

Practice You Would Need Ask for

① a) $x^{\frac{5}{2}}$ b) $y^{\frac{2}{3}}$ c) $(2a)^{\frac{5}{6}}$

d) $x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} \cdot x^{\frac{1}{6}}$

$x^{\frac{1}{2} + \frac{1}{3} + \frac{1}{6}} = \boxed{x}$

② a) $8^{\frac{1}{3}} x^{\frac{7}{3}}$
 $\boxed{2x^{\frac{7}{3}}}$

b) $16^{\frac{3}{4}} x^{\frac{3}{4}}$
 $(2^4)^{\frac{3}{4}} x^{\frac{3}{4}}$
 $\boxed{8x^{\frac{3}{4}}}$

c) $27^{\frac{1}{3}} x^{-\frac{6}{3}} y^{\frac{2}{3}}$
 $\boxed{\frac{3y^{\frac{2}{3}}}{x^2}}$

d) $x^{\frac{1}{4}} \cdot x^{\frac{1}{3}}$
 $\frac{x^{\frac{1}{4}} \cdot x^{\frac{1}{3}}}{x^{\frac{1}{6}}}$

$\frac{x^{\frac{1}{4} + \frac{1}{3}}}{x^{\frac{1}{6}}} = x^{\frac{1}{12} + \frac{1}{12} - \frac{1}{12}} = \boxed{x^{\frac{1}{12}}}$

③ $8^{\frac{2}{3}} = 2^2 = \boxed{4}$

④ $4^{\frac{3}{2}} = 2^3 = \boxed{8}$

⑤ $(-27)^{\frac{1}{3}} = ((-3)^3)^{\frac{1}{3}} = \boxed{-3}$

⑥ $16^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^3 = \boxed{8}$

⑦ $16^{\frac{3}{2}} = 4^3 = \boxed{64}$

⑧ $64^{\frac{3}{2}} = (2^6)^{\frac{3}{2}} = 2^9 = \boxed{512}$

or $(4^2)^{\frac{3}{2}} = 4^3 = \boxed{512}$

⑨ $9^{-\frac{3}{2}} = (3^2)^{-\frac{3}{2}} = 3^{-3} = \frac{1}{3^3} = \boxed{\frac{1}{27}}$

⑩ $25^{-\frac{5}{2}} = (5^2)^{-\frac{5}{2}} = 5^{-5} = \boxed{\frac{1}{3125}}$

$(\sqrt{25})^{-5} = 5^{-5}$

⑪ $\left(\frac{9}{8}\right)^{\frac{3}{2}} = \frac{3^{2(\frac{3}{2})}}{2^{3(\frac{3}{2})}} = \frac{3^3}{2^{\frac{9}{2}}} = \frac{27}{2^4 2^{\frac{1}{2}}}$

$= \frac{27}{16\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{27\sqrt{2}}{32}}$

⑫ $\left(\frac{27}{8}\right)^{\frac{2}{3}} = \left(\frac{3^3}{2^3}\right)^{\frac{2}{3}} = \frac{3^2}{2^2} = \boxed{\frac{9}{4}}$

⑬ $\left(\frac{8}{9}\right)^{-\frac{3}{2}} = \left(\frac{9}{8}\right)^{\frac{3}{2}} = \frac{(3^2)^{\frac{3}{2}}}{(2^3)^{\frac{3}{2}}} = \frac{3^3}{2^{\frac{9}{2}}} = \frac{27}{2^4 2^{\frac{1}{2}}} = \frac{27}{16\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{27\sqrt{2}}{32}}$

⑭ $\left(\frac{8}{27}\right)^{-\frac{2}{3}} = \left(\frac{27}{8}\right)^{\frac{2}{3}} = \frac{(3^3)^{\frac{2}{3}}}{(2^3)^{\frac{2}{3}}} = \frac{3^2}{2^2} = \boxed{\frac{9}{4}}$

d) $\left(\frac{25}{9}\right)^{\frac{3}{2}} = \frac{5^{2(\frac{3}{2})}}{3^{2(\frac{3}{2})}} = \frac{5^3}{3^3} = \boxed{\frac{125}{27}}$

