Max/Min KEYS

WS #1

① 18.5 and 18.5
product = 342.25

② L = 10cM
W = 10cM
Max area = 100 sq.cm

③ Length = 30m
width = 60m
max area = 1800 sq.m

⑤ 175 passengers will produce the max. rental of \$612.50.

WS #2

① L = 40 ft.
W = 40 ft.
max area = 1600 sq ft

② ticket price \$350
max income \$245,000

③ price per copy : 70¢
max income : \$24,500

④ W = 200m
L = 300m
max area = 60,000 sq. m.

⑥ Each piece = 18cm
min. area = 40.5 sq. cM

⑧ ticket price \$3.50
max income \$490

Name KEY Advanced Algebra (H)

Maximum or Minimum

1. Find two numbers whose sum is 37 and whose product is a maximum.

$\boxed{x+y=37}$ $\boxed{P=xy}$

$y=37-x$

$P=x(37-x)$

$P(x)=37x-x^2$

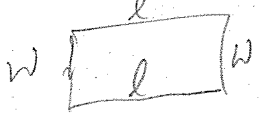
$P(x)-342.25=-(x^2-37x+342.25)$

$P(x)=-(x-18.5)^2+342.25$

$V(18.5, 342.25)$

2 #s are 18.5 and 18.5.

2. Find the dimensions and maximum area of a rectangle if its perimeter is 40 centimeters.



$2w+2l=40$

$wl=A$

$w=\frac{-2l}{2}+\frac{40}{2}$

$w=-l+20$

$A=l(-l+20)$

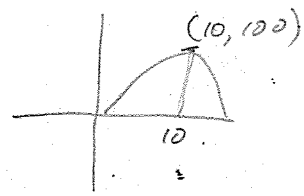
$A(l)=-l^2+20l$

$A(l)-100=-(l^2-20l+100)$

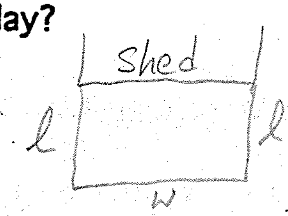
$A(l)=-(l-10)^2+100$

$V(\underset{L}{10},\underset{A}{100})$

L is 10 cm
W is 10 cm



3. Mrs. Werner has 120 meters of fence to make a rectangular pen for her dog Tutu. If a shed is used as one side of the pen, what would be the length and width for Tutu to have the maximum area in which to play?



$2l+w=120$

$w=120-2l$

$A=lw$

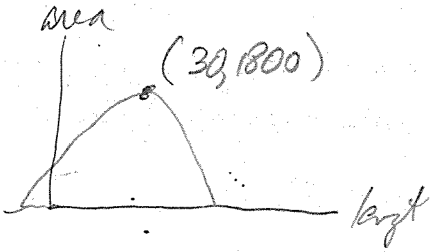
$A(l)=l(120-2l)$

$=-2l^2+120l$ ← complete the sq

$A(l)-1800=-2(l^2-60l+\underline{900})$

$A(l)=-2(l-30)+1800$

Vertex $(\underset{L}{30},\underset{A}{1800})$



max area = 1800 when length = 30 ← plug in to $w=120-2l$

$\boxed{l=30\text{ m},\ w=60\text{ m}}$

4. A taxi service operates between two airports, transporting 300 passengers day. The charge is \$8.00. The owner estimates that 20 passengers will be lost for each \$1 increase in the fare. What charge would be most profitable for the service?

Let x = increments of change

profit = (\$/person)(# people)

$P(x) = (8 + 1x)(300 - 20x)$ ← Set up

$P(x) = 2400 + 300x - 160x - 20x^2$

$P(x) - 2400 = -20x^2 + 140x$

$\frac{-7}{2} = 3.5$

$(3.5)^2 = 12.25$

$P(x) - 2400 - 245 = -20(x^2 - 7x + 12.25)$

$P(x) = -20(x - 3.5)^2 + 2645$ ↖ Do Rt. then left side

$\therefore V(3.5, 2645)$

Then plug 3.5 into $(8+x)$ to get \$11.50 → $\boxed{\$11.50/\text{person}}$

5. The steamship, *The Golden Conifer*, is rented to take 100 campers to Pine Mountain. The fare is \$5 per camper. The steamship company has agreed to reduce the fare by two cents per passenger for every camper over 100. How many passengers will produce a maximum rental for the company?

