

KEYS

Advanced Algebra (H) Solve each problem with 3 equations and 3 unknowns.

1. Amy has \$1, \$5 and \$10 bills in her wallet worth \$87. She has as many \$1 bills as \$5 and \$10 bills combined, and she has 24 bills in all. How many of each denomination has she?
2. Tim has 28 coins made up of nickels, dimes, and quarters. He has four more dimes than nickels and quarters combined. How many of each kind of coin has he if their total value is \$3.20?
3. When three large diamonds are weighed in pairs, the masses of the pairs are found to be 6 carats, 10 carats, and 12 carats. Find the mass of each diamond.
4. The sum of the length, width, and height of a box is 80 cm. The length is 10 cm less than twice the sum of the width and the height, and twice the width exceeds the height by 6 cm. Find the length, width, and height of the box.
5. When Tom rode his bike for one hour, walked for one hour and swam for two hours, he covered a total of 23 km. When he rode for two hours, and walked and swam for one hour each, he covered 33 km. When he rode for one hour and walked for 3 hours, he covered 33 km. Find his speeds of riding, walking, and swimming.
6. A museum charges \$4.00 for regular admission tickets, but admits children for \$2.50 and senior citizens for \$3.00. One day 500 tickets were sold for \$1740. If 100 more regular tickets were sold than children's and seniors' combined, how many of each type of ticket were sold?
7. In a triangle whose perimeter is 60 centimeters, the length of the longest side is 6 centimeters less than the sum of the lengths of the other sides. Three times the length of the shortest side is 40 centimeters minus the difference between the lengths of the other sides. How long is each side of the triangle?
8. At a college basketball game, student tickets cost \$3 each, adult tickets cost \$4.50 each, and children's tickets cost \$2.50 each. Four times as many adult tickets as children's tickets were sold. The total number of children's and adult tickets was half the number of student tickets. If \$8,736.50 was taken in receipts, how many of each type of ticket was sold?

Answers

3x3 wksht

① 12, 9, 3

② 5, 7, 16

③ 2, 4, 8

④ 50, 12, 18

⑤ 12, 7, 2

⑥ 360, 120, 80

⑦ 11.5, 21.5, 27

⑧ 173, 692, 1730

Systems of EQs : 3 EQs and 3 Unknowns

- ① let $x = \#$ of \$1 bills $\rightarrow x = \text{value of } \1 bills
 $f = \#$ of \$5 bills $\rightarrow 5f = \text{value}$
 $T = \#$ of \$10 bills $\rightarrow 10T = \text{value}$

$$\begin{array}{l} \textcircled{A} \quad x + 5f + 10T = 87 \\ \textcircled{B} \quad x + f + T = 24 \\ \textcircled{C} \quad f + T = x \end{array} \quad \left. \begin{array}{l} \textcircled{D} \\ \textcircled{E} \end{array} \right\} \rightarrow \begin{array}{l} 4f + 9T = 63 \\ -4f - 4T = -48 \\ \hline 5T = 15 \\ \textcircled{T = 3} \\ \downarrow \\ \textcircled{D} \quad 4f + 9(3) = 63 \\ 4f = 36 \\ \textcircled{f = 9} \end{array}$$

Ans:

Tens = 3
 Fives = 9
 Ones = 12

- ② let $n = \#$ of nickels $\rightarrow 5n = \text{value in cents}$
 $d = \#$ dimes $\rightarrow 10d = \text{" " "}$
 $q = \#$ quarters $\rightarrow 25q = \text{" " "}$

$$\begin{array}{l} \textcircled{A} \quad d = n + q + 4 \\ \textcircled{B} \quad n + d + q = 28 \\ \textcircled{C} \quad 5n + 10d + 25q = 320 \end{array} \quad \left. \begin{array}{l} \textcircled{A} \\ \textcircled{B} \end{array} \right\} \rightarrow \begin{array}{l} d - n - q = 4 \\ d + n + q = 28 \\ \hline 2d = 32 \\ \textcircled{d = 16} \\ \hline \textcircled{C} \quad 5n + 10(16) + 25q = 320 \\ 5n + 25q = 160 \\ \textcircled{A} \quad -5n - 5q = -60 \\ \hline 20q = 100 \\ \textcircled{q = 5} \\ \textcircled{n = 7} \end{array}$$

dimes = 16
 # quarters = 5
 # nickels = 7

① $16 = n + q + 4$
 $n + q = 12$
 ② $n + q + 16 = 28$
 $n + q = 12$
 same, so must use ③

