to this electron orbital magnetic field, the electron itself is like a tiny intrinsic magnet, which also contributes to the magnetic field produced by the atom.

In atoms with more than one electron, the field contributions produced by the different electrons often cancel each other. This happens because the electrons tend to pair off and orient themselves in opposite directions so that their magnetic field contributions add to zero.

Diamagnetic materials

Examples of diamagnetic materials are water, graphite, bismuth and the pyrolytic carbon in the first experiment in Table 17.5. In all of those materials the contributions to the magnetic field produced by individual electrons in the atoms cancel each other making the total field produced by the atom zero. When such materials are placed in a region with non-zero external $\vec{B}$-field, the motion of the electrons in the individual atoms changes slightly. Remember, the orbiting electrons behave like tiny currents, and therefore the external magnetic field will exert forces on them. The result of this is that the net $\vec{B}_\text{atom}$-field produced by the electrons in each atom is no longer exactly zero, causing the diamagnetic object to be repelled from the external field of the magnet.

To observe this diamagnetic repulsion, push a grape (comprised mostly of water, a diamagnetic material) into each end of a straw and carefully balance the straw on the end of a straight pin that has been stuck into a cork (Fig. 17.52). Now bring the north pole of a magnet near one of the grapes (but do not touch it). The grape slowly moves away from the magnet. Remove the magnet and let the straw with grapes stop turning. Now bring the south pole of the magnet near the same grape. It again starts slowly moving away. The magnet causes the water molecules to become slightly magnetic with their poles in the opposite direction of the magnet. Thus, when the north pole of the magnet approaches, it makes the water molecules into tiny
magnets with their north poles facing the approaching north pole of the approaching magnet. This effect is very small for most materials—about $10^{-5}$ times weaker than the attractive ferromagnetic effect we will discuss shortly. The strongest diamagnetic materials are superconductors, which are repelled very strongly by magnets.

![Diamagnetic repulsion](image)

**Figure 17.52** Diamagnetic repulsion

There are a variety of efforts to use magnetic repulsion in trains that levitate above a track and are pushed forward at speeds up to 300 mph by special magnetic propulsion systems. Such trains are being designed and tested in China and use superconducting electromagnets in the train cars to repel current carrying coils of wire in the tracks that levitate the train several inches above the track. This magnetic levitation is discussed in greater detail in the next chapter.

**Paramagnetic materials**

We mentioned above that in most atoms the contributions to the $\vec{B}$-field of the electron currents, and of the electrons themselves usually cancel. If they don’t, the atom will have a net $\vec{B}$-field resembling that of a small bar magnet. Examples are aluminum, sodium, and oxygen—paramagnetic materials, whose behavior we observed in the second experiment in Table 17.5.

Normally the north-south axes of atoms in paramagnetic materials are oriented randomly (Fig.17.53a). As a result, their contributions to the $\vec{B}$-field add to zero. However, if a paramagnetic material is placed in a region with a $\vec{B}$-field, the atoms behave like tiny (but very weak) compasses and have a tendency to align with that $\vec{B}$-field. The $\vec{B}$-field contributions of the individual atoms no longer cancel each other completely but add together and enhance the $\vec{B}$-field that caused them originally to align (Fig. 17.53b). In most materials this paramagnetic effect produces only a slight enhancement of the $\vec{B}$-field. The thermal energy of the material causes the north-south axes of the atoms to remain mostly randomly oriented. For example, the $\vec{B}$-field contribution of the atoms only enhances the original $\vec{B}$-field by a factor of about $10^{-5}$. Thus paramagnetic materials are attracted only very weakly to magnets and the effect is difficult to measure.
Ferromagnetic materials

Certain materials, such as iron, nickel, cobalt, and gadolinium, have individual atoms with non-zero $\vec{B}$-field contributions; they act like tiny magnets, just like in paramagnetic materials. However, in addition to this, the north-south axes of the atoms have a strong tendency to line up (the third experiment in Table 17.5). This is what makes ferromagnetic materials special. If an iron bar is placed in a region with non-zero $\vec{B}$-field, the north-south axes of the iron atoms strongly align with the original $\vec{B}$-field and enhance it by as much as several thousand times.

Even when not in a region with a magnetic field, neighboring atoms in ferromagnetic materials tend to line up in small, localized regions called domains. In a particular domain the magnetic fields of all atoms point in the same direction. Each domain may include $10^{15}$ to $10^{16}$ atoms and occupy a space less than a millimeter on a side. In a neighboring domain the magnetic fields of the atoms may all point in a different direction. In a piece of iron that is un-magnetized, like a nail, the domains are oriented randomly, and $\vec{B}$-field contributions of the domains add to zero (Fig. 17.54a).

