Answer key with it.

ADVANCED ALGEBRA II (H) **NAME**______________________________

CHAPTER 10 REVIEW **DATE**______________________ **PD**_______

1. How many roots does $9x - 5x^2 + x^3 - 45$ have? What are they?

$(x^3 - 5x^2 + 9x - 45)$ $\quad x^2 = -9$ $\quad \boxed{\text{roots: } 3i, -3i, 5}$

$x^2(x-5) + 9(x-5)$ $\quad x = \pm 3i$

$(x^2+9)(x-5)$ ← factored form

2. Find a polynomial, in lowest degree, which has $1 - \sqrt{2}$, i, and $1 + 4i$ as roots. (no conjugates)

not lowest degree → multiply out

point to examples lowest degree

$(x-(1-\sqrt{2}))(x-i)(x-(1+4i))$

$\boxed{(x-1+\sqrt{2})(x-i)(x-1-4i) = P(x)}$

← factored form.

3. Find a polynomial with rational coefficients, in lowest degree, with $1 + i$ and $2 - \sqrt{5}$ as roots.

Do use conjugates.

$P(x) = (x-(1+i))(x-(1-i))(x-(2-\sqrt{5}))(x-(2+\sqrt{5}))$ → $\left(P(x) = 2x^2 - 6x + 1\right)$

$\boxed{P(x) = (x-1-i)(x-1+i)(x-2+\sqrt{5})(x-2-\sqrt{5})}$

4. Find all roots of: $100 + 4x^2 + 100x + 4x^3 + 25x^2 + x^4 = 0$

$x^4 + 4x^3 + 29x^2 + 100x + 100 = 0$

b: $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20, \pm25, \pm50, \pm100$

$P(-2) = 0$

-2	1	4	29	100	100
		-2	-4	-50	-100
-2	1	2	25	50	0
		-2	0	-50	
	1	0	25	0	

$x^2 + 25$

$x^2 = -25$

$x^2 = (5i)^2$

$(x+5i)(x-5i)$

$\boxed{\text{roots: } -2, -2, -5i, 5i}$

5. Find a polynomial, in lowest degree, which has $4 - \sqrt{3}$ and $9 - 2i$ as roots. no conjugates

$(x-(4-\sqrt{3}))(x-(9-2i))$

$\boxed{(x-4+\sqrt{3})(x-9+2i) = P(x)}$

6. Find the factors of a polynomial with rational coefficients, in lowest degree, with $5 - 12i$ and $3 + \sqrt{2}$ as roots.

$P(x) = (x-(5-12i))(x-(5+12i))(x-(3+\sqrt{2}))(x-(3-\sqrt{2}))$

$\boxed{P(x) = (x-5+12i)(x-5-12i)(x-3-\sqrt{2})(x-3+\sqrt{2})}$

7. Factor completely: $x^3 - 5x^2 + 9x - 5 = 0$

$b = \pm 1, \pm 5$

$b = 1$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 9 & -5 \\ & & 1 & -4 & 5 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

$x = 1$

$x^2 - 4x + 5$

$x = \frac{4 \pm \sqrt{16-20}}{2}$

$= \frac{4 \pm \sqrt{-4}}{2}$

$= 2 \pm i$

$\boxed{(x-1)(x-2+i)(x-2-i)}$ factors

8. Find all the roots of: $x^4 + x^3 + x^2 - 9x - 10 = 0$

$\pm 1, \pm 2, \pm 5, \pm 10$

$b = 2 \rightarrow$

$$\begin{array}{r|rrrrr} 2 & 1 & 1 & 1 & -9 & -10 \\ & & 2 & 6 & 14 & 10 \\ \hline & 1 & 3 & 7 & 5 & 0 \end{array}$$

$x^3 + 3x^2 + 7x + 5$

$$\begin{array}{r|rrrr} -1 & 1 & 3 & 7 & 5 \\ & & -1 & -2 & -5 \\ \hline & 1 & 2 & 5 & 0 \end{array}$$