Let x = # of passengers above 100

$R = (100 + x)(5 - .02x)$

$R = 500 + 5x - 2x - .02x^2$

$= -.02x^2 + 3x + 500$

$R(x) - 500 = -0.02(x^2 - 150x)$

$R(x) - 500 - 112.5 = -0.02(x^2 - 150x + 5625)$

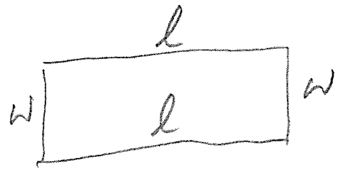
$R(x) = -0.02(x - 75)^2 + 612.5$

$V(75, \underbrace{612.50}_{\$\text{ rental}})$

175 passengers will prod. a max. rental.

WS 2 max/min

(5)



$2l + 2w = 50$

$V = lw5$ ← height

$l = \frac{50}{2} - \frac{2w}{2}$

$l = 25 - w$

$V = 5w(25 - w)$

$V(w) = 125w - 5w^2$

$= -5(w^2 - 25w)$

$V(w) - 781.25 = -5(w^2 - 25w + 156.25)$

$V(w) = -5(w - 12.5)^2 + 781.25$

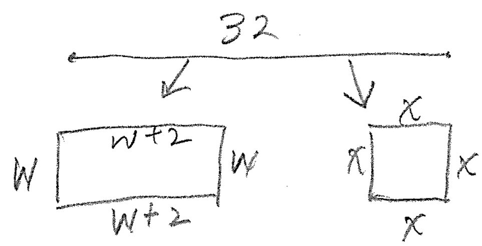
$V(\underset{w}{12.5}, \underset{\text{area}}{781.25})$

$l = 25 - w$

$$\boxed{\begin{array}{c} l = 12.5 \\ w = 12.5 \end{array}}$$

max area = 781.25 cu. cm.

7



$$2w + 2(w+2) + 4x = 32$$
$$2w + 2w + 4 + 4x$$
$$4w + 4x = 28$$
$$w + x = 7 \rightarrow w = 7 - x$$

$A_{rec} + A_{sq}$

$$A = w(w+2) + x^2$$
$$= w^2 + 2w + x^2$$
$$A(x) = (7-x)^2 + 2(7-x) + x^2$$
$$= 49 - 14x + x^2 + 14 - 2x + x^2$$
$$= 2x^2 - 16x + 63$$
$$A(x) - 63 = 2(x^2 - 8x)$$
$$A(x) - 63 + 32 = 2(x^2 - 8x + 16)$$
$$A(x) = 2(x-4)^2 + 31$$

$V(4, 31)$

x area

$W = 7 - 4 = 3$ in.

$L = 5$ in.

$P_{rectangle} = 2(3) + 2(5) = 16$ in.

Each piece is 16 in.

Minimum area = 31 sq. in.

OR

$$\rightarrow x = 7 - w$$
$$A = w(w+2) + x^2$$
$$= w^2 + 2w + (7-w)^2$$
$$= w^2 + 2w + 49 - 14w + w^2$$
$$A(w) = 2w^2 - 12w + 49$$
$$A(w) - 49 = 2(w^2 - 6w)$$
$$A(w) - 49 + 18 = 2(w^2 - 6w + 9)$$
$$A(w) = 2(w-3)^2 + 31$$

$V(3, 31)$

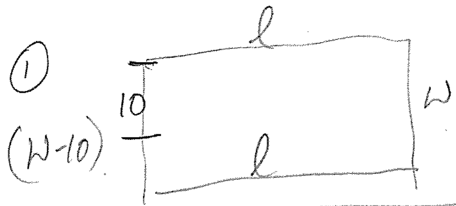
w, area

$W = 3$ in.

$L = 5$ in.

Side = 4 in.

