Geometry Honors date Key

Chapter: Congruent Triangles

Topic: Proofs in Coordinate Geometry

In coordinate geometry, supporting work cannot be a graph. You may use a graph to guide you in your work but it cannot be supporting work for your answers.

What formulas are used in coordinate geometry?________________________________

1. **In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle.**

$M_{\overline{RS}} = \frac{-1+4}{6-1} = \frac{3}{5}$

$M_{\overline{ST}} = \frac{-4-6}{1+5} = \frac{-10}{6} = -\frac{5}{3}$

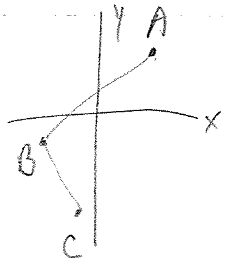
$M_{\overline{RT}} = \frac{-1-6}{6+5} = \frac{-7}{11}$

Since the slopes of $\overline{RS}$ and $\overline{ST}$ are negative reciprocals, $\overline{RS} \perp \overline{ST}$. Because $\perp$ lines form right angles, $\angle S$ is right. Therefore, $\triangle RST$ is right because a right $\triangle$ is a triangle with a right $\angle$.

2. **Triangle ABC has vertices with $A(x,3)$, $B(-3,-1)$, and $C(-1,-4)$.**

Determine and state a value of x that would make triangle ABC a right triangle. Justify why $\triangle ABC$ is a right triangle.

$M_{\overline{BC}} = \frac{-1+4}{-3+1} = \frac{3}{-2}$



$M_{\overline{AB}} = \frac{3+1}{x+3} = \frac{4}{x+3}$ neg reciprocal

$\frac{2}{3} = \frac{4}{x+3}$

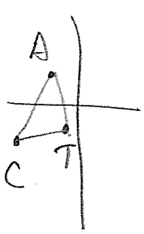
$12 = 2x+6$

$3 = x$

If $x=3$, then the coordinates of A are $(3,3)$. Then the slope of $AB = \frac{4}{6}$ or $\frac{2}{3}$.

Since slope of $\overline{BC}$ is $-\frac{3}{2}$, $\overline{BC} \perp \overline{AB}$, thus making $\angle B$ a right $\angle$ and $\triangle ABC$ a right $\triangle$.

3. Triangle CAT has vertices with C (-10, -2), A(-6, 2), and T(-2, -2). Determine whether this is a right triangle. If so, what kind of a right triangle? Support your conclusion with geometric work.



$M_{\overline{AC}} = \frac{2+2}{-6+10} = \frac{4}{4} = 1$

$M_{\overline{CT}} = \frac{-2+2}{-10+2} = 0$

$M_{\overline{AT}} = \frac{2+2}{-6+2} = \frac{4}{-4} = -1$

$\angle A$ is right, thus we conclude $\overline{CT}$ is hypotenuse. Subsequently, we test if $AC = AT$.

$AC = \sqrt{(-6+10)^2 + (2+2)^2}$

$\sqrt{16+16}$

$= 4\sqrt{2}$

$AT = \sqrt{(-6+2)^2 + (2+2)^2}$

$\sqrt{16+16}$

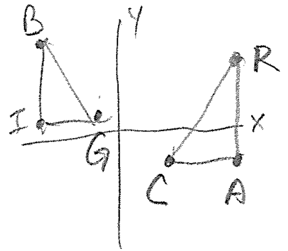
$= 4\sqrt{2}$

Conclusion: Since $\overline{AC}$ + $\overline{AT}$ have slopes that are negative reciprocals we conclude $\overline{AC} \perp \overline{AT}$ and that $\angle A$ is right because $\perp$ lines form rt. $\angle$s. Since $AC = AT$, then $\overline{AC} \cong \overline{AT}$ and $\triangle CAT$ is isosceles. This is an isosceles right $\triangle$.

