Two Approaches to Learning Physics

"I look at all those formulas..." "I'm trying to imagine..."

By David Hammer

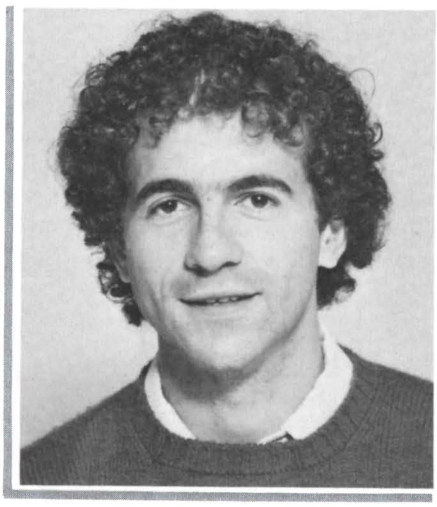
Efforts to understand student misconceptions and difficulties in learning physics generally involve analysis of how students reason about the material.[1] Usually this means observing students learning and working on problems in some specific area such as one-dimensional kinematics,[2] the concept of acceleration,[3] or elementary dynamics.[4] Often it means comparing novices' reasoning with that of experts,[5] or with some ideal model of productive reasoning.[6]

Such analysis is indispensable. It helps teachers design courses by providing information about students and discussion on what outcomes of instruction might be expected or desired. I suggest, however, that in addition to studying how students reason about physics, it is also important to consider what they think reasoning about physics involves.

Students have preconceptions, not only about physical phenomena, but also about schools and learning and about the discipline of physics itself. For some, much of the difficulty may arise out of very general misconceptions of what physics is and how to approach it. For example, if some students conceive of physics as a collection of isolated facts and formulas, it may never occur to them to pay attention to underlying reasoning. They may work in a manner we think of as unreflective, not because they are disinterested or unintelligent, but because that is what they think the subject entails. Moreover, it may be that introductory courses inadvertently support such misconceptions. The belief that physics is formulas, for example, may be supported by the practice of presenting tables of formulas for use in problem solving.

These are not new ideas. The notion of a "hidden curriculum,"[7] of instruction having unintended effects on how students learn, has long been an issue in education. Wertheimer[8] was concerned with the danger that a drill-and-practice method of instruction "mechanizes the mind and uses procedures that change the free and sensible manner of children." Perry[9] studied the development of college students' approach to learning. He delineated stages that progressed, broadly speaking, from those at which students expect teachers to provide absolute truths, to those involving personal commitment to some point of view among many. Perry's work was concerned primarily with liberal arts, but the progression from acceptance of authority to judgment from one's own perspective could also be relevant to physics learning.

More recently, Schoenfeld[10] analyzed a high-school math class, listing a series of unproductive beliefs about mathematics supported by the style of teaching and evalu-



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ation. diSessa[11] compared what he called the "intuitive epistemologies" of two introductory physics students, one who was quite articulate in explaining that the essence to physics was getting the right results and another who emphasized making intuitive sense of the material.

I have observed two introductory courses and interviewed students to study three questions:

1) What general conceptions of physics do students have, if any?

2) How do these conceptions affect their understanding and performance?

3) How are these conceptions affected by the instruction in the course?

For each course, I chose students at random from the roster and asked them to participate in the study—about half of them agreed—until I had five subjects. I met with each separately, five times over the semester, and tape recorded the interviews. Each session took about an hour and involved a variety of tasks: open-ended discussions of the student's impressions of the course and of physics; semidirected tasks such as going through a midterm exam; specific discussions of concepts, and problem solving, both quantitative and qualitative.

This paper concerns two students, Liza and Ellen (pseudonyms), from the same "pre-med" level physics course. They were both planning to go to medical school; both had math SAT scores around 700 and A's in math courses through calculus. Liza's record was the stronger, including a 5 on the BC Calculus Advanced Placement Exam, an A− in the first semester of college chemistry, and an A in the physics course she took in high school. Ellen got a C in the same chemistry course, and this was her first course in physics. I choose to discuss Liza and Ellen because, unlike the other subjects in the study, they both devoted a great deal of time and effort to the course, and, while their grades in the course were similar (Liza: B+, ≈85th percentile; Ellen: B, ≈80th percentile), they had markedly different approaches.

