

Name KEY Advanced Algebra (H)
Maximum or Minimum -

1. Find two numbers whose sum is 37 and whose product is a maximum.

$x + y = 37$
 $\rightarrow y = 37 - x$
 $P = xy$
 $P = x(37 - x)$

$V(18.5, 342.25)$
2 #s are 18.5 and 18.5

$P(x) = 37x - x^2$
 $P(x) - 342.25 = -(x^2 - 37x + 342.25)$
 $P(x) = -(x - 18.5)^2 + 342.25$

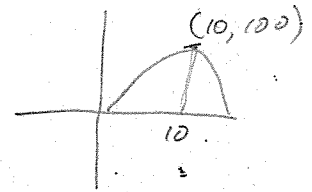
2. Find the dimensions and maximum area of a rectangle if its perimeter is 40 centimeters.



$2w + 2l = 40$
 $wl = A$
 $w = \frac{-2l + 40}{2}$
 $w = -l + 20$

$A = l(-l + 20)$
 $A(l) = -l^2 + 20l$
 $A(l) - 100 = -(l^2 - 20l + 100)$
 $A(l) = -(l - 10)^2 + 100$
 $V(10, 100)$

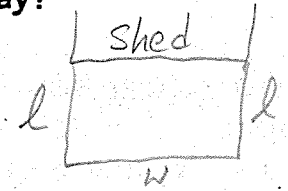
L is 10 cm
W is 10 cm



3. Mrs. Werner has 120 meters of fence to make a rectangular pen for her dog Tutu. If a shed is used as one side of the pen, what would be the length and width for Tutu to have the maximum area in which to play?

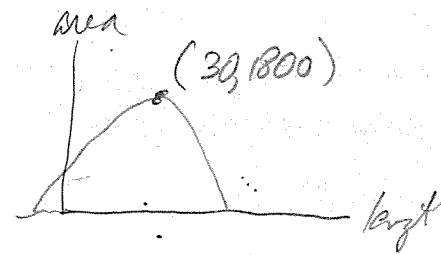
$2l + w = 120$
 $w = 120 - 2l$

$A = lw$
 $A(l) = l(120 - 2l)$



$= -2l^2 + 120l$
 $A(l) - 1800 = -2(l^2 - 60l + 900)$
 $A(l) = -2(l - 30) + 1800$
Vertex $(30, 1800)$
L A

Complete the sq



max area = 1800 when length = 30

plug in to $w = 120 - 2l$

$l = 30m, w = 60m$

4. A taxi service operates between two airports, transporting 300 passengers day. The charge is \$8.00. The owner estimates that 20 passengers will be lost for each \$1 increase in the fare. What charge would be most profitable for the service?

Let $x =$ increments of change

$$\text{profit} = (\$/\text{person}) (\# \text{ people})$$

$$P(x) = (8 + 1x)(300 - 20x) \quad \leftarrow \text{Set up}$$

$$P(x) = 2400 + 300x - 160x - 20x^2$$

$$P(x) - 2400 = -20x^2 + 140x \quad \rightarrow \quad \frac{7}{2} = 3.5$$

$$P(x) - 2400 - \frac{245}{20} = -20(x^2 - 7x + 12.25)$$

$$P(x) = -20(x - 3.5)^2 + 2645 \quad \leftarrow \text{Do Rt. then left side}$$

$$\therefore V(3.5, 2645)$$

Then plug 3.5 into $(8+x)$ to get \$11.50 \rightarrow \$11.50/person

5. The steamship, *The Golden Conifer*, is rented to take 100 campers to Pine Mountain. The fare is \$5 per camper. The steamship company has agreed to reduce the fare by two cents per passenger for every camper over 100. How many passengers will produce a maximum rental for the company?

Let $x =$ # of passengers above 100

$$R = (100 + x)(5 - .02x)$$

$$R = 500 + 5x - 2x - .02x^2$$

$$= -.02x^2 + 3x + 500$$

$$R(x) - 500 = -0.02(x^2 - 150x)$$

$$R(x) - 500 - 112.5 = -0.02(x^2 - 150x + 5625)$$

$$R(x) = -0.02(x - 75)^2 + 612.5$$

$$V(75, 612.50)$$

\$ rental

175 passengers will prod.
a max. rental:

Max/Min KEYS

WS #1

① 18.5 and 18.5
product = 342.25

② $L = 10\text{cm}$
 $W = 10\text{cm}$
Max area = 100 sq. cm

③ Length = 30m
Width = 60m
max area = 1800 sq. m

⑤ 175 passengers
will produce the
max. rental of
 $\$612.50$.

