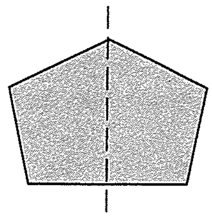
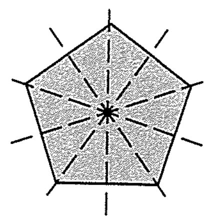
Point, Line, and Rotational Symmetry

Symmetry in the Plane and in Space

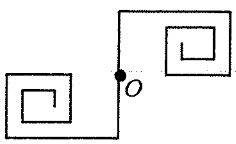
A figure in the plane has **symmetry** if there is an isometry, other than the identity, that maps the figure onto itself. We call such an isometry a *symmetry* of the figure.

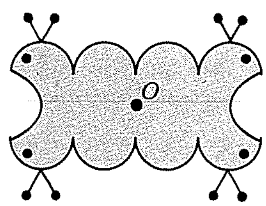
Both of the figures below have **line symmetry.** This means that for each figure there is a symmetry line k such that the reflection R_k maps the figure onto itself. The pentagon at the left has one symmetry line. The regular pentagon at the right has five symmetry lines.

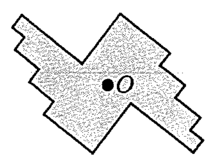




Each figure below has **point symmetry.** This means that for each figure there is a symmetry point O such that the half-turn H_O maps the figure onto itself.



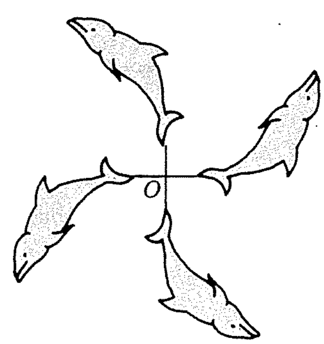




Besides having a symmetry point, the middle figure above has a vertical symmetry line and a horizontal symmetry line.

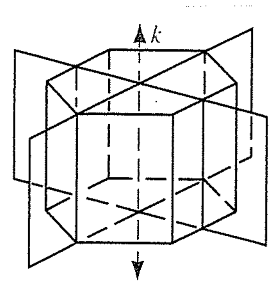
A third kind of symmetry is **rotational symmetry.** The figure below has the four rotational symmetries listed. Each symmetry has center O and rotates the figure onto itself. Note that 180° rotational symmetry is another name for point symmetry.

(1) 90° rotational symmetry: $\mathcal{R}_{O,\,90}$
(2) 180° rotational symmetry: $\mathcal{R}_{O,\,180}$ (or H_O)
(3) 270° rotational symmetry: $\mathcal{R}_{O,\,270}$
(4) 360° rotational symmetry: the identity I

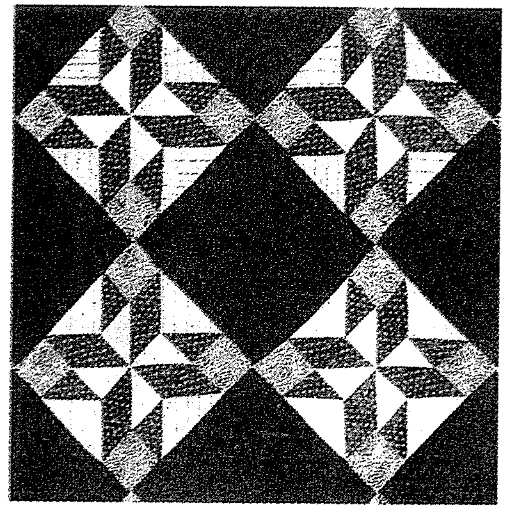


The identity mapping always maps a figure onto itself, and we usually include the identity when listing the symmetries of a figure. However, we do not call a figure *symmetric* if the identity is its only symmetry.

Some geometric solids have more than one symmetry plane. For example, the regular hexagonal prism shown has seven symmetry planes, two of which are shown. It also has six-fold rotational symmetry because rotating it 60°, 120°, 180°, 240°, 300°, or 360° about the line k (called the *axis of symmetry*) maps the prism onto itself.



If a figure or pattern can be rotated $x°$ about a point so that the pattern appears identical to its original position, then the pattern has **$x°$ rotational symmetry.** For example, this quilt has 90°, 180°, and 270° rotational symmetry. It also has 360° rotational symmetry. However, since *any* figure can be rotated 360° onto itself, 360° rotational symmetry is usually not mentioned.

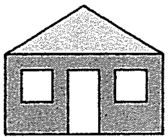


SYMMETRY

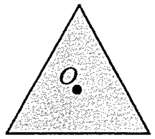
Classroom Exercises

Tell how many symmetry lines each figure has. In Exercise 2, *O* is the center of the equilateral triangle.

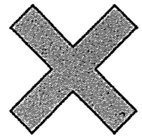
1.



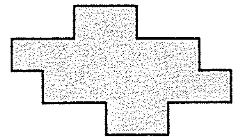
2.



3.



4.



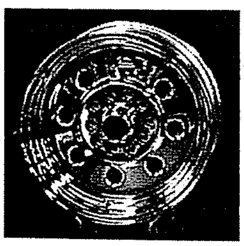
5. Which figures above have point symmetry?
6. Describe all of the rotational symmetries of the figure in Exercise 2.
7. Describe all of the rotational symmetries of the figure in Exercise 3.