- ③ Let $x = \text{diamond}$ #1
 $y = \text{ "}$ #2
 $z = \text{ "}$ #3

Ⓐ	$x + y = 6$	→ Ⓐ	$x = 6 - y$
Ⓑ	$y + z = 10$	→ Ⓑ	$z = 10 - y$
Ⓒ	$x + z = 12$		

Ⓒ $x + z = 12$
 $6 - y + 10 - y = 12$
 $-2y + 16 = 12$
 $4 = 2y$
 $2 = y$

Ⓐ $x + 2 = 6$
 $x = 4$

Ⓒ $4 + z = 12$
 $z = 8$

Ans
(4, 2, 8)

- ④ Let $l = \text{length}$
 $w = \text{width}$
 $h = \text{height}$

Ⓐ	$l + w + h = 80$
Ⓑ	$l = 2(w + h) - 10$
Ⓒ	$2w = h + 6$

Ⓑ $l = 2w + 2h - 10$

$l - 2w - 2h = -10$

Ⓐ $l + w + h = 80$

Ⓒ $-3w - 3h = -90$

Ⓐ $w + h = 30$

Ⓒ $+ (2w - h = 6)$

$3w = 36$

$w = 12$

Ⓐ $12 + h = 30$

$h = 18$

$l = 50$

length = 50
width = 12
height = 18

Ans.

⑦ let l = length of longest
 h = length of shortest
 m = length of middle

$$\begin{aligned} \textcircled{A} \quad & l + h + m = 60 \\ \textcircled{B} \quad & l = (h + m) - 6 \\ \textcircled{C} \quad & 3h = 40 - (l - m) \\ & \quad \quad \quad 40 - l + m \end{aligned}$$

$$\begin{array}{r} \textcircled{A} \quad l + h + m = 60 \\ \textcircled{B} \quad l - h - m = -6 \\ \hline 2l = 54 \\ \hline l = 27 \end{array} \quad \rightarrow \quad \begin{array}{r} \textcircled{A} \quad h + m = 33 \rightarrow -3h - 3m = -99 \\ \textcircled{C} \quad 3h = 13 + m \rightarrow (3h - m = 13) \\ \hline -4m = 86 \\ \hline m = 21.5 \end{array}$$

⑧ let T = # student fix
 A = # adult fix
 C = # children's fix

$$\begin{aligned} \textcircled{A} \quad & A = 4C \\ \textcircled{B} \quad & C + A = \frac{1}{2}T \\ \textcircled{C} \quad & 3T + 4.5A + 2.5C = 8736.50 \end{aligned}$$

$$\begin{aligned} \textcircled{A \& B}: \quad & C + 4C = \frac{1}{2}T \\ & 5C = \frac{1}{2}T \\ & 10C = T \end{aligned}$$

$$\begin{aligned} \textcircled{C} \quad & 3(10C) + 4.5A + 2.5C = 8736.50 \\ & 4.5A + 32.5C = 8736.50 \\ & 4.5(+4C) + 32.5C = 8736.50 \\ & 50.5C = 8736.50 \\ & \quad \quad \quad C = 173 \end{aligned}$$

$$\begin{aligned} \textcircled{B} \quad & C + A = \frac{1}{2}T \\ & C + A = \frac{1}{2}(10C) \\ & C + A = 5C \\ & \quad \quad \quad A = 4C \end{aligned}$$

5) Let R = speed riding bike
 W = speed walking
 m = speed swimming

$$\begin{aligned} \text{A) } R + W + 2m &= 23 \\ \text{B) } 2R + W + m &= 33 \\ \text{C) } R + 3W &= 33 \end{aligned}$$

$$\text{A) } R + W + 2m = 23$$

$$\text{C) } -(R + 3W = 33)$$

$$\text{D) } -2W + 2m = -10$$

$$\text{E) } +2W + 6m = 26$$

$$8m = 16$$

$$m = 2$$

$$\text{A) } -2R - 2W - 4m = -46$$

$$\text{B) } + (2R + W + m = 33)$$

$$\text{E) } -W - 3m = -13$$

$$-W - 6 = -13$$

$$-W = -7$$

$$W = 7$$

$$R = 12$$

Ans =

riding bike	12 km/hr.
swimming	2 km/hr.
walk	7 km/hr.

6) Let R = # regular tick
 C = # child tick
 n = # senior tick

$$\begin{aligned} \text{A) } R + C + n &= 500 \\ \text{B) } 4R + 2.5C + 3n &= 1740 \\ \text{C) } C + n + 100 &= R \end{aligned}$$

$$\text{A) } R + C + n = 500$$

$$\text{C) } + (-R + C + n = -100)$$

$$2C + 2n = 400$$

$$\text{D) } \rightarrow C + n = 200 \rightarrow$$

$$\text{B) } 4R + 2.5C + 3n = 1740$$

$$\text{C) } + (-4R + 4C + 4n = -400)$$

$$\text{E) } 6.5C + 7n = 1340$$

$$\text{D) } -7C - 7n = -1400$$

$$-0.5C = -60$$

$$C = 120$$

120 children's
 80 seniors
 300 regular