WS #2

① $L = 40\text{ ft.}$
 $W = 40\text{ ft.}$
max area = 1600 sq. ft.

② ticket price $\$350$
max income $\$245,000$

③ price per copy: 70%
max income: $\$24,500$

④ $W = 200\text{m}$
 $L = 300\text{m}$
max area = $60,000$
sq. m.

⑥ Each piece = 18cm
min. area = 40.5 sq. cm

⑧ ticket price $\$3.50$
max income $\$490$



WS 2 max/min



$$2l + 2w = 50$$

→ $V = lW$ height

$$l = \frac{50}{2} - \frac{2w}{2}$$

$$l = 25 - w$$

$$V = 5w(25 - w)$$

$$V(w) = 125w - 5w^2$$

$$= -5(w^2 - 25w)$$

$$V(w) - 781.25 = -5(w^2 - 25w + 156.25)$$

$$V(w) = -5(w - 12.5)^2 + 781.25$$

$$V(12.5, 781.25)$$

 w area

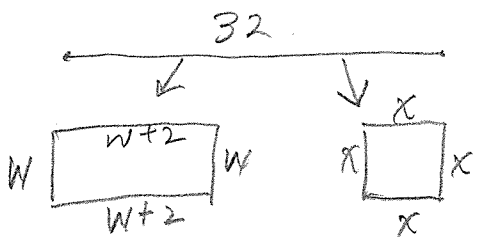
$$l = 25 - w$$

$$l = 12.5$$

$$w = 12.5$$

max area = 781.25 cm. cm.

7



$$2w + 2(w+2) + 4x = 32$$

$$2w + 2w + 4 + 4x$$

$$4w + 4x = 28$$

$$w + x = 7 \rightarrow w = 7 - x$$

$$A = w(w+2) + x^2$$

Area + Area

$$= w^2 + 2w + x^2$$

$$A(x) = (7-x)^2 + 2(7-x) + x^2$$

$$= 49 - 14x + x^2 + 14 - 2x + x^2$$

$$= 2x^2 - 16x + 63$$

$$A(x) - 63 = 2(x^2 - 8x)$$

$$A(x) - 63 + 32 = 2(x^2 - 8x + 16)$$

$$A(x) = 2(x-4)^2 + 31$$

$$V(4, 31)$$

x area

$$w = 7 - 4 = 3 \text{ in.}$$

$$h = 5 \text{ in.}$$

$$P_{\text{rectangle}} = 2(3) + 2(5) = 16 \text{ in.}$$

Each piece is 16 in.

Minimum area = 31 sq. in.

OR

$$\rightarrow x = 7 - w$$

$$A = w(w+2) + x^2$$

$$= w^2 + 2w + (7-w)^2$$

$$= w^2 + 2w + 49 - 14w + w^2$$

$$A(w) = 2w^2 - 12w + 49$$

$$A(w) - 49 = 2(w^2 - 6w)$$

$$A(w) - 49 + 18 = 2(w^2 - 6w + 9)$$

$$A(w) = 2(w-3)^2 + 31$$

$$V(3, 31)$$

w, area

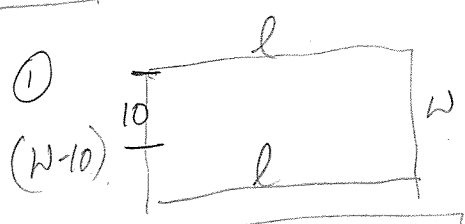
$$w = 3 \text{ in.}$$

$$h = 5 \text{ in.}$$

$$\text{Side} = 4 \text{ in.}$$

Max/min Problems WS #2

Let x = increment of change



$$2l + w + 10 = 150$$

$$A = lw$$

$$\rightarrow 2l + 2w = 160$$

$$l = \frac{160 - 2w}{2}$$

$$l = (80 - w)$$

$$A = w(80 - w)$$

$$= 80w - w^2$$

$$A(w) = -w^2 + 80w$$

$$A(w) - 1600 = -(w^2 - 80w + 1600)$$

$$A(w) = -(w - 40)^2 + 1600$$

$V(40, 1600)$
W Area

Length = 40 feet
Width = 40 ft
Max area = 1600 sq. ft.