Max/min Problems WS #2

① 

$$\boxed{\begin{aligned} 2l + w + w - 10 &= 150 \\ A &= lw \end{aligned}}$$

$2l + 2w = 160$

$l = \frac{160}{2} - \frac{2w}{2}$

$l = (80 - w)$

$A = w(80-w)$

$= 80w - w^2$

$A(w) = -w^2 + 80w$

$A(w) - 1600 = -1(w^2 - 80w + 1600)$

$A(w) = -1(w-40)^2 + 1600$

$V(\underset{w}{40}, \underset{\text{Area}}{1600})$

Length = 40 feet

Width = 40 ft

Max area = 1600 sq. ft.

Let x = increment of change

② $I = (\text{tix cost/person})(\#\text{people})$

$I = (300 + 5x)(800 - 10x)$

$I(x) = 240{,}000 + 1000x - 50x^2$

$I(x) - 240000 = -50(x^2 - 20x)$

$I(x) - 240000 - 5000 = -50(x^2 - 20x + 100)$

$I(x) = -50(x-10)^2 + 245{,}000$

$V(\underset{x}{10}, \underset{\text{income}}{245000})$

$300 + 5(10) =$ ticket price

\$350 = ticket price

max income = \$245,000

③ Let x = increment of change

$I = (50{,}000 - 5000x)(40 + 10x)$

$I(x) = 2{,}000{,}000 + 300{,}000x - 50000x^2$

$I(x) - 2000{,}000 = -50{,}000(x^2 - 6x)$

$I(x) - 2000{,}000 - 450{,}000 = -50{,}000(x^2 - 6x + 9)$

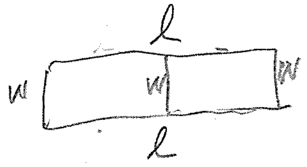
$I(x) = -50{,}000(x-3)^2 + 2450000$

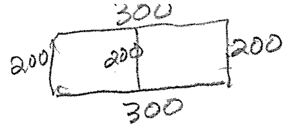
$V(\underset{x}{3}, \underset{\text{max income}}{2450000})$

Price per copy: 70¢

Max income: \$24,500

(4)





$3w + 2l = 1200$

$A = lw$

$l = \frac{1200}{2} - \frac{3w}{2}$

$l = 600 - \frac{3}{2}w$

$A(w) = w(600 - \frac{3}{2}w)$

$= 600w - \frac{3}{2}w^2$

$A(w) - 60{,}000 = -\frac{3}{2}(w^2 - 400w + 40{,}000)$

$A(w) = -\frac{3}{2}(w - 200)^2 + 60{,}000$

$V(\underset{w}{200}; \underset{\text{area}}{60{,}000})$

width = 200m
length = 300m

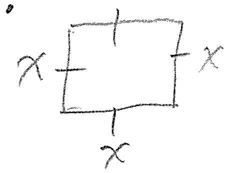
max area = 60,000 sq. m

(6)

36

$x \swarrow \quad \searrow$

SQ_1:



$\frac{36-4x}{4}$

$\downarrow$

$9 - x$ = each side

Perimeter:
$4x + 36 -$

$A(x) = x^2 + (9-x)^2$

$= x^2 + 81 - 18x + x^2$

$= 2x^2 - 18x + 81$

$A(x) - 81 + 40.5 = 2(x^2 - 9x + 20.25)$

$A(x) = 2(x - 4.5)^2 + 40.5$

$L_1 = 18$

$L_2 = 18$

$V(\underset{\text{side of } SQ_1}{4.5}, \underset{\text{Area}}{40.5})$

Each piece = 18cm

min. Area = 40.5 sq. cm

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(8) Let x = increment of change

$I = (\text{price/person})(\#\text{people})$

$I(x) = (2 + .25x)(200 - 10x)$

$= 400 + 30x - 2.5x^2$

$I(x) - 400 = -2.5x^2 + 30x$

$I(x) - 400 - 90 = -2.5(x^2 - 12x + 36)$

$I(x) = -2.5(x-6)^2 + 490$

vertex $(\underset{x}{6}, \underset{\text{income}}{490})$

$\downarrow$

$2 + .25(6) = \$3.50$

ticket price \$3.50

max income \$490

Ck: $= (3.50)(200 - 10(6))$

$= \$490$