Naturally, I do not claim there are only two kinds of students. Still, Liza and Ellen do not seem to be idiosyncratic. Other subjects in my study exhibited similar views, and, while there is not a precise correspondence with diSessa's students, there are certainly similarities. I expect many instructors will recognize some of their own students in these accounts.

General Conceptions

The first question is whether it is even possible to identify students' general conceptions of physics. If not, this sort of analysis would obviously not be useful.

For most of my subjects, it has not been difficult to see such conceptions, apparent throughout the interviews and consistent from one context of discussion to another. Consistency is important to the claim, because without it one may argue that a student's response merely reflected her interpretation of that particular task, or that the views themselves are not involved enough to have any significant effect. It will be difficult to convey consistency here, however, even for only two students. Due to space constraints, I will be limited to citing a few representative excerpts.

Liza

Throughout her transcripts there are both explicit and implicit indications that Liza's approach to the course was to learn formulas and facts based on the authority of the instructor and text. To Liza, the formulas were the physics, and that the professor said it in lecture or that it was in the book constituted sufficient justification.

She almost always said she understood the lectures or the reading; it was only when I pressed for explanations beyond citations that she "didn't think about it" or was "not sure":

Liza: ...he defines the, um, acceleration of circular motion is this formula, and then later on he proves it.
Interviewer: OK, how does he prove it?
L: Um, (flips pages) ah, this acceleration is referred to, um, inward acceleration.
I: Why is the acceleration inward?
L: He said there is an inward acceleration for circular motion.
I: Does that make sense?
L: (pause) I didn't think about it (laughs).

She solved problems by "figuring out which formula to use":

L: I look at all those formulas, say I have velocity, time, and acceleration, and I need to find distance, so maybe I would use a formula that would have those four things.

But having an explanation for the formula itself did not seem relevant:

L: This is v f equals v oh plus a t ($v_f = v_0 + at$).
I: OK, and where did that come from?
L: It came from here (laughs and indicates book).
I: Why is it true?
L: Well, there is an initial velocity, and there is a final velocity, we know what the a is which is the acceleration, and I just use this to find what time it takes.

Liza felt no need to check presented facts for consistency:

L: The frequency (of a pendulum) doesn't depend on mass.
I: But it does with a spring.
L: Right.
I: OK, why is it different?
L: (15-second pause, flips through the book) Well, isn't a spring dealing with linear motion and here you are dealing with angular motion?

An instructor might take such interactions as evidence of laziness or lack of aptitude. To a physicist, it may seem obvious that to have no justification for facts and equations, other than an appeal to authority, is to have a limited and unsatisfactory understanding. To a student, however, it may be that knowing formulas is the same sort of thing as, say, knowing state capitals: whether it "makes sense" that acceleration is inward for circular motion might seem as odd a question as whether it "makes sense" that Boston is the capital of Massachusetts. (Of course, in some contexts the latter would be a perfectly reasonable question. The issue here is whether, to Liza, asking if centripetal acceleration makes sense is reasonable in the context of a physics course.)

It is worth stressing that Liza put in a great deal of time and effort, working extra problems, attending every lecture, discussion section, and lab. Liza was trying to learn physics, and, for the most part, she felt she was succeeding. In the last interview I asked if she had liked the course. She answered: "Not really...it was kind of boring...all those formulas and derivations." When I asked whether she was describing the course or physics in general, she said: "Both."

Ellen

Ellen's attitude was more independent, and, to her, the formalism was only one way of looking at the physics. She did want to make sense of the material, to integrate it with her own intuitions, and was able to do so for the first few weeks.

Ellen criticized lectures for failing to connect "theory" with "reality," for emphasizing the formalism over simple explanations. She felt, for example, that the professor's treatment of finding the range of a projectile was needlessly complicated:

> *Ellen*: ...obviously when it stops being in the air it stops going horizontally. ...it seems like we spent a couple of lectures just trying to get that through people's heads, and I'm sure if you just say that to someone they'll (say) well obviously. ...I guess that's what it is, we get theory with theory, and then we get application with application.
> *Interviewer*: What do you mean by theory?
> *E*: It means formulas, ...let's use this formula because it has the right variable, ...instead of saying ok, we want to know how fast the ball goes in this direction, because if we know that, all we have to do is find out how long it goes in that direction before it hits the ground and we can find out how far.