$x^2 + 2x + 5$

$x = \frac{-2 \pm \sqrt{4 - 4(5)}}{2}$

$= -1 \pm 2i$

$\boxed{\text{roots } \{-1, 2, -1+2i, -1-2i\}}$

9. Find a polynomial, in lowest degree, which has $3 - \sqrt{5}$ and $8 + 3i$ as roots.

$\boxed{P(x) = (x - 3 + \sqrt{5})(x - 8 - 3i)}$

10. Find factors of a polynomial with rational coefficients, in lowest degree, with $3 - 5i$ and $8 + \sqrt{5}$ as roots.

$\boxed{P(x) = (x-3-5i)(x-3+5i)(x-8-\sqrt{5})(x-8+\sqrt{5})}$

11. Use $P(x) = x^4 - 5x^3 + 7x^2 - 5x + 6$ to answer the following:

The roots of P(x) are $i, -i, 3, 2$

a) Which of these roots, if any, are rational? 3 and 2

b) Which of these roots, if any, are not rational? $i, -i$

12. Use $P(x) = x^5 - 2x^4 - 3x^3 + 6x^2 - 4x + 8$ to answer the following:

a) How many roots does this polynomial have? How do you know?

5 by using the degree

b) Given the roots: -2, $-i$, and 2 (multiplicity 2), how many more roots does P(x) have? Find them.

one more; it must be $-i$.

c) How did you find the missing root(s) in part b?

The coefficients are all rational. So, if one root is complex, then there must be another root that is complex, and is its conjugate.

$$x = \frac{6 \pm \sqrt{36-52}}{2}$$

$$= \frac{6}{2} \pm \frac{\sqrt{-16}}{2}$$

$$= 3 \pm 2i$$

13. Factor completely: $z^3 - 5z^2 + 7z + 13 = 0$

$b = \pm 1, \pm 13$

$P(-1) = 0$

$$\begin{array}{r|rrrr} -1 & 1 & -5 & 7 & 13 \\ & & -1 & 6 & -13 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

$x^2 - 6x + 13$

$\boxed{\text{Factors: } (z+1)(z-3-2i)(z-3+2i)}$

14. Find all the roots of $x^4 - 3x^3 + 7x^2 + 21x - 26 = 0$.

$b = \pm 1, \pm 2, \pm 13, \pm 26$

$P(1) = 0$

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & 7 & 21 & -26 \\ & & 1 & -2 & 5 & 26 \\ \hline & 1 & -2 & 5 & 26 & \underline{|0} \end{array}$$

$x^3 - 2x^2 + 5x + 26$

$$\begin{array}{r|rrrr} -2 & 1 & -2 & 5 & 26 \\ & & -2 & 8 & -26 \\ \hline & 1 & -4 & 13 & \underline{|0} \end{array}$$

$x^2 - 4x + 13$

$$x = \frac{4 \pm \sqrt{16-52}}{2}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$= 2 \pm 3i$$

$\boxed{\text{roots } \{1, -2, 2+3i, 2-3i\}}$

15. If $P(x) = x^3 - x^2 + x - 1$ and one of the roots is equal to 1, find the other roots of $P(x)$.

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 1 & -1 \\ & & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & \underline{|0} \end{array}$$

$x^2 + 1$

$x^2 - -1 \rightarrow x^2 - \sqrt{-1}^2$

$(x+i)(x-i)$

roots: $1, i, -i$

$\boxed{\text{other roots: } 1, i, -i}$

16. Find a polynomial of degree 5 with -1 as a root (multiplicity 3), 0 as a root (multiplicity 1), and 4 as a root (multiplicity 1). * does not say "lowest degree"