Draw each figure on the chalkboard and describe all of its symmetries.

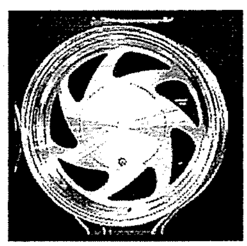
8. isosceles triangle
9. parallelogram
10. rectangle
11. rhombus

WHEEL HUBS Describe the rotational symmetry of the wheel hub.

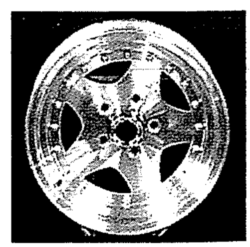
36.



37.



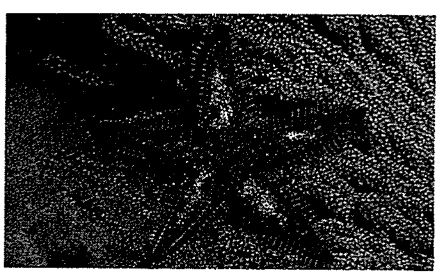
38.



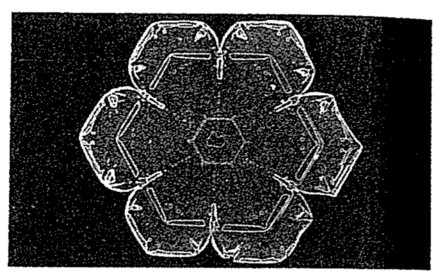
Written Exercises

Consider the object shown in each photograph as a plane figure.
a. State how many symmetry lines each figure has.
b. State whether or not the figure has a symmetry point.
c. List all the rotational symmetries of each figure between 0° and 360°.

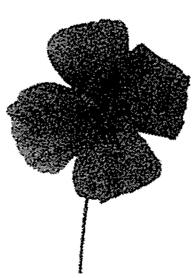
1.



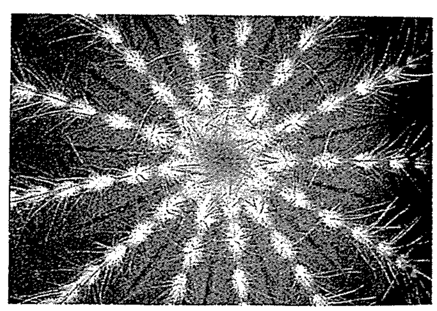
2.



3.

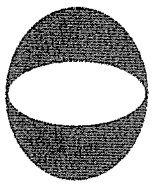


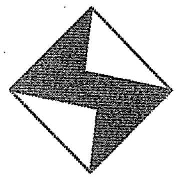
4.

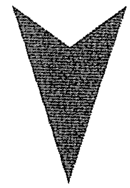


5. Which capital letters of the alphabet have just one line of symmetry? (One answer is "D".)
6. Which capital letters of the alphabet have two lines of symmetry?
7. Which capital letters of the alphabet have a point of symmetry?

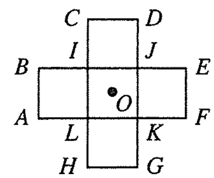
Determine whether the figure has rotational symmetry. If so, descril rotations that map the figure onto itself.

10. 

11. 

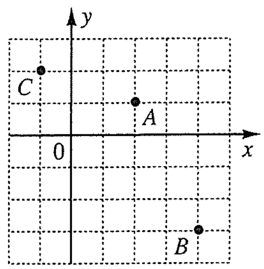
12. 

This pattern of five squares is centered at O. Find the 90° counterclockwise rotation image about O of each of the following.



15. point A
16. square $ABIL$
17. rectangle $ABEF$
18. rectangle $CDKL$
19. square $IJKL$
20. $\overline{AE}$

Find the 90° counterclockwise rotation image about (0, 0) of each point.

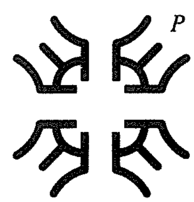


21. $A(2, 1)$ 22. $B(4, -3)$ 23. $C(-1, 2)$

Find the 90° clockwise rotation image about (0, 0) of each point.

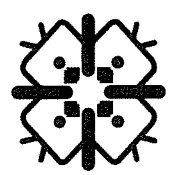
24. $A(2, 1)$ 25. $B(4, -3)$ 26. $C(-1, 2)$

Trace the figure and determine whether each statement is true or false. Explain your reasoning.



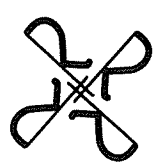
27. If P' is the half-turn image of point P, then P is the half-turn image of P'.
28. The figure has 45° rotational symmetry.
29. The figure has 90° rotational symmetry.
30. The figure has 270° rotational symmetry.

Trace the figure and use the tracing to answer each question.



31. Does the figure have 90° rotational symmetry?
32. Does the figure have 45° rotational symmetry?
33. Does the figure have 270° rotational symmetry?
34. Does the figure have 60° rotational symmetry?
35. Does the figure have reflectional symmetry?

Trace the figure and use the tracing to answer each question.



36. Does the figure have 180° rotational symmetry?
37. Does the figure have 135° rotational symmetry?
38. Does the figure have 90° rotational symmetry?
39. Does the figure have reflectional symmetry?