⑮ a) $\frac{3^{2(\frac{1}{2})}}{5^{2(\frac{1}{2})}} = \boxed{\frac{3}{5}}$

b) $\left(\frac{25}{9}\right)^{\frac{1}{2}} = \frac{5^{2(\frac{1}{2})}}{3^{2(\frac{1}{2})}} = \boxed{\frac{5}{3}}$

c) $\frac{3^{2(\frac{5}{2})}}{5^{2(\frac{5}{2})}} = \frac{3^5}{5^5} = \boxed{\frac{243}{3125}}$

↑

$$\textcircled{16} \text{ a) } \left(\frac{27}{8}\right)^{\frac{1}{3}} = \frac{3^{3(\frac{1}{3})}}{2^{3(\frac{1}{3})}} = \boxed{\frac{3}{2}} \quad \text{b) } \left(\frac{27}{8}\right)^{\frac{2}{3}} = \frac{3^{3(\frac{2}{3})}}{2^{3(\frac{2}{3})}} = \frac{3^2}{2^2} = \boxed{\frac{9}{4}}$$

$$\text{c) } \left(\frac{27}{8}\right)^{-\frac{2}{3}} = \left(\frac{8}{27}\right)^{\frac{2}{3}} = \frac{(2^3)^{\frac{2}{3}}}{(3^3)^{\frac{2}{3}}} = \frac{2^2}{3^2} = \boxed{\frac{4}{9}} \quad \text{d) } 1$$

$$\textcircled{17} \left(16^{-\frac{3}{5}}\right)^{\frac{5}{4}} \Rightarrow \left(2^4\right)^{-\frac{3}{4}} \Rightarrow 2^{-3} \Rightarrow \frac{1}{2^3} \Rightarrow \boxed{\frac{1}{8}}$$

$$\textcircled{18} \left(25^{-\frac{1}{2}}\right)^{-\frac{3}{2}} \Rightarrow (25)^{\frac{1}{2}} \Rightarrow \boxed{5}$$

$$\textcircled{19} \left(81^{\frac{1}{2}} - 9^{\frac{1}{2}}\right)^2 \Rightarrow (9 - 3)^2 \Rightarrow 6^2 \Rightarrow \boxed{36}$$

$$\textcircled{20} \left(3^{-2} + 4^{-2}\right)^{-\frac{1}{2}} \Rightarrow \left(\frac{1}{9} + \frac{1}{16}\right)^{-\frac{1}{2}} \Rightarrow \left(\frac{16}{144} + \frac{9}{144}\right)^{-\frac{1}{2}} \Rightarrow \left(\frac{25}{144}\right)^{-\frac{1}{2}} \Rightarrow \left(\frac{144}{25}\right)^{\frac{1}{2}} \Rightarrow \boxed{\frac{12}{5}}$$

$$\textcircled{21} \left(8a^{-6}\right)^{-\frac{2}{3}} \Rightarrow 8^{-\frac{2}{3}} (a^{-6})^{-\frac{2}{3}} \Rightarrow (2^3)^{-\frac{2}{3}} a^4 \Rightarrow 2^{-2} a^4 \Rightarrow \boxed{\frac{a^4}{4}}$$

$$\textcircled{22} \left(9n^{-5}\right)^{-\frac{3}{2}} \Rightarrow 9^{-\frac{3}{2}} n^{-5(-\frac{3}{2})} \Rightarrow 3^{2(-\frac{3}{2})} n^{\frac{15}{2}} \Rightarrow 3^{-3} n^{\frac{15}{2}} \Rightarrow \boxed{\frac{n^{\frac{15}{2}}}{27}}$$