② $I = (\text{tik cost/person})(\# \text{people})$

$$I = (300 + 5x)(800 - 10x)$$

$$I(x) = 240000 + 1000x - 50x^2$$

$$I(x) - 240000 = -50(x^2 - 20x)$$

$$I(x) - 240000 - 5000 = -50(x^2 - 20x + 100)$$

$$I(x) = -50(x - 10)^2 + 245,000$$

$V(10, 245000)$
x income

$$300 + 5(10) = \text{ticket price}$$

$$\text{\$}350 = \text{ticket price}$$

$$\text{max income} = \text{\$}245,000$$

③ Let x = increment of change

$$I = (50,000 - 5000x)(40 + 10x)$$

$$I(x) = 2,000,000 + 300,000x - 50,000x^2$$

$$I(x) - 2,000,000 = -50,000(x^2 - 6x)$$

$$I(x) - 2,000,000 = -50,000(x^2 - 6x + 9)$$

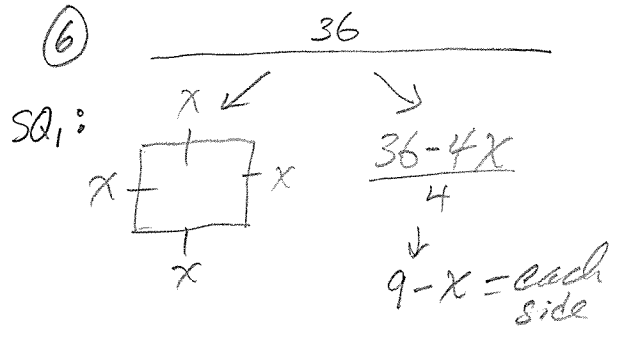
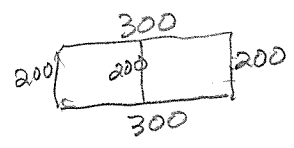
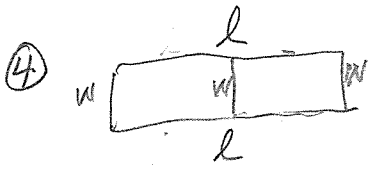
$$-450,000$$

$$I(x) = -50,000(x - 3)^2 + 2,450,000$$

$V(3, 2450000)$
x max income

price per copy: 70¢

Max income: \$24,500



$$3w + 2l = 1200$$

$$A = lw$$

$$l = \frac{1200 - 3w}{2}$$

$$l = 600 - \frac{3}{2}w$$

$$A(w) = w(600 - \frac{3}{2}w)$$

$$= 600w - \frac{3}{2}w^2$$

$$A(w) - 60,000 = -\frac{3}{2}(w^2 - 400w + 40,000)$$

$$A(w) = -\frac{3}{2}(w - 200)^2 + 60,000$$

V(200, 60,000)
w area

width = 200m
length = 300m

max area = 60,000 sq.m

Perimeter:

$$4x + 36 =$$

$$A(x) = x^2 + (9-x)^2$$

$$= x^2 + 81 - 18x + x^2$$

$$= 2x^2 - 18x + 81$$

$$A(x) - 81 + 40.5 = 2(x^2 - 9x + 20.25)$$

$$A(x) = 2(x - 4.5)^2 + 40.5$$

$$L_1 = 18$$

$$L_2 = 18$$

V(4.5, 40.5)
side of Area.
sq.

Each piece = 18cm
min. Area = 40.5 sq.cm

⑧ let x = increment of change

$$I = (\text{price/person})(\# \text{ people})$$

$$I(x) = (2 + .25x)(200 - 10x)$$

$$= 400 + 30x - 2.5x^2$$

$$I(x) - 400 = -2.5x^2 + 30x$$

$$I(x) - 400 - 90 = -2.5(x^2 - 12x + 36)$$

$$I(x) = -2.5(x - 6)^2 + 490$$

Vertex (6, 490)

x income



$$2 + .25(6) = \$3.50$$

ticket price \$3.50

max income \$490

$$ck = (3.50)(200 - 10(6))$$

$$= \$490$$