After the first few weeks she became frustrated, finding it difficult to reconcile different parts of the formalism with each other and with her intuition:

> *E*: Well, things like friction, things like that, I'm trying to imagine. I know that weight cancels out the normal to a plane, but then we consider friction, and something isn't canceled out is it? ...And so, in my mind it would make the force on this car less, but according to the equation that was on the board, it doesn't affect the amount of force, on this rope.

Eventually she compromised her standards:

> *E*: I'd rather know why for real.
> *I*: What do you mean by "know why for real?"
> *E*: Well, sometimes I rationalize things, and I know that it's probably not the soundest rationalization, or something, it's just that I can kind of make it make sense for the time being, if I don't have time to really find out everything about it...even though I don't really know how it was derived.

Like Liza, Ellen was able to apply prescribed methods to solve problems, but for her this was not sufficient:

> *I*: So part b, can you do it?
> *E*: No. (laughs) I can pretend to. I can do it the way I know it's supposed to be done, but I pretty much already did that.

Her attitude was one some might perceive as belligerent. She wanted everything to make sense *to her*, unsatisfied with only the professor's word, and complained of being able only to "reiterate" what she heard in lecture. Like Liza, Ellen also spent a great deal of time on the course, but she did not feel she got much out of it. While she became quite comfortable with most of kinematics, there remained large gaps in her understanding of dynamics, both by my observations and by her own appraisal.

Effects on Understanding and Performance

While one may make aesthetic judgments on the relative merits of the two students' conceptions, the question here is what were the practical effects.

Broadly speaking, the effect of Liza's general conceptions was that she was satisfied with learning physics as a set of relatively unrelated pieces, isolated both from each other and from her own everyday experience. Her understanding remained incoherent and fragmented, as naive physics has been characterized in general,[12] but it served her comparatively well on problem sets and examinations. Ellen, when her approach was successful, was able to bring fragments of her understanding together, to integrate different pieces from the course with each other and with her intuitive knowledge. Unfortunately, it was not successful after the first few weeks.

More specifically, there was evidence for the different conceptions having effects on each student's ability to solve problems, both those from the course and the qualitative questions I introduced, and on the forming or retaining of intuitive misconceptions.

Course problems

An assigned problem asked: "One ball is thrown horizontally with a velocity v_0 from a height h, and another is thrown straight down with the same initial speed. Which ball will land first?"[13] Liza wrote out $x = x_0 + v_0 t + (1/2) at^2$ for the horizontal and vertical components of each ball, substituting appropriate initial positions, speeds, and accelerations. She then used the vertical component equations to show, not without some difficulty, that the second ball travels a greater vertical distance after time t than the first, so it hits first. Ellen, in contrast, said immediately that the answer was "obvious," and went on to explain qualitatively that the second ball hits first because it has a greater speed downward to cover the same vertical distance.

An exam question asked: In which direction "is the force on (the driver) exerted" when a car turns in a circle? To Ellen, it was "obvious" that the acceleration was inward and so there had to be a force inward, but she did not feel this force was "exerted," apparently confused by the connotation of effort being ascribed to a passive force. The driver, she felt, would be "exerting" a force outward "to hold himself in and have the car pull him," so she answered outward. Liza answered inward, and, when I asked her to explain why, she said simply: "Because for centripetal force it's inward."

These two examples illustrate the advantages and disadvantages in each approach for solving course problems. When Ellen was able to make sense of the material, her intuitions helped her understand the formalism and guided her solutions. She understood projectile kinematics well enough, in fact, to help her friend in a more rigorous course with his problem set. After the first few weeks, however, Ellen's attitude interfered with her ability to answer the course questions. For a while she continued to try to reason based on her own sense of things, rather than accepting what she was told or answering by rote, but her intuitions were not generally in accord with Newtonian physics.

