~~Section 5.7~~
~~p. 210~~

2. ***Bathtub Problem*** Assume that the number of liters of water remaining in the bathtub varies quadratically with the number of minutes which have elapsed since you pulled the plug.

a. If the tub has 38.4, 21.6, and 9.6 liters remaining at 1, 2, and 3 minutes respectively, since you pulled the plug, write an equation expressing liters in terms of time.

b. How much water was in the tub when you pulled the plug?

c. When will the tub be empty?

d. In the real world, the number of liters would never be negative. What is the lowest number of liters the *model* predicts? Is this number reasonable?

e. Draw a graph of the function in the appropriate domain.

f. Why is a quadratic function more reasonable for this problem than a linear function would be?

(time, liters) (1, 38.4) (2, 21.6) (3, 9.6)

$L(t) = (\_\_)^2 + (\_\_) + \_\_$

a)

$$\boxed{\begin{aligned} 38.4 &= a1^2 + b + c \\ 21.6 &= a4 + 2b + c \\ 9.6 &= 9a + 3b + c \end{aligned}}$$

$$\begin{aligned} 21.6 &= 4a + 2b + c \\ -9.6 &= -9a - 3b - c \\ \hline 12 &= -5a - b \end{aligned}$$

$$\begin{aligned} 38.4 &= a + b + c \\ -21.6 &= -4a - 2b - c \\ \hline 16.8 &= -3a - b \\ -12.0 &= 5a + b \\ \hline 4.8 &= 2a \\ 2.4 &= a \end{aligned}$$

$$\begin{aligned} 12 &= -5(2.4) - b \\ 24 &= -b \\ -24 &= b \end{aligned}$$

$9.6 = 9(2.4) + 3(-24) + c$

$c = 60$

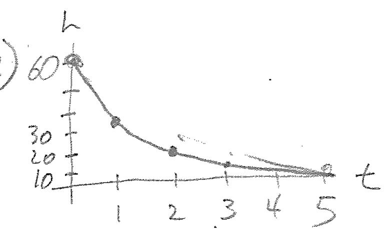
$\boxed{L(t) = 2.4x^2 - 24x + 60}$ ← can use calc. to do regression.

b) 60L $t=0$

c) $0 = 2.4x^2 - 24x + 60$ use Quad. Form. Ans ≈ 5 min.

d) 0 liters, reasonable b/c all water drained.

e)




f) more $H_2O$ → more pressure → faster
less $H_2O$ → less pressure → slower
$H_2O$ not drain at constant rate

4. *Cost of Operating a Car Problem* The number of cents per kilometer it costs to drive a car depends on how fast you drive it. At low speeds the cost is high because the engine operates inefficiently, while at high speeds the cost is high because the engine must overcome high wind resistance. At moderate speeds the cost reaches a minimum. Assume, therefore, that the number of cents per kilometer varies *quadratically* with the number of kilometers per hours (kph).

a. Suppose that it costs 28, 21, and 16 cents per kilometer to drive at 10, 20, and 30 kph, respectively. Write the particular equation for this function.

b. How much would you spend to drive at 150 kph?

c. Between what two speeds must you drive to keep your cost no more than 13 cents per kilometer?

d. Is it possible to spend only 10 cents per kilometer? Justify your answer.

e. The *least* number of cents per kilometer occurs when you get the *most* kilometers per liter of gas. If your tank were nearly empty, at what speed should you drive to have the best chance of making it to a gas station before you run out? → 50Kph

a) (speed, cost) (10, 28¢) (20, 21¢) (30, 16¢)
kph

$C = \_\_ S^2 + \_\_ S + \_\_$

regression calc:

$\boxed{C = .01S^2 - S + 37}$

b) \$1.12/KM ← driving fast is expensive!
at 150 kph

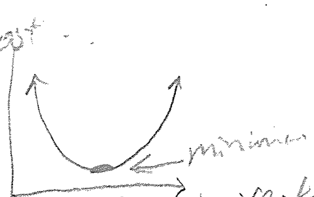
c) $C \leq 13$, 60 kph
40

d) $C = 10$ $\quad 10 = .01S^2 - S + 37$ → $0 = S^2 - 100S + 2700$

$a = 1,\ b = -100,\ c = 2700$

$S = \dfrac{100 \pm \sqrt{100^2 - 4(2700)}}{2}$

$\dfrac{100 \pm \sqrt{-800}}{2}$ → NO, 10¢/km is not possible

e) 


vertex (speed, cost) $\boxed{S = 50\text{ Kph}}$

$x = \dfrac{-1}{2(.01)} = 50$

$y = 12¢/\text{km}$