

Advanced Algebra II Honors: 1.2 Transformations, Stretching and Shrinking Graphs

Warm-Up!



Compare the parent graph $y = x^2$ to the following. How are they shifted?

a. $y = (x-2)^2$

Shift right 2x

b. $y - 1 = (x-3)^2$

$y = (x-3)^2 + 1$
 shift rt 3x, shift 1 1x

c. $y - 2 = x^2$

shift up 2x

d. $y + 2 = (x-3)^2$

shift right 3x
 shift down 2x

e. $y - 3 = -(x+1)^2$

shift left 1x, reflect over y-axis
 shift up 3x

f. $y + 1 = (-x-2)^2$

reflect over y-axis
 shift right 2x
 shift down 1x

What effect does each of the following have on the parent graph?

1. $y = f(x-h)$ shift right h units

2. $y = -f(x)$ reflect over x-axis

3. $y = f(x+k)$ shift up k units

4. $y = f(-x)$ reflect over y-axis

Example 1 Let $f(x) = 4x - 9$

a. Write a function g whose graph is a translation 5 units up of the graph of f .

$g(x) = f(x) + 5 \rightarrow g(x) = (4x-9) + 5 \rightarrow g(x) = 4x - 4$

b. Write a function h whose graph is a translation 1 unit to the right of the graph of f .

$h(x) = f(x-1) \rightarrow g(x) = 4(x-1) - 9 \rightarrow g(x) = 4x - 13$

Example 2 Let $f(x) = |x-5| - 4$

a. Write a function h whose graph is a reflection in the x-axis of the graph of f .

$h(x) = -f(x) \rightarrow h(x) = -(|x-5| - 4) \rightarrow h(x) = -|x-5| + 4$

b. Write a function g whose graph is a reflection in the y-axis of the graph of f .

$g(x) = f(-x) \rightarrow g(x) = |-x-5| - 4$

Try Write a function g whose graph represents the indicated transformation of the graph of f .

1. $f(x) = -(x+2)^2 - 1$; Translate 3 units right and reflection in the x-axis.

$g(x) = -f(x-3) \rightarrow g(x) = -[-(x-3+2)^2 - 1] \rightarrow g(x) = (x-1)^2 + 1$

2. $f(x) = \sqrt{x-2} - 3$; Translate 5 units down and reflection in the y-axis.

$g(x) = f(-x) - 5 \rightarrow g(x) = +\sqrt{-x-2} - 3 - 5 \rightarrow g(x) = +\sqrt{-x-2} - 8$

Stretching and Shrinking:

Core Concept

Horizontal Stretches and Shrinks

The graph of $y = f(ax)$ is a horizontal stretch or shrink by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

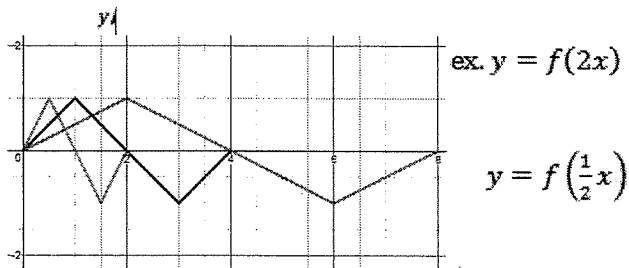
Multiplying the **inputs** by a before evaluating the function stretches the graph horizontally (away from the y -axis) when $0 < a < 1$, and shrinks the graph horizontally (toward the y -axis) when $a > 1$.

Vertical Stretches and Shrinks

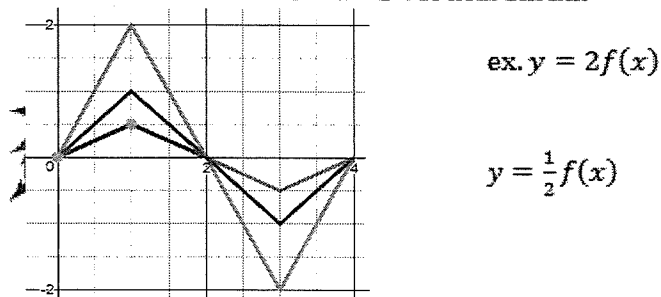
The graph of $y = a \cdot f(x)$ is a vertical stretch or shrink by a factor of a of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

Multiplying the **outputs** by a stretches the graph vertically (away from the x -axis) when $a > 1$, and shrinks the graph vertically (toward the x -axis) when $0 < a < 1$.

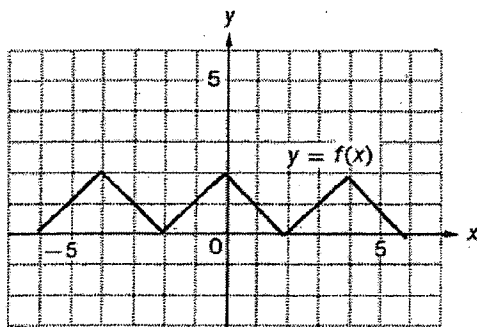
$y = f(ax)$ if $a > 1$ horizontal shrink
if $0 < a < 1$ horizontal stretch



$y = af(x)$ if $a > 1$ vertical stretch
if $0 < a < 1$ vertical shrink



Example 3 Consider the graph of $y = f(x)$. (Hint: List some coordinates of $f(x)$ first.)