$$\textcircled{23} \left(4x^{-3}\right)^{-\frac{1}{2}} \cdot 4x^{\frac{1}{2}} \rightarrow 4^{-\frac{1}{2}} x^{-3(-\frac{1}{2})} \cdot 4x^{\frac{1}{2}} \rightarrow \frac{1}{2} x^{\frac{3}{2}} (4) x^{\frac{1}{2}} \rightarrow \boxed{2x^2}$$

$$\textcircled{24} \left(4a^3\right)^{\frac{1}{3}} \div \left(4a^3\right)^{-\frac{2}{3}} \rightarrow \frac{4^{\frac{1}{3}} a}{4^{-\frac{2}{3}} a^{-2}} \rightarrow 4^{\frac{1}{3} + \frac{2}{3}} a \cdot a^2 \rightarrow \boxed{4a^3}$$

$$\textcircled{25} x^{\frac{3}{4}} x^{\frac{1}{3}} x^{-\frac{1}{2}} \rightarrow x^{\frac{9}{12} + \frac{4}{12} + -\frac{6}{12}} \rightarrow \boxed{x^{\frac{7}{12}}}$$

$$\textcircled{26} x^{\frac{2}{3}} x^{\frac{1}{2}} x^{-\frac{1}{4}} \rightarrow x^{\frac{4}{6} + \frac{3}{6} - \frac{1}{4}} \rightarrow x^{\frac{14}{12} - \frac{3}{12}} \rightarrow \boxed{x^{\frac{11}{12}}}$$

$$\textcircled{27} \left(x^3 y^6\right)^{\frac{1}{3}} \rightarrow \boxed{xy^2} \quad \textcircled{28} x^{4(\frac{3}{4})} y^{8(\frac{3}{4})} \rightarrow \boxed{x^3 y^6}$$

$$\textcircled{29} x^{\frac{2}{3}} y^{\frac{1}{3}} \cdot x^{\frac{2}{3}} y^{\frac{4}{3}} \rightarrow \boxed{x^{\frac{4}{3}} y^{\frac{5}{3}}}$$

$$\textcircled{30} x^{\frac{1}{4}} y^{\frac{1}{4}} x^{2(\frac{1}{2})} y^{2(\frac{1}{2})} \rightarrow x^{\frac{1}{4}} y^{\frac{1}{4}} \rightarrow \boxed{x^{\frac{5}{4}} y^{\frac{5}{4}}}$$

Practice You Would Never Ask For...

$$(31) 16^{\frac{3}{4}} x^{2(\frac{3}{4})} y^{-\frac{1}{4}(\frac{3}{4})} \rightarrow 2^{4(\frac{3}{4})} x^{\frac{3}{2}} y^{-\frac{1}{4}} \rightarrow 8x^{\frac{3}{2}} y^{-\frac{1}{4}} \rightarrow \boxed{\frac{8x^{\frac{3}{2}}}{y^{\frac{1}{4}}}}$$

$$(32) 4^{\frac{3}{2}} x^{-\frac{3}{2}} y^{\frac{1}{2}} \rightarrow 2^{2(\frac{3}{2})} x^{-\frac{3}{2}} y^{\frac{1}{2}} \rightarrow 8x^{-\frac{3}{2}} y^{\frac{1}{2}} \rightarrow \boxed{\frac{8y^{\frac{1}{2}}}{x^{\frac{3}{2}}}}$$

$$(33) a^2 - 2a \quad (34) 2n^{\frac{1}{3}} n^{\frac{2}{3}} + 2n^{\frac{1}{3}} n^{-\frac{1}{3}} \rightarrow 2n + 2n^0 \rightarrow \boxed{2n + 2}$$

$$(35) x^{-\frac{1}{2}} x^{\frac{5}{2}} - 2x^{\frac{3}{2}} x^{-\frac{1}{2}} \rightarrow \boxed{x^2 - 2x}$$

