1. *Phoebe Small's Rocket Problem*

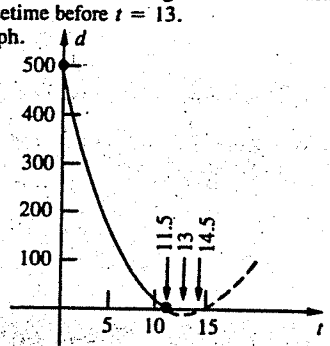
a. General Equation is $d = at^2 + bt + c$.
Ordered pairs: (1, 425), (2, 356), (3, 293)
$a + b + c = 425$
$4a + 2b + c = 356$
$9a + 3b + c = 293$
Solving gives $a = 3$, $b = -78$, $c = 500$.
$\therefore \underline{d = 3t^2 - 78t + 500}$

b. *d*-intercept is *500*. So Phoebe was 500 km away when she *started* firing her rocket engine.

c. $t = 15$: $d = 3(225) - 78(15) + 500 = 5$ km.
$t = 16$: $d = 3(256) - 78(16) + 500 = 20$ km.
So Phoebe appears to be *pulling away* when $t = 16$.

d. Completing the square, $d + 7 = 3(t - 13)^2$. So the vertex is at $(13, -7)$, meaning that Phoebe *crashed* into the surface sometime before $t = 13$.

e. Graph.



f. The model is reasonable until $d = 0$. If $d = 0$, then
$3t^2 - 78t + 500 = 0$

$$t = \frac{78 \pm \sqrt{84}}{6}$$

≈ 14.5 or 11.5
Domain ends when *d first* equals 0; that is, when $t = 11.5$.
$\therefore$ Domain $= \underline{\{t: 0 \le t \le 11.5\}}$

3. *Car Insurance Problem*

a. Let t = no. of years old.
Let A = no. of accidents/100 million km.
General equation is $A = at^2 + bt + c$.
Ordered pairs are (20, 440), (30, 280), (40, 200).
$400a + 20b + c = 440$
$900a + 30b + c = 280$
$1600a + 40b + c = 200$
Solving gives: $a = 0.4$, $b = -36$, $c = 1000$.
$\therefore \underline{A = 0.4t^2 - 36t + 1000}$

b. $t = 80$: $A = 0.4(80)^2 - 36(80) + 1000$
$= \underline{680 \text{ accidents/100 million km.}}$

c. $t = 16$: $A = 0.4(16)^2 - 36(16) + 1000 = 526.4$
$t = 70$: $A = 0.4(70)^2 - 36(70) + 1000 = 440$
$\therefore$ <u>70-year-olds appear to be safer.</u>

d. Safest age is minimum value of A. Since the graph will open upwards, the minimum value of A occurs at the vertex. The vertex is at:

$$t = -\frac{b}{2a} = -\frac{-36}{2(0.4)} = 45.$$

So <u>45-year-olds</u> appear to be the safest.

e. $A = 830$: $0.4t^2 - 36t + 1000 = 830$

$$0.4t^2 - 36t + 170 = 0$$

$$t^2 - 90t + 425 = 0$$

$$t = \frac{90 \pm \sqrt{6400}}{2} = 85 \quad \text{or} \quad 5.$$

Since insurance applies only to *licensed* drivers, t must be greater than or equal to the minimum driving age, often 16 years.
$\therefore$ Domain $= \underline{\{t: 16 \le t \le 85\}}$.

5. *Artillery Problem*

a. Since the path of the projectile is *parabolic*, the function is *quadratic*.
General equation is $y = ax^2 + bx + c$.
Ordered pairs are $(-2, 50)$, $(-1, 410)$, $(3, 250)$.
$4a - 2b + c = 50$
$a - b + c = 410$
$9a + 3b + c = 250$
Solving gives: $a = -80$, $b = 120$, $c = 610$.
$\therefore \underline{y = -80x^2 + 120x + 610}$

b. $x = 2$: $y = -80(2^2) + 120(2) + 610 = \underline{530 \text{ meters}}$
$x = 0$: $y = \underline{610 \text{ meters}}$