Liza, following the algorithms provided, often failed to apply her own knowledge from everyday life. She almost certainly had the common sense to know that the ball thrown down would hit first, but she did not think to make use of it in this context. At no time did she acknowledge that this was the answer one would expect. However, with her methodical adherence to the procedures, she was more reliable than Ellen in answering correctly. Liza was able to solve more course problems correctly during the interviews, and her scores on problem sets and exams were slightly higher overall.[14]

Qualitative problems

Ellen did better than Liza, however, on the qualitative problems I presented, which mainly involved concepts from the first few weeks of the course. One problem, for example, had a monkey and an equivalent weight suspended at the same height from opposite sides of a massless, frictionless pulley: If the monkey climbs on the rope, will it or the weight end up higher?

Both Liza and Ellen had trouble with this at first, so I gave an easier one: With the monkey and weight on frictionless ice at opposite ends of a rope, which, if either, moves more when the monkey pulls?

(I provided sketches of each situation. In both problems, the force of the rope on the monkey is equal in magnitude to the force of the rope on the weight. So the monkey and weight have the same magnitude accelerations, and, therefore, the same speeds since they both start from rest.)

They were both eventually able to answer the second question. Ellen then went on to relate it to the pulley problem, saying she thought the same argument "should" apply but that she also "felt" the answers should be different. Eventually, she decided there was a problem with her intuition here, saying she thought it might derive from her experience of climbing a rope tied to a rock thrown over a tree branch. She could "just see (the monkey) just sitting there, not going anywhere," because, she remembered, she could "hang on the rope and...balance off, ...but if I tried to climb up, I'd climb up a little bit and then the rope would slip." Ellen accounted for her experience by saying that there was "friction in the tree," and "I guess I weighed more than the rock." She said she "felt better" about the first problem, answering that the monkey and the weight would end up at the same height. Although her intuitions disagreed at first, she was able to account for the conflict.

Liza was not able to relate the two situations. She saw the one as involving rotational motion, and therefore torques, while the other was "an $F = ma$ problem." While it is possible to solve the pulley problem in terms of torques, Liza was not able to do so. What hampered her was the lack of integration in her understanding, which, it seems, was partly a result of her overall approach to physics.

Misconceptions

Ellen's solution to the pulley problem is a good example of a more general advantage to her approach. It involved her in a process of reconciliation between the Newtonian physics she learned in the course and the intuitions she acquired through experience. This had her examining those intuitions, along with the formalism, providing a mechanism for recognition and modification of naive misconceptions.

I never observed Liza going through such a process. For her, understanding constituted literal knowledge of the fact or formula and the ability to apply it to problems. Knowing that centripetal acceleration is directed inward with magnitude v^2/r, Liza was able to solve problems and had no reason to give the matter further thought.

This is not to say Liza's intuition was never involved in her reasoning. To the contrary, in discussing the content and in solving qualitative problems, she showed many of the familiar misconceptions. The claim is that Liza's intuition was not involved in any deliberate or conscious manner. At no time in any context did Liza refer to her own experience to justify any physical argument. What was missing for Liza, I believe, was conscious recognition of the conflicting intuition and the knowledge that it is appropriate to resolve the disagreement.

My impression is that Liza showed more misconceptions than Ellen on the early material. On the later material, Ellen showed many as well: while she was aware of discrepancies, her success in resolving them was limited. Unfortunately, this study was not systematic enough in this regard to allow substantive comparison. (Analysis of transcripts from the subsequent study—of an introductory course geared for engineers and physical science majors—is just beginning, but the preliminary indications are that the students who try to involve their intuitive knowledge in the course do show fewer misconceptions.)

There were also several incidents in which Liza's uncritical acceptance of authority seemed to result in her acquiring misconceptions during the course. In the first lab of the course, students were asked to confirm or refute Galileo's statement that the period of a pendulum is independent of its amplitude. In fact, the statement is only approximately true for small amplitudes; it was intended that students would find a deviation for which experimental error could not account, showing that even Galileo could be wrong. But, because it was Galileo, Liza concluded the statement was correct, although this was not supported by her data.

Liza was also at risk during lectures if she misinterpreted the professor's words or if, on occasion, he misspoke. In one instance, the professor, finishing a calculation of the gravitational force between two people one meter apart, commented that the attraction for the other person is nine orders of magnitude less than the attraction to the earth, "which is why you don't go smashing into them." Liza interpreted this to mean that, without the earth, people would "smash" into each other. This would contradict what she had learned about Newton's second law, as well as what she might have known about what happens to astronauts, but she did not make such comparisons.

