

1. Solve for x:

$$(\log_p 2p^3)(\log_m p)(\log_{2p^2} 3x+4) = \log_m 5x$$

$$\frac{\log 2p^3}{\log p} \cdot \frac{\log p}{\log m} \cdot \frac{\log (3x+4)}{\log 2p^2} = \log_m 5x$$

$$\frac{\log (3x+4)}{\log m} = \log_m 5x$$

$$\log_m (3x+4) = \log_m 5x$$

$$3x+4 = 5x$$

$$2 = x$$

2. Simplify:

$$\log_4 2^4 - \log_3 \left( \frac{1}{27} \right)$$

$$\log_4 16 - \log_3 \frac{1}{27}$$

$$2 - -3$$

$$\boxed{5}$$

3. Expand:

$$\ln \left( \frac{3x+5}{x^2-3x-4} \right)^3$$

$$3 \left[ \ln(3x+5) - \ln(x^2-3x-4) \right]$$

$$3 \left[ \ln(3x+5) - \ln(x-4)(x+1) \right]$$

$$3 \left[ \ln(3x+5) - (\ln(x-4) + \ln(x+1)) \right]$$

$$3 \ln(3x+5) - 3 \ln(x-4) - 3 \ln(x+1)$$

4. Express as a single log and in simplified form:

$$\frac{1}{2} \log_q (x^2 + 1) - 3 \log_q \frac{1}{3} - \frac{1}{2} [\log_q (x - 4) + \log_q x]$$

$$\log_q (x^2 + 1)^{\frac{1}{2}} - \log_q \left(\frac{1}{3}\right)^3 - \log_q [x(x - 4)]^{\frac{1}{2}}$$

$$\log_q \sqrt{x^2 + 1} - \left( \log_q \frac{1}{27} + \log_q \sqrt{x^2 - 4x} \right)$$

$$\log_q \frac{\sqrt{x^2 + 1}}{\frac{1}{27} \sqrt{x^2 - 4x}}$$

$$\log_q \frac{27 \sqrt{x^2 + 1}}{\sqrt{x^2 - 4x}}$$

or

$$\log_q \frac{27 \sqrt{x^2 + 1}}{(x^2 - 4x)^{\frac{1}{2}}}$$