c. $y = 130$:

$$-80x^2 + 120x + 610 = 130$$

$$-80x^2 + 120x + 480 = 0$$

$$2x^2 - 3x - 12 = 0$$

$$x = \frac{3 \pm \sqrt{105}}{4} \approx 3.31 \quad \text{or} \quad -1.81$$

Projectile is at $\underline{x \approx 3.31 \text{ km} \quad \text{or} \quad -1.81 \text{ km.}}$

d. *Solution 1:* Set $y = 660$ and see if there are any real values of x.
$y = 660$: $-80x^2 + 120x + 610 = 660$
$-80x^2 + 120x - 50 = 0$
$8x^2 - 12x + 5 = 0$
Discriminant $= -16$, so there are *no* real solns.
There is *no danger of being hit* because there are no values of x for which $y = 660$.

Solution 2: Find the vertex and see if it is above or below 660.

Vertex is at $x = -\dfrac{b}{2a} = \dfrac{-120}{-160} = \dfrac{3}{4}$

$$x = \frac{3}{4}: y = -80\left(\frac{3}{4}\right)^2 + 120\left(\frac{3}{4}\right) + 610 = 655.$$

There is *no danger of being hit* because the projectile gets no higher than 655 m, and the plane is at 660 m.

Turn Over →

7. *Rectangular Field Problem*

a. x = no. of yards wide, for roadway
Let L = no. of yards long for interior.
Let W = no. of yards wide for interior.
$\underline{\underline{L = 500 - 2x}}$

$\underline{\underline{W = 300 - 2x}}$

These are *linear* functions.

b. Let $A(x)$ no. of sq. yards area.
Area = LW
$\therefore A(x) = (500 - 2x)(300 - 2x) = \underline{\underline{150{,}000 - 1600x + 4x^2}}$
This is a *quadratic function.*

c. $A(5) = \underline{\underline{142{,}100}}$

$A(10) = \underline{\underline{134{,}400}}$

$A(15) = \underline{\underline{126{,}900}}$

d. $100{,}000 = 150{,}000 - 1600x + 4x^2$
$4x^2 - 1600x + 50{,}000 = 0$

$x^2 - 400x + 12{,}500 = 0$

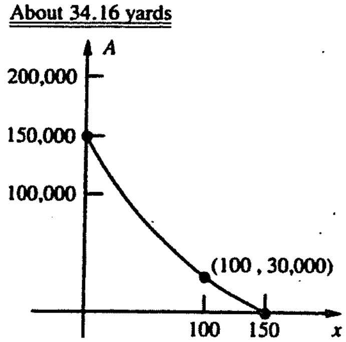
$$x = \frac{400 \pm \sqrt{110{,}000}}{2}$$

$x = \sout{365.831\ldots}$ → out of domain

or 34.168...

About 34.16 yards

e.



f. If area of roadway equals area of field, then each has area $\frac{1}{2}(300)(500)$, or 75,000 sq. yd.

$75{,}000 = 150{,}000 - 1600x + 4x^2$

$x^2 - 400x + 18{,}750 = 0$

$$x = \frac{400 \pm \sqrt{85{,}000}}{2}$$

$x = \sout{345.77\ldots}$ → out of domain

or 54.226...

About 54.27 yd.

9. *Loan Problem*

a. Let d = no. of dollars still owed.
Let w = no. of weeks.
$d = 544 - 17(w - 13)$
$\underline{\underline{d = -17w + 765}}$

b. d varies *linearly* with w because the equation has the form $y = mx + b$.

c. $w = 20$: $d = -17(20) + 765 = 425$
$\underline{\underline{\$425}}$

d. $d < 100$
$-17w + 765 < 100$
$w > 39.117\ldots$
After 40 weeks.

e. d-intercept = $\underline{\underline{765}}$

You originally borrowed \$765.

f. w-intercept:
$0 = -17w + 765$
$w = \underline{\underline{45}}$

It takes 45 weeks to pay off the loan.

g.