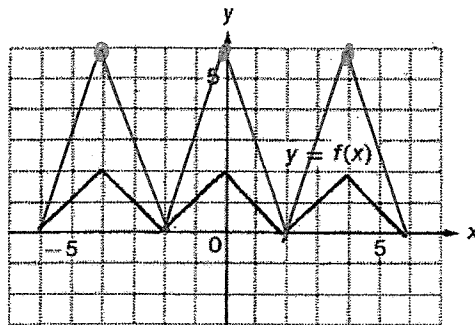


a. Sketch a graph of $y = 3f(x)$

vertical stretch by factor of 3

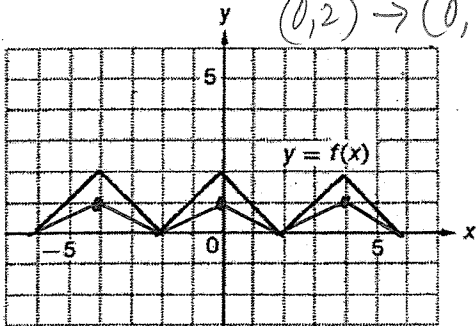
$$(0, 2) \rightarrow (0, 6)$$

$$(2, 0) \rightarrow (2, 0)$$



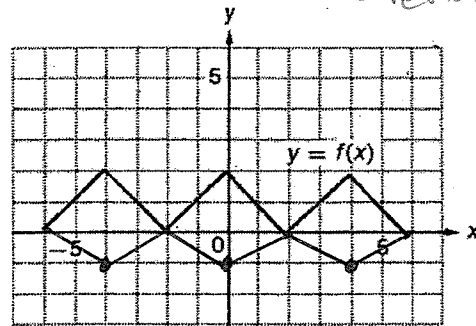
b. Sketch a graph of $y = \frac{1}{2}f(x)$

vertical shrink by factor of $\frac{1}{2}$
 $(0, 2) \rightarrow (0, 1)$

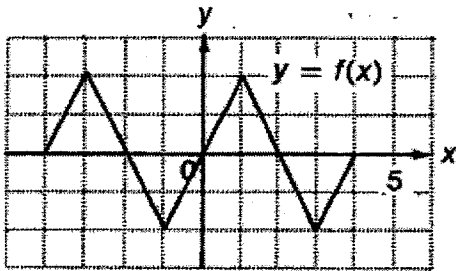


c. Sketch a graph of $y = -\frac{1}{2}f(x)$

① vertical shrink by factor of $\frac{1}{2}$
 ② reflected over x-axis



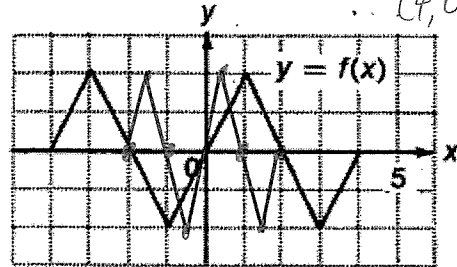
Example 4 Consider the graph of $y = f(x)$.



a. Sketch a graph of $y = f(2x)$

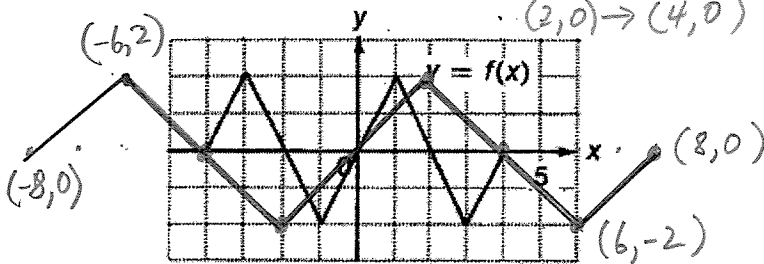
horizontal shrink by factor $\frac{1}{2}$

factor $\frac{1}{2}$
 $(2, 0) \rightarrow (1, 0)$
 $(4, 0) \rightarrow (2, 0)$



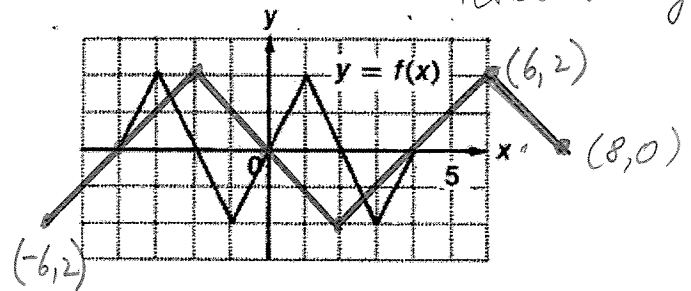
b. Sketch a graph of $y = f(\frac{1}{2}x)$

horizontal stretch by factor of 2
 $(1, 0) \rightarrow (2, 0)$
 $(2, 0) \rightarrow (4, 0)$

