Section II: Do #1-33 odds, #34

For Problems 1 through 20, simplify and write in terms of i.

1. i^5	2. i^7	3. i^{55}	4. i^{25}
5. i^{62}	6. i^{74}	7. i^{300}	8. i^{180}
9. i^0	10. i^{-2}	11. i^{-7}	12. i^{-25}
13. i^{-38}	14. i^{-54}	15. $\sqrt{-16}$	16. $\sqrt{-25}$
17. $\sqrt{-18}$	18. $\sqrt{-48}$	19. $\sqrt{-7}$	20. $\sqrt{-3}$

For Problems 29 through 32, find (a) $z_1 + z_2$, (b) $z_1 - z_2$, (c) $z_1 z_2$, and (d) $\frac{z_1}{z_2}$.

29. $z_1 = 2 + 3i$, $z_2 = 4 - 5i$

30. $z_1 = 6 + 2i$, $z_2 = 5 - 7i$

31. $z_1 = -1 + 2i$, $z_2 = 6 + 7i$

32. $z_1 = -3 - i$, $z_2 = 4 + 8i$

33. Find the product of the complex number $a + bi$ and its complex conjugate, and explain why the answer is a *real* number.

34. Use the results of Problem 33 to factor the *sum* of two squares, $x^2 + y^2$.

1. i
2. $-i$
3. $-i$
4. i
5. -1
6. -1
7. 1
8. 1
9. 1
10. -1
11. i
12. $-i$
13. -1
14. -1
15. $4i$
16. $5i$
17. $3i\sqrt{2}$
18. $4i\sqrt{3}$
19. $i\sqrt{7}$
20. $i\sqrt{3}$
21–28. See Additional Answers.
29. a. $6 - 2i$ **b.** $-2 + 8i$
c. $23 + 2i$ **d.** $\frac{-7}{41} + \frac{22}{41}i$
30. a. $11 - 5i$ **b.** $1 + 9i$
c. $44 - 32i$ **d.** $\frac{8}{37} + \frac{26}{37}i$
31. a. $5 + 9i$ **b.** $-7 - 5i$
c. $-20 + 5i$ **d.** $\frac{8}{85} + \frac{19}{85}i$
32. a. $1 + 7i$ **b.** $-7 - 9i$
c. $-4 - 28i$ **d.** $-\frac{1}{4} + \frac{1}{4}i$
33. $(a + bi)(a - bi) = a^2 - b^2i^2$
$= a^2 + b^2$.
Since a and b are real numbers, and the set of real numbers is *closed* under multiplication and addition, $a^2 + b^2$ is also a real number.
34. $(x + yi)(x - yi)$

Section I: Do #1–25, odds

For Problems 1 through 12,
a. Solve the equation.
b. Check one of the solutions.

1. $x^2 - 14x + 58 = 0$
2. $x^2 - 6x + 73 = 0$
3. $x^2 - 10x + 26 = 0$
4. $x^2 - 14x + 50 = 0$
5. $9x^2 + 12x + 68 = 0$
6. $9x^2 + 90x + 226 = 0$
7. $2x^2 - 3x - 5 = 0$
8. $4x^2 - 21x - 18 = 0$
9. $x^2 - 3x + 41 = x + 12$
10. $x^2 + 5x + 50 = 3x - 15$
11. $3x(x + 5) + 2x^2 = 8x - 11$
12. $8(x - 1)^2 = 7x - 32$

23. ***Complex Conjugates Problem***
   a. Write the complex conjugate of $4 + 7i$.
   b. Write the complex conjugate of $3 - 8i$.
   c. Do the multiplying: $(7 + 3i)(7 - 3i)$. What do you notice about the answer?
   d. Do the addition: $(11 + 5i) + (11 - 5i)$. What do you notice about the answer?
   e. Do the subtraction: $(6 + 10i) - (6 - 10i)$. What do you notice about the answer?
   f. Prove that the sum and the product of two complex conjugates is always a *real* number, and the difference between a complex number and its conjugate is always a pure imaginary number.

25. ***Powers of* i *Problem*** The definition of i makes i^2 equal to -1. Since $i^3 = i^2 \cdot i$, it follows that $i^3 = -i$.
   a. Evaluate each positive integer power of i from i^4 through i^{10}.
   b. Describe the pattern that shows up in the powers of i.
   c. Show that i and i^0 both fit the pattern in part (a).
   d. Quick! Tell what i^{100} will equal.
   e. What will i^{2001} equal? What will i^{137} equal? What will i^{50} equal?

25. *Powers of* i *Problem*
a. $i^4 = 1$
$i^5 = i$
$i^6 = -1$
$i^7 = -i$
$i^8 = 1$
$i^9 = i$
$i^{10} = -1$
b. For an integer n:
$i^{4n} = 1$
$i^{4n+1} = i$
$i^{4n+2} = -1$
$i^{4n+3} = -i$
c. $i = i^{4(0)+1} = i$
$i^0 = i^{4(0)} = 1$ The zero power of any number is 1.
d. 1
e. $i^{2001} = i$
$i^{137} = i$
$i^{50} = -1$

23. *Complex Conjugates Problem*
a. $4 - 7i$
b. $3 + 8i$
c. 58
The product is real.
d. 22
The sum is real.
e. $20i$
The difference is imaginary.
f. $(a + bi)(a - bi)$
$= a^2 + abi - abi - b^2i^2$
Distributive axiom applied twice
$= a^2 - b^2i^2$
Combine like terms
$= a^2 + b^2$
Definition of i
$a^2 + b^2$ is real
Closure of reals under addition and multiplication
Q.E.D.
$(a + bi) - (a - bi)$
$= a + bi - a + bi$
Distributive axiom
$= 2bi$
Combine like terms
$2b$ is real

1. **a.** $\{7 + 3i, 7 - 3i\}$
b. $0 = 0$
2. **a.** $\{3 + 8i, 3 - 8i\}$
b. $0 = 0$
3. **a.** $\{5 + i, 5 - i\}$
b. $0 = 0$
4. **a.** $\{7 + i, 7 - i\}$
b. $0 = 0$
5. **a.** $\left\{\frac{-2 + 8i}{3}, \frac{-2 - 8i}{3}\right\}$
b. $0 = 0$
6. **a.** $\left\{-5 + \frac{1}{3}i, -5 - \frac{1}{3}i\right\}$
b. $0 = 0$
7. **a.** $\{2.5, -1\}$
b. $0 = 0$
8. **a.** $\left\{6, -\frac{3}{4}\right\}$
b. $0 = 0$

9. **a.** $\{2 + 5i, 2 - 5i\}$
b. $14 + 5i = 14 + 5i$ ✓
10. **a.** $\{-1 + 8i, -1 - 8i\}$
b. $-18 - 24i = -18 - 24i$ ✓
11. **a.** $\left\{\frac{-7 + i\sqrt{171}}{10}, \frac{-7 - i\sqrt{171}}{10}\right\}$
b. $-16.6 + 8i\sqrt{1.71} =$
$-16.6 + 8i\sqrt{1.71}$ ✓
12. **a.** $\left\{\frac{23 + i\sqrt{751}}{16}, \frac{23 - i\sqrt{751}}{16}\right\}$
b. $-21.93\ldots + 7i\sqrt{2.93\ldots} =$
$-21.93\ldots + 7i\sqrt{2.93\ldots}$ ✓