Ellen, in contrast, was constantly complaining that she did not see how various statements made sense based on what she had already learned or simply believed. She did not understand the lectures and, in fact, tended to blame this on the professor.

Effects of the Course on General Conceptions

It is not likely that the course created these general conceptions of physics, as students come with ideas of their own about what the subject will involve. The question here is what conceptions did the course support and what did it challenge.

This was a standard introductory physics course. The method of instruction was to present material, demonstrate its validity, provide examples of its application, and assign further problems for students to do on their own. Lectures paralleled the text,[13] at a pace of about a chapter per week. Newton's laws, for example, were introduced beginning with an explanation of *forces* as pushes or pulls, and the statement that weight is the gravitational force, mass is the weight divided by *g*. The laws were stated in turn, discussed qualitatively, and then applied to several quantitative sample problems. There was no substantive discussion of their origins, only the statement that they are "a set of brilliant and precise rules" Newton discovered through "observation."

There are several points to be made. The flow of reasoning was always from the theory to the phenomena, the flow of information always from the professor and text to the students. The students applied the theoretical concepts of the course, but they were not involved in or even witnesses to the formation of those concepts. The laws were simply provided, and students were to become familiar with them through practice in solving problems.

Similarly, only two out of ten labs involved determination of some empirical rule, including the pendulum lab mentioned earlier. All others concerned verification of known results or the use of known results as tools to measure some quantity. In every case, the procedures were specified.

With the two exceptions in the labs, then, there were no activities in the course by which conjectures were evaluated through discourse or experimentation. The implicit purpose of the course, made explicit as well on occasion, was for students to be able to solve the problems. Although the lectures and the text both contained some qualitative discussion, the ultimate goal in each was to apply the material to problems. Both modeled solution techniques consisting of choosing formulas by the variables they contain, finding as many equations as unknowns and then performing the algebraic manipulations. The solution sets handed out after assignments and exams consisted almost exclusively of algebra.

The emphasis on formalism and the goal of problem-solving facility seem consonant with conceptions of physics as a collection of facts and formulas. The style of instruction, with knowledge passing from instructor to student, procedures and results to experimentation specified in advance, seems consistent with a reliance on authority. Furthermore, the pace of the course, with a chapter of reading and ten problems due every week, would not allow much independent exploration, except perhaps for those students who find the material easy. On the whole then, the course seemed to foster Liza's approach of learning facts and formulas provided by authority and to challenge Ellen's of trying to develop an integrated, intuitive understanding.

These assertions are supported by the observations 1) that Liza's methods were more effective on the course problems and 2) that Ellen eventually found it necessary to abandon her approach and do "what everybody else does" to get through the assignments and the exams.

Discussion

Most physics instructors have stories of students who either lack common sense or, at least, fail to apply it to a physics course. And there is extensive documentation of those who continue to hold beliefs directly in contradiction to principles they not only have encountered in lectures, demonstrations, and readings but have applied correctly on problem sets and exams.[15] The argument here is that this may be a result of misconceptions, not only of specific elements of physics knowledge, but of the general nature of what physics knowledge is, and of what reasoning and learning in physics involves.

Of course, any conclusions drawn from case studies such as these must be tentative, both for the small sample size and for the inexactness of the process of deriving information from student comments. If, nevertheless, there is merit to these results, then there are implications for teaching.

Instructors tend to deal with student difficulties at the level of content. In choosing textbooks, preparing lectures, and assigning problems and labs, we concentrate on clear presentation. If students have trouble understanding centripetal acceleration, we add another particularly convincing demonstration, provide supplemental reading, or assign more problems. Part of the difficulty for some students, however, may be in their conceptions of what understanding centripetal acceleration means. We need to pay attention not only to our treatment of the content but to the general intellectual environment of the course and the kinds of reasoning it supports.