$$(36) 2n^{-\frac{2}{3}} n^{\frac{8}{3}} - 2n^{-\frac{2}{3}} \cdot 3n^{\frac{5}{3}} \rightarrow \boxed{2n^2 - 6n}$$

$$(37) \frac{x^{\frac{1}{2}}}{x^{-\frac{1}{2}}} - \frac{2x^{-\frac{1}{2}}}{x^{-\frac{1}{2}}} \rightarrow x^{\frac{1}{2} + \frac{1}{2}} - 2x^{-\frac{1}{2} + \frac{1}{2}} \rightarrow \boxed{x - 2}$$

$$(38) y^{-\frac{1}{3} + \frac{4}{3}} - 3y^{\frac{2}{3} + \frac{4}{3}} \rightarrow \boxed{y - 3y^2}$$

$$(39) \frac{2n^{\frac{1}{3}}}{2n^{-\frac{2}{3}}} - \frac{4n^{-\frac{2}{3}}}{2n^{-\frac{2}{3}}} \rightarrow n^{\frac{1}{3} + \frac{2}{3}} - 2n^{-\frac{2}{3} + \frac{2}{3}} \rightarrow \boxed{n - 2}$$

$$(40) x^{-\frac{1}{2} + \frac{3}{2}} (2x^{\frac{1}{2}} - x^{-\frac{1}{2}}) \rightarrow x^1 (2x^{\frac{1}{2}} - x^{-\frac{1}{2}}) \rightarrow \boxed{2x - 1}$$

$$(41) \frac{4ab^{-\frac{1}{2}} - 2ab^{\frac{1}{2}}}{(a^2b)^{-\frac{1}{2}}}$$

$$\frac{4ab^{-\frac{1}{2}}}{a^{-1}b^{-\frac{1}{2}}} - \frac{2ab^{\frac{1}{2}}}{a^{-1}b^{-\frac{1}{2}}}$$

$$4a \cdot ab^{-\frac{1}{2} + \frac{1}{2}} - 2a \cdot ab^{\frac{1}{2} - -\frac{1}{2}}$$

