Justifying ASA Triangle Congruence

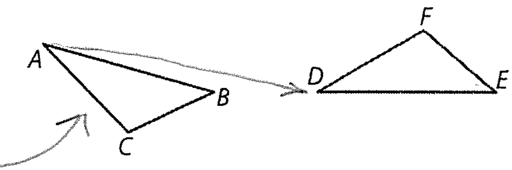
Explain the results of Explore 1 using transformations.

A Use tracing paper to make two copies of the triangle from Explore 1 as shown. Identify the corresponding parts you know to be congruent and mark these congruent parts on the figure.

Given: $\angle A \cong$ $\angle D$

$\angle B \cong$ $\angle E$

$\overline{AB} \cong$ $\overline{DE}$

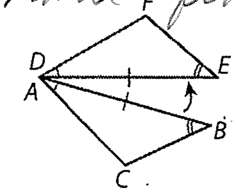


B What can you do to show that these triangles are congruent?

Find a sequence of rigid motions that maps one △ onto the other △.

C Translate $\triangle ABC$ so that point A maps to point D. What translation vector did you use?

the vector with initial point A and terminal point D ($\overrightarrow{AD}$)



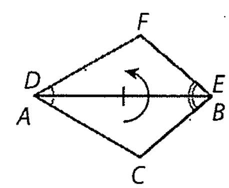
D Use a rotation to map point B to point E. What is the center of the rotation? What is the angle of the rotation?

Center of rotation is pt. A (or pt. A)

∡ of rotation is m∡EDB.

E How do you know the image of point B is point E?

It's given that $\overline{AB} \cong \overline{DE}$, so the image of pt B is pt. E.



F What rigid motion do you think will map point C to point F?

reflection across $\overleftrightarrow{DE}$

G To show that the image of point C is point F, notice that $\angle A$ is reflected across $\overleftrightarrow{DE}$, so the measure of the angle is preserved. Since $\angle A \cong \angle D$ you can conclude that the image of $\overline{AC}$ lies on $\overrightarrow{DF}$. In particular, the image of point C must lie on $\overrightarrow{EF}$. By similar reasoning, the image of $\overline{BC}$ lies on $\overrightarrow{FE}$ and the image of point C must lie on $\overrightarrow{FE}$. The only point that lies on both $\overline{DF}$ and $\overline{EF}$ is pt. F.

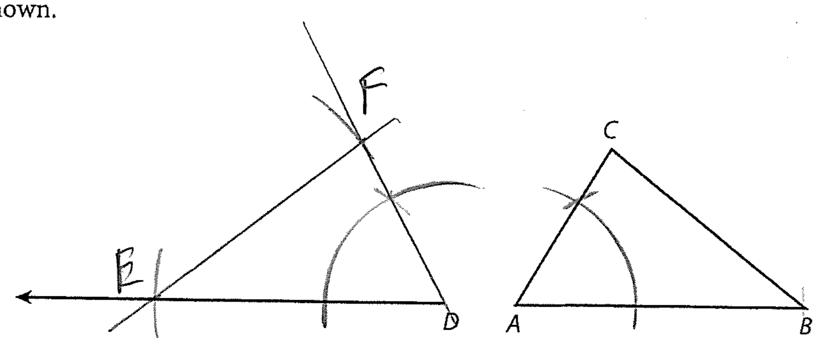
H Describe the sequence of rigid motions used to map $\triangle ABC$ to $\triangle DEF$.

A translation with $\overrightarrow{AD}$ followed by a rotation about ∡EDB followed by a reflection across $\overleftrightarrow{DE}$.

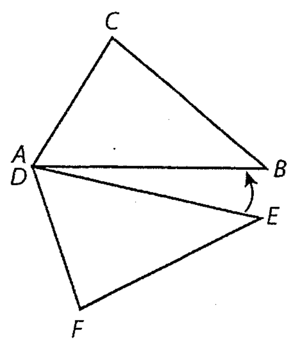
Explore 2 Justifying SAS Triangle Congruence

You can explain the results of Explore 1 using transformations.

(A) Construct $\triangle DEF$ by copying $\angle A$, side $\overline{AB}$, and side $\overline{AC}$. Let point D correspond to point A, point E correspond to point B, and point F correspond to point C, and place point E on the segment shown. → So: $\angle A \cong \angle D$, $\overline{DE} \cong \overline{AB}$, $\overline{DF} \cong \overline{AC}$



(B) The diagram illustrates one step in a sequence of rigid motions that will map $\triangle DEF$ onto $\triangle ABC$. Describe a complete sequence of rigid motions that will map $\triangle DEF$ onto $\triangle ABC$.



Possible answer: Translate $\triangle DEF$ so that pt D maps to pt A. Then rotate $\triangle DEF$ 180° counterclockwise about pt. A. Pt. E will map to pt. B b/c DE = AB. Then reflect $\triangle DEF$ across $\overline{AD}$. Pt. F will map to pt C b/c $\angle A \cong \angle D$ and DF = AC.

(C) What can you conclude about the relationship between $\triangle ABC$ and $\triangle DEF$? Explain your reasoning.

$\triangle ABC \cong \triangle DEF$ b/c there is a sequence of rigid motions that maps one onto the other.