4. Determine whether $\Delta BIG \cong \Delta CAR$? Include an explanation with your work.

$B(-7,7), I(-7,3), G(-2,3)$ and $C(4,-1), A(8,-1),$ and $R(8,4)$

Can use & test for SSS, SAS, or Hy Leg.



$m_{\overline{BI}}$ = undefined (or use slope formula)

$m_{\overline{IG}} = 0$

Because slopes are negative reciprocals, we conclude that $\overline{BI} \perp \overline{IG}$ & $\measuredangle I$ is right because $\perp$ lines form rt $\measuredangle$s, and ΔBIG is a right Δ.

$m_{\overline{RA}}$ = undefined

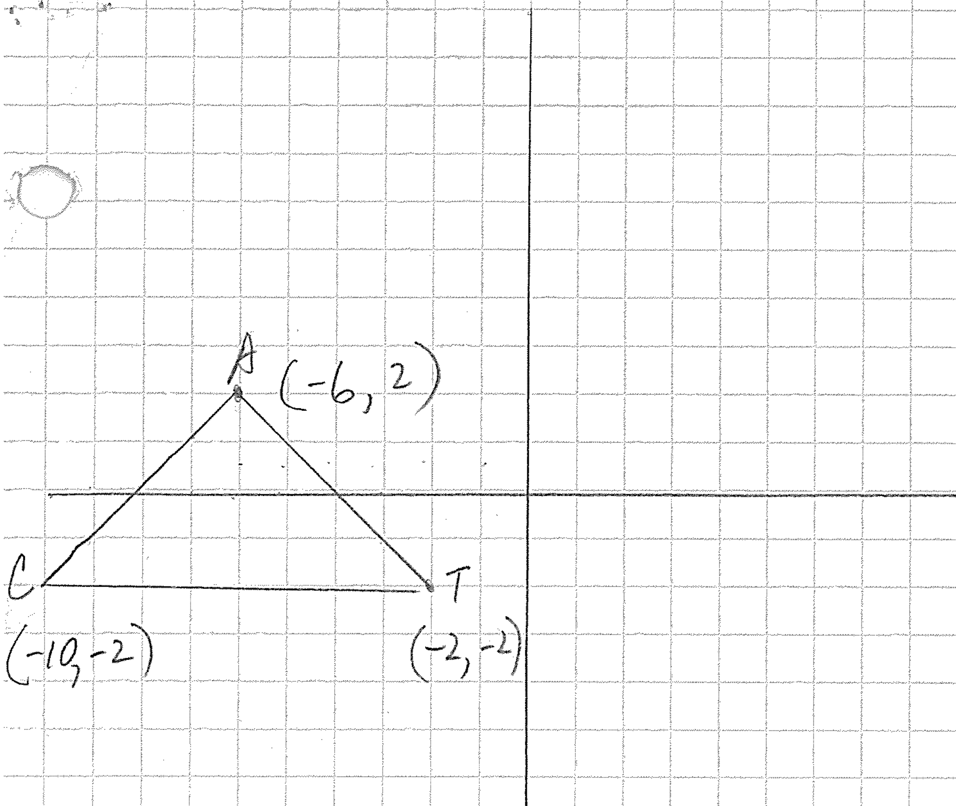