$$4a^2b^0 - 2a^2b$$

$$\boxed{4a^2 - 2a^2b}$$

$$(42) \frac{(4a)^{\frac{2}{3}}}{(4a)^{\frac{1}{6}}}$$

$$(4a)^{\frac{2}{3} - \frac{1}{6}}$$

$$(4a)^{\frac{4}{6} - \frac{1}{6}}$$

$$(4a)^{\frac{1}{2}}$$

$$(2^2a)^{\frac{1}{2}}$$

$$\boxed{2a^{\frac{1}{2}}} \text{ or } \boxed{2\sqrt{a}}$$

$$(43) \frac{(2x)^{\frac{5}{2}}}{(2x)^{\frac{9}{2}}}$$

$$(2x)^{\frac{5}{2} - \frac{9}{2}}$$

$$(2x)^{-\frac{4}{2}}$$

$$(2x)^{-2}$$

$$\boxed{\frac{1}{4x^2}}$$

$$\textcircled{44} \quad 8^x = 2^6$$

$$2^{3x} = 2^6$$

$$\boxed{x=2}$$

$$\textcircled{45} \quad 9^{4x} = 81$$

$$9^{4x} = 9^2$$

$$4x=2$$

$$\boxed{x=\frac{1}{2}}$$

$$\textcircled{46} \quad 8^{x-1} = 2^{x+1}$$

$$2^{3(x-1)} = 2^{x+1}$$

$$3x-3 = x+1$$

$$2x=4$$

$$\boxed{x=2}$$

$$\textcircled{47} \quad 3^{2x} = 3^{10}$$

$$\boxed{x=5}$$

$$\textcircled{48} \quad 2^{3x} = 2^7 \cdot 2^{18}$$

$$3x=25$$

$$\boxed{x=\frac{25}{3}}$$

$$\textcircled{49} \quad 3^{3(1-x)} = 3^{-2(2-x)}$$

$$3-3x = -4+2x$$

$$7=5x$$

$$\boxed{x=\frac{7}{5}}$$

this method only
for odd exponents

$$\textcircled{50} \text{ a) } (8x)^{-3(-\frac{1}{3})} = 64^{-\frac{1}{3}}$$

$$8x = 4^{3(-\frac{1}{3})}$$

$$x = \frac{4^{-1}}{8}$$

$$\boxed{x = \frac{1}{32}}$$

$$\textcircled{50b} \quad x^{-3} = 8$$

$$x^{-3(-\frac{1}{3})} = 2^{3(-\frac{1}{3})}$$

$$x = 2^{-1}$$

$$\boxed{x = \frac{1}{2}}$$

$$\textcircled{50c} \quad (8+x)^{-3(-\frac{1}{3})} = 4^{3(-\frac{1}{3})}$$

$$8+x = 4^{-1}$$

$$x = \frac{1}{4} - 8$$

$$\boxed{x = -7\frac{3}{4}}$$

$$\textcircled{51} \text{ a) } (2x)^{-2} = 16$$

$$2x = 4^{2(-\frac{1}{2})}$$

$$2x = 4^{-1}$$

$$\boxed{x = \pm \frac{1}{8}}$$

OR

$$((2x)^{-1})^2 = 4^2$$

$$(2x)^{-1} = \pm 4$$

$$2x = \pm \frac{1}{4}$$

$$\boxed{x = \pm \frac{1}{8}}$$

$$\text{b) } x^{-2} = 8$$

$$x = 2^{3(-\frac{1}{2})}$$

$$x = 2^{-\frac{3}{2}}$$

$$x = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\boxed{x = \frac{\sqrt{2}}{4}}$$

OR

$$(x^{-1})^2 = 8$$

$$x^{-1} = \pm 8^{\frac{1}{2}}$$

$$x = \pm \frac{1}{\sqrt{8}} = \pm \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\pm \frac{\sqrt{2}}{4}}$$

$$\text{c) } 4(x-2)^{-2} = 16$$

$$(x-2)^{-2} = 4$$

$$(x-2)^{-1} = \pm 2$$

$$x-2 = \pm \frac{1}{2}$$

$$\boxed{x = 2\frac{1}{2}} \quad \boxed{x = -\frac{1}{2} + 2}$$

$$\boxed{x = 1\frac{1}{2}}$$

$$(54) \quad 2^{x^2-x} = 2^6$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$(x=3) \quad (x=-2)$$

$$(55) \quad \frac{5^{x^2}}{(5^x)^2} = 125$$

$$5^{x^2-2x} = 5^3$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$(x=3) \quad (x=-1)$$