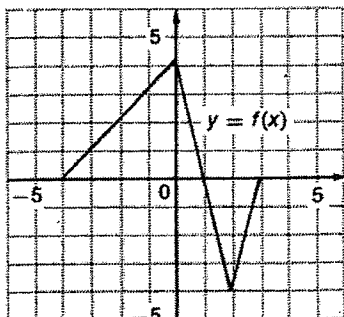


c. Sketch a graph of $y = f(-\frac{1}{2}x)$

horizontal stretch by factor of 2
 reflect over y-axis

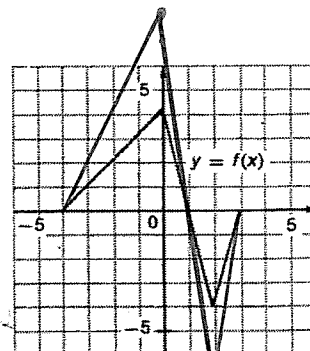


Try Consider the graph of $y = f(x)$. Sketch graphs by transforming this one.

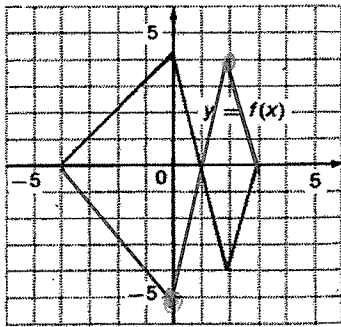


a. $y = 2f(x)$

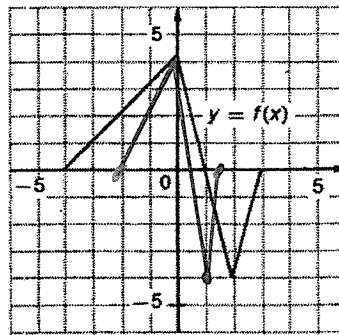
vertical stretch by factor of 2



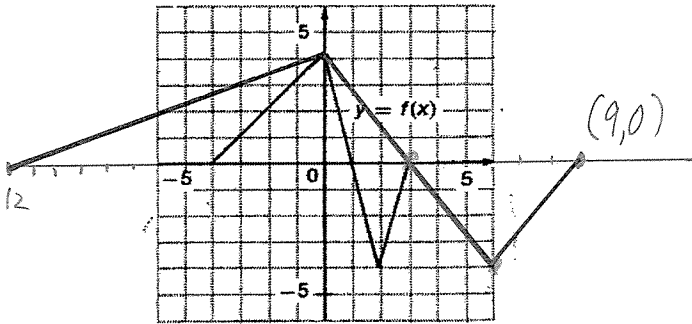
b. $y = -f(x)$



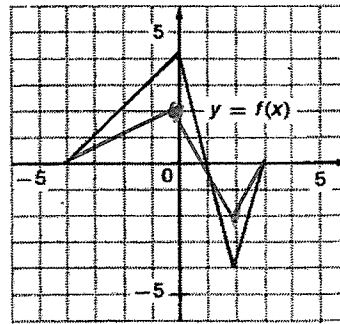
c. $y = f(2x)$ horizontal shrink by factor of $\frac{1}{2}$



d. $y = f(\frac{1}{3}x)$ horizontal stretch by factor of 3

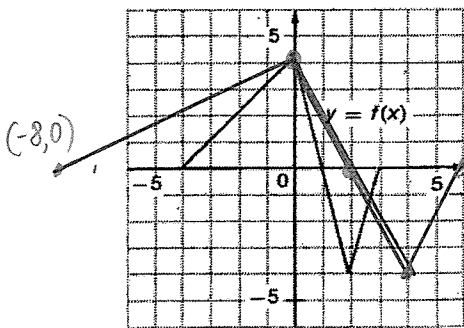


e. $y = \frac{1}{2}f(x)$ vertical shrink by factor of 2



f. $y = f(\frac{1}{2}x)$

horizontal stretch by factor of 2



Example 5 Let $f(x) = |x - 1| + 2$

- a. Write a function g whose graph is a horizontal stretch of the graph of f by a factor of 5.

inside $\frac{1}{5}$

$$g(x) = f\left(\frac{1}{5}x\right) \rightarrow g(x) = \left|\frac{1}{5}x - 1\right| + 2$$

- b. Write a function h whose graph is a vertical shrink of the graph of f by a factor of 0.25.

outside $\frac{1}{4}$

$$h(x) = \frac{1}{4}(|x - 1| + 2)$$

$$h(x) = \frac{1}{4}|x - 1| + \frac{1}{2}$$

or $h(x) = 0.25|x - 1| + 0.5$