$m_{\overline{CA}} = 0$

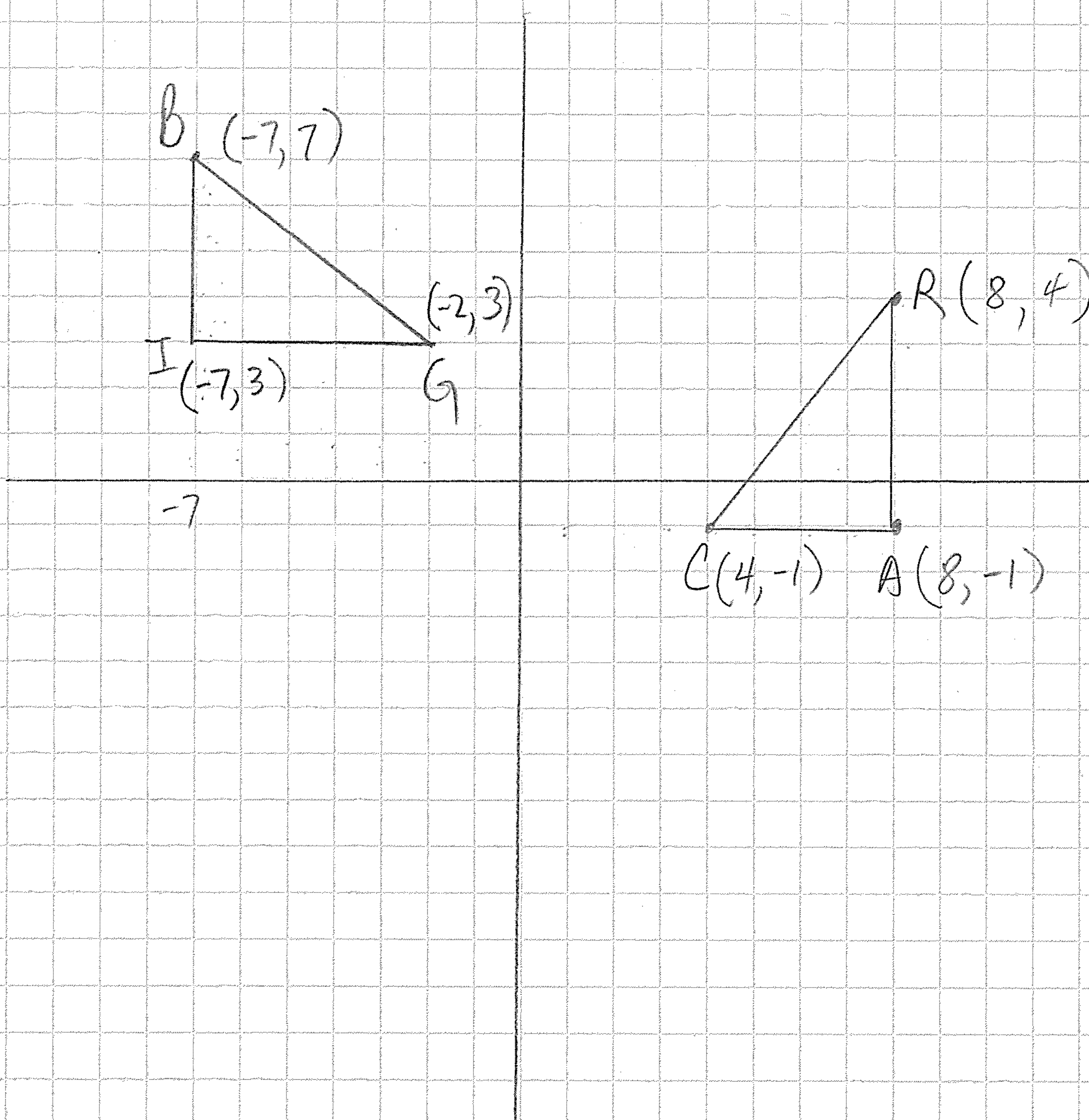
Because slopes are negative reciprocals, we conclude that $\overline{RA} \perp \overline{CA}$ and $\measuredangle A$ is right because $\perp$ lines form rt $\measuredangle$s and ΔCAR is right.

$BI = 4$, $IG = 5$, $CA = 4$, and $RA = 5$, thus, $BI = CA$ and $IG = RA$, which means $\overline{BI} \cong \overline{CA}$ and $\overline{IG} \cong \overline{RA}$.

Since all right $\measuredangle$s $\cong$ and 2 pairs of sides $\cong$,

$\Delta BIG \cong \Delta CAR$ by SAS.

