

The Perpendicular Bisector

You must be able to write the equation (find) of the perpendicular bisector to a line segment. Below is an example. Study it carefully and complete the problems.

Example: Find the perpendicular bisector of \overline{AB} given the following coordinates:

A (1, 7) B (13, 9)

Step 1: Find the midpoint of \overline{AB} . (Why? Since the perpendicular bisector intersects the segment at its midpoint, we have to know the midpoint.) **Important!** This is the point you must use in step 3 when you are looking for the y-intercept.

$$\text{Midpoint} = \left(\frac{1+13}{2}, \frac{7+9}{2} \right) \\ = (7, 8)$$

Step 2: Find the slope of \overline{AB} . (Why? Since the perpendicular bisector is *perpendicular* to the segment, we can find the slope of it by finding the slope of \overline{AB} and using its negative reciprocal.

$$m_{\overline{AB}} = \frac{9-7}{13-1} = \frac{2}{12} = \frac{1}{6} \quad m_{\perp} = -6$$

Step 3: Find the y-intercept. Remember to use the midpoint from step 1 and the m_{\perp} from step 2.

$$y = mx + b \quad b = 50 \\ 8 = -6(7) + b \\ 8 = -42 + b$$

This is the perpendicular bisector $y = -6x + 50$

Find the perpendicular bisector for each line segment below.

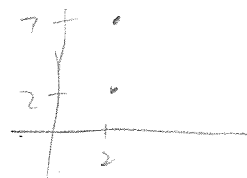
1. \overline{AB} if A (2, 2) and B (2, 7)

① Midpt $\left(\frac{2+2}{2}, \frac{2+7}{2} \right)$ ② slope: $m = \frac{7-2}{2-2} = \frac{5}{0}$

$$\left(2, \frac{9}{2} \right)$$

$$m_{\perp} = 0$$

$$y = \frac{9}{2}$$



2. \overline{AB} if A (-4, -3) and B (6, 2)

$$\text{midpt of } \overline{AB} \left(\frac{-4+6}{2}, \frac{-3+2}{2} \right) \left\{ \begin{array}{l} M_{\overline{AB}} = \frac{-3-2}{-4-6} = \frac{-5}{-10} = \frac{1}{2} \\ M_{\perp} = -2 \end{array} \right.$$
$$(1, -\frac{1}{2})$$

$$y = mx + b$$
$$-\frac{1}{2} = -2(1) + b$$
$$b = 1\frac{1}{2}$$

$$y = -2x + 1\frac{1}{2}$$

3. \overline{AB} if A (-6, -4) and B (1, -4)

$$\text{midpt of } \overline{AB} \left(\frac{-6+1}{2}, \frac{-4-4}{2} \right) \left\{ \begin{array}{l} M_{\overline{AB}} = \frac{-4+4}{-6-1} = 0 \\ M_{\perp} = \text{undefined} \end{array} \right.$$
$$\left(-\frac{5}{2}, -4 \right)$$

↑
x

vertical line
 $x = \#$

$$x = -\frac{5}{2}$$

4. \overline{AB} if A (-7, -2) and B (-1, -7)

$$\text{midpt } \left(\frac{-7-1}{2}, \frac{-2-7}{2} \right) \left\{ \begin{array}{l} M_{\overline{AB}} = \frac{-2+7}{-7+1} = \frac{5}{-6} \\ M_{\perp} = +\frac{6}{5} \end{array} \right.$$
$$\left(-4, -\frac{9}{2} \right)$$
$$y = mx + b$$
$$-\frac{9}{2} = \frac{6}{5}(-4) + b$$
$$-\frac{9}{2} + \frac{24}{5} = b$$
$$\frac{3}{10} = b$$

$$y = \frac{6}{5}x + \frac{3}{10}$$

5. \overline{AB} if A (-8, 4) and B (-4, 7)

$$\text{midpt } \left(\frac{-8-4}{2}, \frac{4+7}{2} \right) \left\{ \begin{array}{l} M_{\overline{AB}} = \frac{4-7}{-8+4} = \frac{-3}{-4} = \frac{3}{4} \\ M_{\perp} = -\frac{4}{3} \end{array} \right.$$
$$\left(-6, \frac{11}{2} \right)$$

$$y = mx + b$$
$$\frac{11}{2} = -\frac{4}{3}(-6) + b$$
$$\frac{11}{2} = 8 + b$$
$$-\frac{5}{2} = b$$

$$y = -\frac{4}{3}x - \frac{5}{2}$$