$$(56) \quad \frac{9^{(x+3)^{\frac{1}{2}}}}{27^{x(\frac{1}{2})}} = 3^4$$

$$\frac{3^{2(\frac{1}{2})(x+3)}}{3^{3(\frac{1}{2})x}} = 3^4$$

$$x+3 - \frac{3}{2}x = 4$$

$$-\frac{1}{2}x = 1$$

$$(x=-2)$$

$$(57) \quad \frac{8^{(x+1)(\frac{1}{3})}}{16^{x(\frac{1}{3})}} = 2^5$$

$$\frac{2^{3(\frac{1}{3})(x+1)}}{2^{\frac{4}{3}x}} = 2^5$$

$$x+1 - \frac{4}{3}x = 5$$

$$-\frac{1}{3}x = 4$$

$$(x=-12)$$

$$(52) \quad a) \quad (4x)^3 = 9^6$$

$$((4x)^3)^{\frac{1}{3}} = (9^6)^{\frac{1}{3}}$$

$$4x = 9^2$$

$$\boxed{x = \frac{81}{4}}$$

$$b) \quad 3^{4x} = 9^6$$

$$3^{4x} = 3^{2(6)}$$

$$4x = 12$$

$$\boxed{x=3}$$

$$(53) \quad a) \quad (4-x)^{\frac{1}{2}} = 8$$

$$((4-x)^{\frac{1}{2}})^2 = 8^2$$

$$4-x = 64$$

$$\boxed{-60=x}$$

$$b) \quad \left(\frac{1}{2}\right)^{4-x} = 8$$

$$2^{-1(4-x)} = 2^3$$

$$-4+x = 3$$

$$\boxed{x=7}$$

$$(58a) \quad a^{\frac{3}{2}}b^{\frac{1}{2}} - a^{\frac{1}{2}}b^{\frac{3}{2}} \rightarrow a^{\frac{1}{2}}b^{\frac{1}{2}}(a-b)$$

III. Factor

$$(58b) \quad a^{\frac{1}{2}}b^{-\frac{1}{2}} - a^{\frac{3}{2}}b^{\frac{1}{2}} \quad \begin{array}{l} \swarrow y \quad \leftarrow -\frac{1}{2} + y = \frac{1}{2} \\ \quad \quad \quad y = \frac{2}{2} \end{array}$$

$$a^{\frac{1}{2}}b^{-\frac{1}{2}}(1 - a^{\frac{2}{2}}b^{\frac{2}{2}}) \rightarrow \boxed{a^{\frac{1}{2}}b^{-\frac{1}{2}}(1-ab)}$$

$$(59a) \quad (x-1)^{\frac{1}{2}} - x(x-1)^{-\frac{1}{2}}$$

$$(x-1)^{-\frac{1}{2}}((x-1)^{\frac{2}{2}} - x) \rightarrow (x-1)^{-\frac{1}{2}}(x-1-x) \rightarrow (x-1)^{-\frac{1}{2}}(-1)$$

$$\hookrightarrow -(x-1)^{-\frac{1}{2}}$$

$$(59b) \quad (x+1)^{\frac{3}{2}} - 4(x+1)^{\frac{1}{2}}$$

$$(x+1)^{\frac{1}{2}}((x+1)^{\frac{2}{2}} - 4)$$

$$(x+1)^{\frac{1}{2}}(x+1-4)$$

$$(x+1)^{\frac{1}{2}}(x-3)$$

$$(60a) \quad (x^2+1)^{\frac{3}{2}} - x^2(x^2+1)^{\frac{1}{2}}$$

$$(x^2+1)^{\frac{1}{2}}((x^2+1)^{\frac{2}{2}} - x^2)$$

$$(x^2+1)^{\frac{1}{2}}(x^2+1-x^2)$$

$$\boxed{(x^2+1)^{\frac{1}{2}}}$$

$$(60b) \quad (x^2+2)^{\frac{1}{2}} - x^2(x^2+2)^{-\frac{1}{2}}$$

$$(x^2+2)^{-\frac{1}{2}}((x^2+2)^{\frac{2}{2}} - x^2)$$

$$(x^2+2)^{-\frac{1}{2}}(x^2+2-x^2)$$

$$\boxed{2(x^2+2)^{-\frac{1}{2}}}$$

$$(61a) \quad (2x+1)^{\frac{2}{3}} - 4(2x+1)^{-\frac{1}{3}}$$

$$(2x+1)^{-\frac{1}{3}}((2x+1) - 4)$$

$$\boxed{(2x+1)^{-\frac{1}{3}}(2x-3)}$$

$$(61b) \quad (1+x^2)^{-\frac{3}{2}} - (1+x^2)^{-\frac{1}{2}}$$

$$(1+x^2)^{-\frac{3}{2}}(1 - (1+x^2))$$

$$(1+x)^{-\frac{3}{2}}(1-1-x^2)$$

$$\boxed{-x^2(1+x)^{-\frac{3}{2}}}$$