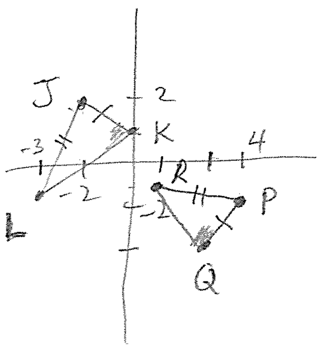

A (-6, 2)
C
T
(-10, -2)
(-2, -2)


B (-7, 7)
(-2, 3)
I (-7, 3)
G
R (8, 4)
-7
C (4, -1)
A (8, -1)

[illegible]

⑤ Given: $J(-2,2)$, $K(0,1)$, $L(-3,-1)$
$P(4,-2)$, $Q(3,-4)$, $R(1,-1)$

Prove: $\angle JKL \cong \angle PQR$



step 1 - verify SSS by using distance formula.

$JK = \sqrt{(-2)^2+(2-1)^2} \rightarrow \sqrt{5}$

$JL = \sqrt{(-2+3)^2+(2+1)^2} \rightarrow \sqrt{1+9} = \sqrt{10}$

$LK = \sqrt{(+3)^2+(2)^2} \rightarrow \sqrt{(9)+4} = \sqrt{13}$

$PQ = \sqrt{(1)^2+(-2+4)^2} \rightarrow \sqrt{5}$

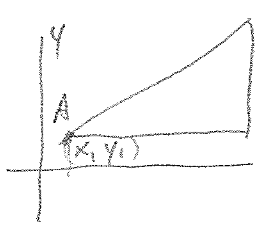
$QR = \sqrt{(3-1)^2+(-4+1)^2} \rightarrow \sqrt{4+9} = \sqrt{13}$

$RP = \sqrt{(4-1)^2+(-2+1)^2} \rightarrow \sqrt{10}$

So, $\overline{JK} \cong \overline{PQ}$, $\overline{JL} \cong \overline{RP}$, and $\overline{LK} \cong \overline{QR}$.

Therefore, $\triangle JKL \cong \triangle PQR$ by the SSS $\triangle$ Congruence Thm and $\angle JKL \cong \angle PQR$ by CPCTC.

⑥ Derive the distance formula



← what are the coordinates? (x_2, y_1)

Pyth. Thm: $(AB)^2+(BC)^2=(AC)^2$

$(x_2-x_1)^2+(y_2-y_1)^2=(AC)^2$

$$\boxed{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = AC}$$

Given: $A(x_1, y_1)$
$C(x_2, y_2)$
where $x_1 \neq x_2$
$y_1 \neq y_2$

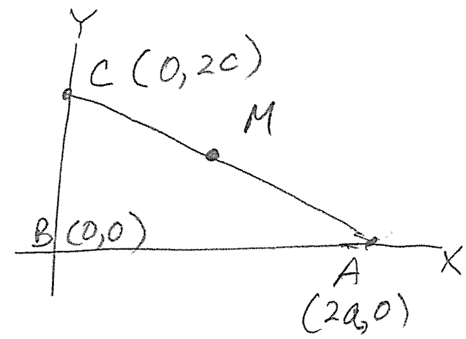
Prove: $AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

FROM "Evaluate : Homework" on p. 453 (text)

① Proof

Given: $\angle B$ is right $\angle$ in $\triangle ABC$
M midpt of $\overline{AC}$

Prove: M is equidistant from all 3 vertices of $\triangle ABC$



$$M: \left(\frac{0+2a}{2}, \frac{2c+0}{2}\right) \to M(a,c)$$

$$MC = \sqrt{(a-0)^2+(c-2c)^2} \to \sqrt{a^2+c^2}$$

$$MA = \sqrt{(a-2a)^2+(c-0)^2} \to \sqrt{a^2+c^2}$$

$$MB = \sqrt{(a-0)^2+(c-0)^2} \to \sqrt{a^2+c^2}$$

So, $MC = MA = MB$. Therefore, M is equidistant from all 3 vertices of $\triangle ABC$.