$P(x) = (x+1)^3 x(x-4)$

$= (x^3 + 3x^2 + 3x + 1)(x^2 - 4x)$

$= x^5 + 3x^4 + 3x^3 + x^2 - 4x^4 - 12x^3 - 12x^2 - 4x$

$\boxed{P(x) = x^5 - x^4 - 9x^3 - 11x^2 - 4x}$

17. If $P(x) = x^4 - 5x^3 + 10x^2 - 20x + 24$, then find all the roots given that $2i$ is a root. real coeff $\Rightarrow$ complex conjugat

$$\begin{array}{r|rrrrr} 2i & 1 & -5 & 10 & -20 & 24 \\ & & 2i & -4-10i & 20+12i & -24 \\ \hline & 1 & -5+2i & 6-10i & 12i & \underline{|0} \end{array}$$

$$\begin{array}{r|rrrr} -2i & 1 & -5+2i & 6-10i & 12i \\ & & -2i & 10i & -12i \\ \hline & 1 & -5 & 6 & \underline{|0} \end{array}$$

Conjugate

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0.$

$\boxed{\text{roots: } 2i, -2i, 3, 2}$

18. Find all the roots of $x^3 - 7x^2 + 17x - 15$ if $2 - i$ is one root.

Done in 2 ways:

$P(x) = (x-2-i)(x-2+i) \cdot Q(x)$

$= ((x-2)^2 - i^2) \cdot Q(x)$

$= (x^2 - 4x + 4 + 1) \cdot Q(x)$

$= (x^2 - 4x + 5) \cdot Q(x)$

$\leftarrow$ quotient

$$\begin{array}{r} x - 3 \\ x^2-4x+5 \overline{) \, x^3 - 7x^2 + 17x - 15} \\ - \; x^3 - 4x^2 + 5x \\ \hline -3x^2 + 12x - 15 \\ -3x^2 + 12x - 15 \\ \hline 0 \end{array}$$

OR

$$\begin{array}{r|rrrr} 2-i & 1 & -7 & 17 & -15 \\ & & 2-i & 3i-11 & \\ \hline & 1 & -5-i & 3i+6 & \end{array}$$

$\boxed{\text{roots } (2-i), (2+i), (3)}$

19. Find all the **possible** rational roots of: $P(x) = x^4 + 2x^3 + 5x^2 + 34x + 30$

$b = \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

actual: $-1, -3, 1+3i, 1-3i$

20. Find a polynomial of degree 3 with roots 0, 1 and *i*.

$$P(x) = (x-0)(x-1)(x-i)$$

$$x(x^2 - x - ix + i)$$

$$x^3 - x^2 - ix^2 + ix \longrightarrow \boxed{P(x) = x^3 - (1+i)x^2 + ix}$$

Just a division problem

21. $(2x^4 + 7x^3 + x - 12) \div (x + 3)$

-3	2	7	0	1	-12
		-6	-3	9	-30
	2	1	-3	10	-42

$$\boxed{P(x) = 2x^3 + x^2 - 3x + 10 - \frac{42}{(x+3)}}$$

22. $P(x) = x^3 + 7x^2 - 12x - 3$. Find $P(-3)$, $P(-2)$, and $P(1)$.

-3	1	7	-12	-3
		-3	-12	72
	1	4	-24	69

-2	1	7	-12	-3
		-2	-10	44
	1	5	-22	41

1	1	7	-12	-3
		1	8	-4
	1	8	-4	-7

$\boxed{P(-3) = 69 \qquad P(-2) = 41 \qquad P(1) = -7}$

For questions 23-25, find the roots stating the multiplicity of each.

23. $(x^2 - 7x + 12)^2 = 0$

$$[(x-4)(x-3)]^2 = 0$$

$$(x-4)^2(x-3)^2 = 0$$

$\boxed{3 \text{ (multiplicity 2)}, 4 \text{ (multiplicity 2)}}$

24. $x^3(x - 1)(x + 4) = 0$

$\boxed{0 \text{ (multiplicity 3)}, 1 \text{ (multiplicity 1)}, -4 \text{ (multiplicity 1)}}$

25. $-8(x - 3)^2(x + 4)^3 x^4 = 0$

$\boxed{0 \text{ (multiplicity 4)}, -4 \text{ (multiplicity 3)}, 3 \text{ (multiplicity 2)}}$