If our main goal is for students to be familiar with a given body of knowledge, and to be able to apply that knowledge reliably to solve some classes of problems, then we probably want to encourage them to follow procedures we provide. It was Liza's adherence to the techniques she was taught that gave her an advantage in solving course problems. The disadvantage to her approach was that, although the techniques derived from a coherent theory, for her they were a collection of isolated formulas essentially divorced from physical meaning. She could not apply her knowledge flexibly, because it was not well organized[16] or integrated with her intuitions. We need to design procedures more carefully, based not only on what we know about physics but also on what we know about learning. We can use education research to anticipate intuitive misconceptions and address them directly. Rather than providing formulas for algebraic manipulations, we should prescribe coherent procedures that incorporate the meaning of physical concepts explicitly.[17]

Labudde, Reif, and Quinn,[17] for example, tested a prescriptive approach to the concept of acceleration and found it to be successful in helping students develop a coherent understanding, substantially improving performances on qualitative problems. They specified the following procedure: find the velocity of the particle at the time of interest and at a short time later; find the difference by vector subtraction and divide by the time interval. For a sufficiently small interval, the result is the acceleration. Students who had studied acceleration in an introductory course were able to answer an average of 40 percent of the questions on a pretest correctly. They answered an average of 95 percent of post-test questions correctly, after only two half-hour sessions of instruction in the procedure.

This sort of approach, then, can be far more effective than conventional instruction at enabling students to solve physics problems, but it still may support reliance on the instructor to provide correct procedures. If we want students to learn to reason independently, to be able to form and test hypotheses, to learn to evaluate evidence and come to their own conclusions, then we should support an approach like Ellen's and challenge conceptions of physics reasoning as the application of techniques provided by authority. Then we need to involve students in the formation of the theoretical concepts of the course, not only in applications, making their ideas a substantive part of the development process and encouraging them to come up with their own procedures. In contrast to conventional courses, we should use experiments and demonstrations to discover physical phenomena and to test hypotheses, rather than simply to illustrate the validity of known results.[18]

Thornton,[19] for example, uses microcomputers to teach one-dimensional kinematics through real-time measurement of position, velocity, and acceleration, largely involving the students' own motions. The lab curriculum has students devising their own techniques and constructing their own knowledge, making use of their predictions and preconceptions through guided discovery. For example, one exercise has them predict the shape of a position-vs-time graph, given a description of someone's movements. They discuss their predictions, try to resolve disagreements, and perform experiments until they are convinced by some result. Students who had studied the material in a standard course have shown substantially improved performances on kinematics questions after, generally, two labs lasting between two and three hours.

In the best of all worlds, instruction would be tailored to each student's needs, but that is not generally practicable. What we can do is to use a variety of approaches. Prescribing procedures, done well, can be an effective way to teach the content of introductory physics, but it does not involve students in much creative thought. Other methods involve student invention, but the techniques students initially devise will seldom be very efficient. Thornton's results are a demonstration that students can learn traditional content through discovery, but the process is slow. Laws[20] describes an entire curriculum with no lectures, but with an intentional reduction in coverage. There seems to be a trade-off in the extent to which we can promote independent reasoning vs the amount of material we can cover. We need to consider our priorities.

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References

1. L.C. McDermott, "Research on conceptual understanding in mechanics," *Phys. Today* **37**, 24 (1984).
2. D.E. Trowbridge and L.C. McDermott, "Investigation of student understanding of the concept of velocity in one dimension," *Am. J. Phys.* **49**, 242 (1981); F.M. Goldberg and J.H. Anderson, "Student difficulties with graphical representations of negative values of velocity," *Phys. Teach.* **27**, 254 (1989).
3. F. Reif, "Instructional design, cognition, and technology: applications to the teaching of scientific concepts," *J. Res. Sci. Teach.* **24**, 309 (1987).

