*Do not round K

*Only round final answer.

Show calculator work w/ class – should get same #s as me.

Lesson 70

APPLICATIONS OF LOGARITHMS

Pull up TI-84 on computer

I. Compound Interest

The amount A that principal P will be worth after t years at interest rate r, compounded annually, is given by the formula: $A = P(1+r)^t$

State formula: $A = P(1+r)^t$

Example: Suppose \$4000 principal is invested at 6% interest and yields \$5353. For how many years was it invested?

State your values:
$A = 5353$
$P = 4000$
$r = .06$
$t = ?$

$$A = P(1+r)^t$$
$$5353 = 4000(1+.06)^t$$
$$1.33825 = 1.06^t$$
$$\log(1.33825) = t \log 1.06$$
$$t = \frac{\log(1.33825)}{\log(1.06)}$$
$$= 5.000313197$$
$$\approx 5 \text{ yrs.}$$

II. Population Growth

One mathematical formula for describing population growth is the formula:

$$P = P_0 e^{kt}$$

initial

P = pop. after time t has passed

Students: You must get same numbers as me.

where P_0 is the number of people at time 0, **P** is the number of people at time **t**, and **k** is a positive constant depending on the situation.

Example: The population of the United States in 1970 ($t=0$) was 208 million. In 1980 ($t=10$) it was 225 million. Use the data to find the value of k and then use the model to predict the population in 2000.

$P = 225$
$P_0 = 208$
K
$t = 10$

$$P = P_0 e^{kt}$$
$$225 = 208 e^{k(10)}$$

use calc:

$$\frac{225}{208} = e^{10K}$$

do not round

$$\ln\left(\frac{225}{208}\right) = \ln e^{10K}$$
$$\ln\left(\frac{225}{208}\right) = 10K \leftarrow \text{work: skip}$$
$$K = \frac{\ln\left(\frac{225}{208}\right)}{10}$$
$$K = .00785\ldots$$

keep

*round only final ans.

$$P = P_0 e^{kt}$$
$$= 208 e^{(.00785\ldots)(30)}$$
$$= 263.2818278$$
$$\boxed{\approx 263.28 \text{ mill people}}$$

(already in) × 30
e^(ANS)
Ans × 208

Calculator keys

II. Population Growth

One mathematical formula for describing population growth is the formula:

where P_0 is the number of people at time 0, **P** is the number of people at time **t**, and **k** is a positive constant depending on the situation.

Example: The population of the United States in 1970 was 208 million. In 1980 it was 225 million. Use the data to find the value of k and then use the model to predict the population in 2000.

Values:

$P = 225$

$P_0 = 208$

K

$t = 10$

State Formula: $P = P_0 e^{Kt}$

$$225 = 208e^{K(10)}$$

$$\frac{225}{208} = e^{10K}$$

$$\ln\left(\frac{225}{208}\right) = \ln e^{10K}$$

$$\ln\left(\frac{225}{208}\right) = 10K$$

$$K = \frac{\ln\left(\frac{225}{208}\right)}{10}$$

$$K = 0.00785\ldots$$

Keep in calculator

To find pop'n in 2000:

$$P = P_0 e^{Kt}$$

$$= 208e^{(\quad)(30)}$$

$$\approx 263.2818278$$

* round only FINAL answer.

$\Rightarrow \approx 263.28$ million people

$\Rightarrow \approx 263$ million people (nearest # people)

III. Radioactive Decay

In a radioactive element, some of the atoms are always transforming themselves into other elements. Thus the amount of a radioactive substance decreases. This is called radioactive decay. A model for radioactive decay is as follows:

$$N = N_0 e^{-kt}$$

where **N_0** is the amount of a radioactive substance at time 0, **N** is the amount at time t, and **k** is a positive constant depending on the rate that a particular element decays.

Part I **Example:** Strontium-90, a radioactive substance, has a half-life of 25 years. This means that half of a sample of the substance will remain as the original element in 25 years. Find k in the formula and then use the formula to find how much of a 36 gram sample will remain after 100 years.
Part II

2 types of problems: (1) Find the ½ life.
(2) Find an amount.

(I) Find K:

values:
$N = \frac{1}{2}$
$N_0 = 1$
K
$t = 25$

$$N = N_0 e^{-kt}$$
$$\tfrac{1}{2} N_0 = N_0 e^{-K 25}$$
$$\frac{1}{2}\frac{N_0}{N_0} = e^{-K25}$$
$$0.5 = e^{-25K}$$
$$\ln(0.5) = -25K$$
$$K = \frac{\ln(0.5)}{-25}$$

notice positive K → $K \approx 0.0277\ldots$

Keep in calculator

(II) Amt after 100 years

$N = ?$
$N_0 = 36$
$K = 0.0277\ldots$
$t = 100$

$$N = N_0 e^{-kt}$$
$$= 36 e^{-0.0277\ldots(100)}$$

Calculator work here:
"K" in calcul.
multiply (−)(100)
$e^{\wedge(\text{ans})} \Rightarrow 0.0625$ (exact)
$(\text{ans}) * 36 \Rightarrow 2.25\text{g}$

Not 2 grams!
Not 2.2!

Notes to go with Applic of Logs Lesson 70

Announce quiz!

4 main formulas

$A = P(1+r)^t$

$P = P_0 e^{kt}$

$N = N_0 e^{-kt}$

$V_n = P(1+r)^n$, $r = +$ appreciation, $r = -$ depreciation

(WS #1)

$\boxed{\text{Prob. \#5}}$ – Find its half-life. (Reverse the steps in notes)

Step 1

$N = N_0 e^{-kt}$

$4.3 = 34580\, e^{-k(18)}$

$N = 4.3$

$N_0 = 34{,}580$

K

$t = 18$

$\frac{4.3}{34580} = e^{-18k}$

$\ln(\text{—}) = \ln e^{-18k}$

$\ln(\text{—}) = -18K$

$K = \frac{\ln\left(\frac{4.3}{34580}\right)}{-18}$

$K = 0.499\ldots$

Step 2

$N = N_0 e^{-kt}$

$\frac{1}{2} N_0 = N_0 e^{(.499\ldots)t}$

$\frac{1}{2} = e^{(-.499\ldots)t}$

$\ln(.5) = \ln e^{(-.499\ldots)t}$

$\ln(.5) = -0.499\ldots t$

$t = \frac{\ln(.5)}{-.499\ldots} = 1.387463571$

$\boxed{\approx 1.39 \text{ days}}$

over →

(9) $V_n = 50{,}000(1 \pm r)^n$

$25{,}000 = 50{,}000(1 - .10)^n$

$0.5 = 0.9^n$

$\log(.5) = n \log(.9)$

$n = \dfrac{\log(.5)}{\log(.9)}$

$n \approx 6.5788$

$n \approx 6.58$ years