AA2H * Go over b, e, 2, 10

Tell class: given radical, end up radical; given rat exp, end up rat exp

Name KEY

Chapter: Rational Exponents and Radical Functions

date___________

## (5.2) Using Properties of Rational Exponents and Radicals

Warm-Up: Use the properties to simplify each expression.

**a.** $7^{1/4} \cdot 7^{1/2} =$ $7^{\frac{3}{4}}$

Go over with class

**b.** $(6^{1/2} \cdot 4^{1/3})^2 =$ $6 \cdot 4^{\frac{2}{3}} \to 6\cdot\sqrt[3]{16} = 6\sqrt[3]{8\cdot 2} = \boxed{12\sqrt[3]{2}} = \boxed{12\cdot 2^{\frac{1}{3}}}$

$6\cdot(2^2)^{\frac{2}{3}} = 6\cdot 2^{\frac{4}{3}}$

$6\cdot 2\cdot 2^{\frac{1}{3}}$

$\boxed{12\cdot 2^{\frac{1}{3}}}$

**c.** $(4^5 \cdot 3^5)^{-1/5} =$ $4^{-1}\cdot 3^{-1} = \boxed{\frac{1}{12}}$

**d.** $\frac{5}{5^{1/3}} = \frac{5^1}{5^{1/3}} =$ $5^{\frac{2}{3}}$

**e.** $\left(\frac{42^{1/3}}{6^{1/3}}\right)^2 =$ $\left(\frac{42}{6}\right)^{\frac{2}{3}} = \boxed{7^{\frac{2}{3}}}$

distributive property

### Using Properties of Radicals

Go over rationalizing

An expression involving a radical with index $n$ is in **simplest form** when these three conditions are met.

- No radicands have perfect $n$th powers as factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

To meet the last two conditions, rationalize the denominator by multiplying the expression by an appropriate form of 1 that eliminates the radical from the denominator.

Adding and subtracting: add or subtract only radicals with like index numbers and like radicands.

### Writing Radicals in Simplest Form

1. $\sqrt[3]{135} =$ $\sqrt[3]{27\cdot 5}$
   $\boxed{3\sqrt[3]{5}}$

2. $\sqrt[5]{7} \div \sqrt[5]{8}$ $\quad \underbrace{\frac{\sqrt[5]{7}}{\sqrt[5]{8}}}_{\sqrt[5]{32}\ \leftarrow \text{step 1}} \cdot \frac{\sqrt[5]{4}}{\sqrt[5]{4}} = \frac{\sqrt[5]{28}}{\sqrt[5]{32}} = \boxed{\frac{\sqrt[5]{28}}{2}}$

   step 2

3. $\frac{1}{2+\sqrt[3]{3}}$ $\cdot\frac{(2-\sqrt{3})}{(2-\sqrt{3})} = \frac{2-\sqrt{3}}{4-3} = \boxed{2-\sqrt{3}}$

4. $\sqrt[4]{10} + 7\sqrt[4]{10}$ $= \boxed{8\sqrt[4]{10}}$

5. $2(8^{\frac{1}{5}}) + 10(8^{\frac{1}{5}})$ $= \boxed{12(8^{\frac{1}{5}})}$

   $12\cdot 2^{3(\frac{1}{5})}$

   $12\cdot 2^{\frac{3}{5}}$

6. $\sqrt[3]{54} - \sqrt[3]{2}$

   $\sqrt[3]{27\cdot 2} - \sqrt[3]{2}$

   $3\sqrt[3]{2} - \sqrt[3]{2}$

   $\boxed{2\sqrt[3]{2}}$

## Simplifying Variable Expressions

Go over

$\sqrt[n]{x^n} = x$

$\sqrt[n]{x^{n+y}} = x\sqrt[n]{x^y}$

ex: $\sqrt[3]{x^7} = x^2\sqrt[3]{x}$

The properties of rational exponents and radicals can also be applied to expressions involving variables. Because a variable can be positive, negative, or zero, sometimes absolute value is needed when simplifying a variable expression.

| | Rule | Example |
|---|---|---|
| When $n$ is odd | $\sqrt[n]{x^n} = x$ | $\sqrt[7]{5^7} = 5$ and $\sqrt[7]{(-5)^7} = -5$ |
| When $n$ is even | $\sqrt[n]{x^n} = \lvert x \rvert$ | $\sqrt[4]{3^4} = 3$ and $\sqrt[4]{(-3)^4} = 3$ |

Absolute value is not needed when all variables are assumed to be positive.

7. $\sqrt[3]{64y^6} = 4y^2$

8. $\sqrt[4]{\dfrac{x^4}{y^8}} = \boxed{\dfrac{x}{y^2}}$ $\quad \dfrac{|x|}{y^2}$

9. $\sqrt[5]{4a^8b^{14}c^5}$

$ab^2c\sqrt[5]{4a^3b^4}$

10. $\dfrac{x}{\sqrt[3]{y^8}} = \dfrac{x}{y^2\sqrt[3]{y^2}} \cdot \dfrac{\sqrt[3]{y}}{\sqrt[3]{y}}$

$= \boxed{\dfrac{x\sqrt[3]{y}}{y^3}}$

11. $\dfrac{14xy^{1/3}}{2x^{3/4}z^{-6}}$

$\boxed{7x^{\frac{1}{4}}y^{\frac{1}{3}}z^6}$

**COMMON ERROR**

You must multiply both the numerator *and* denominator of the fraction by $\sqrt[3]{y}$ so that the value of the fraction does not change.

**SOLUTION**

a. $\sqrt[5]{4a^8b^{14}c^5} = \sqrt[5]{4a^5a^3b^{10}b^4c^5}$ Factor out perfect fifth powers.

$= \sqrt[5]{a^5b^{10}c^5} \cdot \sqrt[5]{4a^3b^4}$ Product Property of Radicals

$= ab^2c\sqrt[5]{4a^3b^4}$ Simplify.

b. $\dfrac{x}{\sqrt[3]{y^8}} = \dfrac{x}{\sqrt[3]{y^8}} \cdot \dfrac{\sqrt[3]{y}}{\sqrt[3]{y}}$ Make denominator a perfect cube.

$= \dfrac{x\sqrt[3]{y}}{\sqrt[3]{y^9}}$ Product Property of Radicals

$= \dfrac{x\sqrt[3]{y}}{y^3}$ Simplify.

c. $\dfrac{14xy^{1/3}}{2x^{3/4}z^{-6}} = 7x^{(1-3/4)}y^{1/3}z^{-(-6)} = 7x^{1/4}y^{1/3}z^6$

27·2

12. $5\sqrt{y} + 6\sqrt{y} = 11\sqrt{y}$

13. $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2}$

$12z\sqrt[3]{2z^2} - 3z\sqrt[3]{2z^2}$

$\boxed{9z\sqrt[3]{2z^2}}$