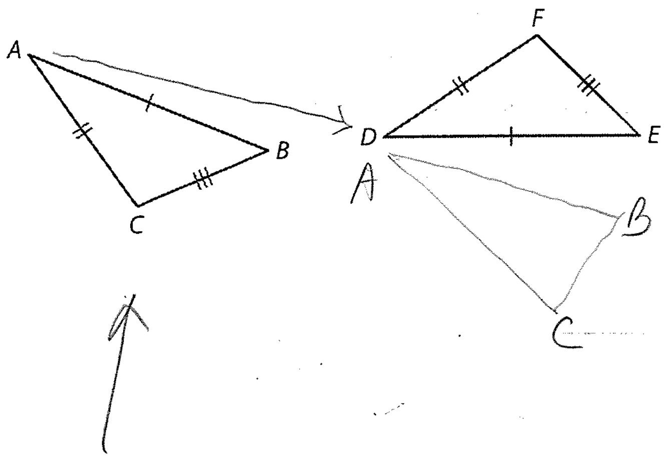
Justifying SSS Triangle Congruence

You can use rigid motions and the converse of the Perpendicular Bisector Theorem to justify this theorem.

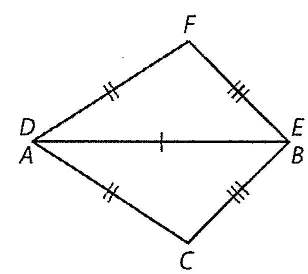
SSS Triangle Congruence Theorem

If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

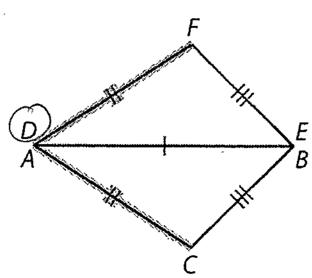
Example 1 **In the triangles shown, let $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\overline{BC} \cong \overline{EF}$. Use rigid motions to show that $\triangle ABC \cong \triangle DEF$.**



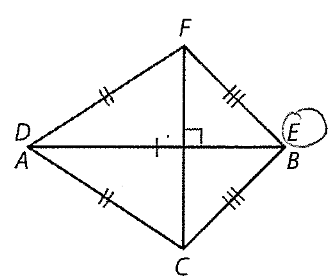
(A) Transform $\triangle ABC$ by a translation along $\overrightarrow{AD}$ followed by a rotation about point D, so that $\overline{AB}$ and $\overline{DE}$ coincide. The segments coincide because they are the same length.



Does a reflection across $\overline{AB}$ map point C to point F? To show this, notice that $DC = DF$, which means that point D is equidistant from point C and point F.

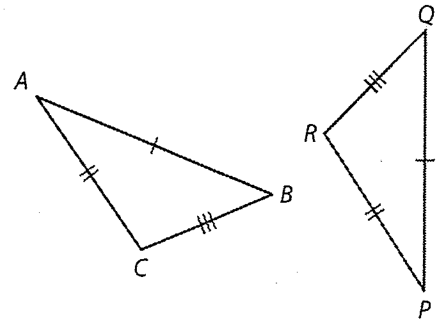


Therefore, point D lies on the perpendicular bisector of $\overline{CF}$ by the converse of the perpendicular bisector theorem. Because $EC = EF$, point E also lies on the perpendicular bisector of $\overline{CF}$.

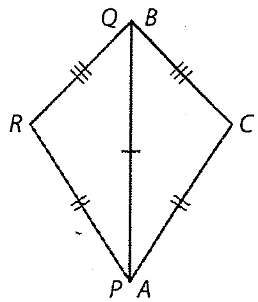


Since point D and point E both lie on the perpendicular bisector of $\overline{CF}$ and there is a unique line through any two points, $\overleftrightarrow{DE}$ is the perpendicular bisector of $\overline{CF}$. By the definition of reflection, the image of point C must be point F. Therefore, $\triangle ABC$ is mapped onto $\triangle DEF$ by a translation, followed by a rotation, followed by a reflection, and the two triangles are congruent.

Ⓑ Show that $\triangle ABC \cong \triangle PQR$.



Triangle ABC is transformed by a sequence of rigid motions to form the figure shown below. Identify the sequence of rigid motions. (You will complete the proof on the following page.)

	1. Translation along $\overrightarrow{AP}$ 2. Rotation about P so that $\overline{PQ}$ & $\overline{AB}$ coincide. 3. Reflection across $\overline{PQ}$.

Complete the explanation by filling in the blanks with the name of a point, line segment, or geometric theorem.

Because $\overline{QR} \cong$ $\overline{QC}$, point Q is equidistant from R (or C) and C (or R). Therefore, by the converse of the ⊥ bisector Theorem, point Q lies on the ⊥ bisector of $\overline{RC}$. Similarly, $\overline{PR} \cong$ $\overline{PC}$. So point P lies on the perpendicular bisector of $\overline{RC}$. Because two points determine a line, the line $\overleftrightarrow{PQ}$ is the ⊥ bisector of $\overline{RC}$.

By the definition of reflection, the image of point C must be point R. Therefore, $\triangle ABC \cong \triangle PQR$ because $\triangle ABC$ is mapped to $\triangle PQR$ by a translation, a rotation, and a reflection.