4. J. Clement, "Students' preconceptions in elementary mechanics," *Am. J. Phys.* **50**, 66 (1982); L. Viennot, "Spontaneous reasoning in elementary mechanics," *Eur. J. Sci. Ed.* **1**, 205 (1979).
5. J. Larkin, "The role of problem representation in physics," in *Mental Models*, edited by D. Gentner and A. Stevens (Lawrence Erlbaum, Hillsdale, NJ, 1983).
6. F. Reif, "Scientific approaches to science education," *Physics Today*, (Nov. 1986).
7. B.R. Snyder, *The Hidden Curriculum* (Knopf, New York, 1971).
8. Max Wertheimer, *Productive Thinking*, Enlarged edition, edited by Michael Wertheimer (University of Chicago Press, Chicago, 1982, originally published in 1945).
9. W.B. Perry, *Forms of Intellectual and Ethical Development in the College Years: A Scheme* (Holt, Rinehart, and Winston, New York, 1970).
10. A. Schoenfeld, "When good teaching leads to bad results: the disasters of well taught mathematics classes," in T.P. Carpenter and P.L. Peterson, "Learning mathematics from instruction," a special issue of *Educational Psychologist* **23**, 145 (1988); A. Schoenfeld, "On mathematics as sense-making: an informal attack on the unfortunate divorce of formal and informal mathematics," in *Informal Reasoning and Education*, edited by D.N. Perkins, J. Segal, and J. Voss (Lawrence Erlbaum, Hillsdale, NJ, 1987).
11. A. diSessa, "Learning about knowing," in *Children and Computers, New Directions for Child Development (28)*, edited by E.L. Klein (Jossey-Bass, San Francisco, 1985).
12. A. diSessa, "Knowledge in pieces," in *Constructivism in the Computer Age*, edited by G. Forman and P. Putall (Lawrence Erlbaum, Hillsdale, NJ, 1988).
13. M.M. Sternheim and J.W. Kane, *General Physics* (Wiley, New York, 1986), p. 38.
14. Liza: first midterm 45/50 (<x> = 39, σ = 9); second midterm 72/100 (<x> = 56, σ = 20); final 62/100 (<x> = 52, σ = 18); grade B+.
Ellen: first midterm 46/50 (<x> = 39, σ = 9); second midterm 54/100 (<x> = 56, σ = 20); final 62/100 (<x> = 52, σ = 18); grade B. (The pattern of Ellen's scores may be worth noting. Her worst performance was on the second midterm, which may be because she was still trying to use her intuition. Later, she adopted an approach more like Liza's, which may be reflected in her performance on the final.)
15. P.C. Peters, "Even honors students have conceptual difficulties with physics," *Am. J. Phys.* **50**, 510 (1981); I.A. Halloun and D. Hestenes, "Common sense concepts about motion," *Am. J. Phys.* **53**, 1056 (1985).
16. B. Eylon and F. Reif, "Effects of knowledge organization on task performance," *Cognition and Instruction* **1**, 5 (1984).
17. P. Labudde, F. Reif, L. Quinn, "Facilitation of scientific concept learning by interpretation procedures and diagnosis," *Eur. J. Sci. Ed.* **10**, 81 (1988).
18. R. Karplus, "Teaching for the development of reasoning," in *The Psychology of Teaching for Creative Thinking and Creativity*, edited by A.E. Lawson, 1980 AETS Yearbook, (ERIC/SMEAC, Columbus, OH, 1979); J. Minstrell, "Teaching science for understanding," in *Toward the Thinking Curriculum: Current Cognitive Research*, edited by L. Resnick and L. Klupfer (1989 Yearbook of the Association for Supervision and Curriculum).
19. R. Thornton, "Tools for scientific thinking: microcomputer-based laboratories for physics teaching," *Phys. Educ.* **22**, 230 (1987).
20. P. Laws, "Workshop physics: replacing lectures with real experience," in *Computers in Physics Instruction: Proceedings*, edited by E.F. Redish and J.S. Risley (Addison-Wesley, Reading, MA, 1989).

Road Sign

In the early 1970s Australia changed from feet-and-inches measurements to the metric system. Many road signs were replaced, but some were simply modified.

The sign at the entrance to a National Park in New South Wales originally read:

KOSCIUSKO NATIONAL PARK
ENTRANCE STATION 300 YDS
REDUCE SPEED TO 10 M.P.H.

It was modified by placing a "275 m" plate over the 300 YDS and a "15 km/h" plate over the 10 M. As you can see, (Fig. 1) they got it wrong. Instead of speed, they have acceleration!

$$15 \text{ km/h.P.H.} = 1.16 \times 10^{-3} \text{ m/s/s.}$$

I first spotted the mistake in 1975 when I was a school student studying physics. Thirteen years later I'm on the other side of the front desk as a high-school physics teacher, and the sign is still there. For how much longer, I don't know. ♦

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