If an un-magnetized piece of iron is placed in a region with a non-zero $\vec{B}$-field, the domains oriented in the direction of the $\vec{B}$-field increase in size, while those oriented in other directions decrease in size, or shrink. When the $\vec{B}$-field is removed, the domains remain aligned and the iron now produces its own strong $\vec{B}$-field (Fig. 17.54b.) This is how permanent magnets are created.
We can now explain the legend about Magnes that you read at the beginning of the chapter. The lodestone that he found contains iron oxide, $\text{Fe}_3\text{O}_4$, a natural ferromagnetic material which acquires its magnetic properties due to being in the external magnetic field of Earth for billions of years. We can also explain why a magnet split in two pieces has both poles on each part. Each piece has its domains aligned as they were before splitting. Thus, each piece has its own north and south pole.

Understanding ferromagnetism helps also explain how a device called an *electromagnet* works. Picture in your mind a wire that has been shaped into solenoid, a so-called air-filled solenoid. A current through this wire produces a $\vec{B}$-field that resembles the field of a bar magnet. Now, insert a bar of iron into the solenoid. The $\vec{B}$-field produced by the wire causes the magnetic domains within the iron to line up. The now magnetized iron produces its own contribution to the $\vec{B}$-field, that is up to several thousands of times stronger than the $\vec{B}$-field produced by the current. This is an electromagnet.

A magnetized piece of iron can be unmagnetized; for example, if you drop a permanent magnet, its magnetization sometimes weakens significantly. Why? When the magnet hit the hard surface, the collision caused violent vibrations that shook the atoms causing their north-south axes to become randomly oriented. When the vibrations stopped, the magnetic domains formed again but were more randomly aligned with a smaller amount of magnetization. By similar reasoning, increasing the temperature of a ferromagnetic material should also disrupt the magnetic domains by increasing the randomness in the orientation of the atoms. We can test this hypothesis by magnetizing an iron nail near a strong magnet so it attracts iron filings or steel paper clips. Then we put the nail above a candle until it gets hot. Once it gets very hot, it stops attracting the iron filings and paper clips. The temperature at which a ferromagnetic material loses its magnetization completely is called its Curie temperature (in honor of a French physicist Pierre Curie who discovered this phenomenon). The Curie temperature for iron is $753^{\circ}\text{C}$, Nickel’s is $365^{\circ}\text{C}$.

Ferromagnetic materials have many practical applications. Hard disk drives use the permanent magnetization of ferromagnetic materials to store data. Airport metal detectors, transformers, electric motors, loudspeakers, and electric generators all depend on the magnetization of ferromagnetic materials. And finally all permanent magnets that were mentioned in this chapter were all made from ferromagnetic materials.

**Review Question 17.8**

Why is there a difference in the behavior of paramagnetic and diamagnetic materials when they are placed in a region with non-zero $\vec{B}$-field?
### Summary

<table>
<thead>
<tr>
<th>Word Representation</th>
<th>Sketch and Diagrammatic Representation</th>
<th>Mathematical Representation</th>
</tr>
</thead>
</table>
| **Magnets** always have a north pole and a south pole, completely different from positive and negative electric charge. | ![Diagram of magnet poles] | If an object used to detect a magnetic field is a current carrying wire, the magnitude of the \( \vec{B} \)-field vector can be found as  
\[
B = \frac{F_{\text{max}}}{IL} \tag{17.2}
\]  |
| **\( \vec{B} \)-field lines** represent the \( \vec{B} \)-field in a region. The \( \vec{B} \)-field vector at a point is tangent to the direction of the \( \vec{B} \)-field line in the vicinity of that point. The separation of lines in a region represents the magnitude of the \( \vec{B} \)-field in that region—the closer the lines the stronger the \( \vec{B} \)-field. | ![Diagram of magnetic field lines] |  |
| **Right-hand rule for the magnetic force:** Point the fingers of your open right hand in the direction of the magnetic field. Orient your hand so that your thumb points along the direction of motion of the charged particle or the electric current. If the particle is positively charged, the magnetic force exerted on it is in the direction your palm is facing. If the particle is negatively charged, it points in the opposite direction. | ![Diagram of right-hand rule] |  |
| **Magnitude of magnetic force exerted on a current in a wire, and on an individual charged object:** See the diagrams and equations. | ![Diagram of magnetic force on current] |  
\[
F_{\text{mag}} = ILB \sin \theta \tag{17.3}
\]  
where \( \theta \) is the angle between \( I \) and \( \vec{B} \).  
\[
F_{\text{mag}} = |q| \vec{v} B \sin \theta \tag{17.5}
\]  
where \( \theta \) is the angle between \( \vec{v} \) and \( \vec{B} \).  |
| **Magnetic torque on current carrying coil:** A magnetic field exerts forces on the current passing through the wires of a coil resulting in a torque on the coil. | ![Diagram of magnetic torque on coil] |  
\[
\tau_{\text{mag}} = NBA \sin \theta \tag{17.4}
\]  |