Geometry (H) date KEY
Section 11-4 name _____

AREAS OF REGULAR POLYGONS

Study text pages 440 – 442 to complete the definitions below. Also study examples 1 & 2.

Given any circle, you can inscribe in it a regular polygon of any number of sides.

It is also true that if you are given any regular polygon, you circumscribe a circle about it. This relationship between circles and regular polygons leads to the following definitions:

Center of a regular polygon is the center of the circumscribed circle.

Radius of a regular polygon is the distance from the center to a vertex

Central angle of a regular polygon is an ∠ formed by 2 radii drawn to consecutive vertices.

Apothem of a regular polygon is the ⊥ distance from the center of the polygon to a side.

Name each part:

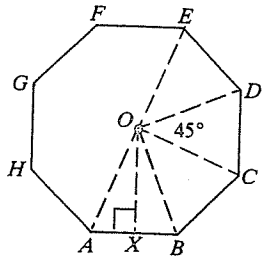
Center of regular octagon: O

Radii: $\overline{OE}$, $\overline{OD}$, $\overline{OC}$, $\overline{OB}$, $\overline{OA}$

Central angles: ∠AOB, ∠BOC, ∠DOC

Measure of the central angle; $\frac{360}{8} = 45$

Apothem: $\overline{OX}$



If you know the apothem and the perimeter of a regular polygon, you can find the area of the polygon by using the following theorem:

The area of a regular polygon is equal to half the product of the apothem and the perimeter.

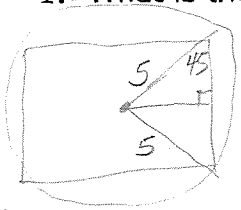
Formula: $A = \frac{1}{2}ap$

Complete the problems on the following pages. Come in with questions.
Work must be done on separate paper with a diagram for each problem.

Geometry Honors
(11.4) Area of Regular Polygons

Name:

1. What is the area of a square inscribed in a circle with a radius of 5 cm?



$x\sqrt{2} = 5$

$x = \frac{5}{\sqrt{2}} \cdot = \frac{5\sqrt{2}}{2}$

Side $= \frac{10\sqrt{2}}{2} = \frac{10}{\sqrt{2}}$

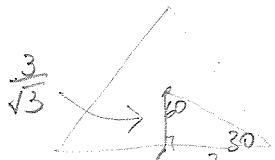
$A = \left(\frac{10}{\sqrt{2}}\right)^2$

$= \frac{100}{2} = 50$ sq. cm

OR $A = \frac{1}{2}\frac{5}{\sqrt{2}}\frac{40}{\sqrt{2}}$

$= \frac{100}{2} = 50$

2. What is the area of an equilateral triangle with side length of 6 cm?



$x\sqrt{3} = 3$

$x = \frac{3}{\sqrt{3}}$

$A = \frac{1}{2}ap$

$= \frac{1}{2}\left(\frac{3}{\sqrt{3}}\right)(18)$

$= \frac{27}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 9\sqrt{3}$ sq. cm

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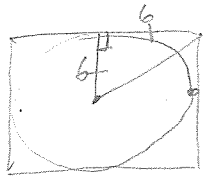
OR $A = \frac{1}{2}(6)\left(\frac{3}{\sqrt{3}}\right)$

$= \frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 3\sqrt{3}$

$A_2 = 3 \times 3\sqrt{3} = 9\sqrt{3}$

3. Find the area of a square circumscribed about a circle with a radius of 6 cm.



Side $= 12$

$A = \frac{1}{2}ap$

$= \frac{1}{2}(6)(48)$

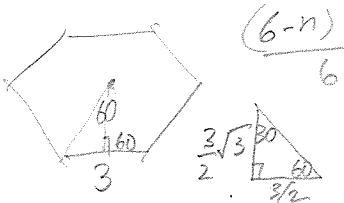
$= 144$

OR $A_\Delta = \frac{1}{2}(6)(12)$

$= 36$

$A_2 = 144$

4. What is the area of a regular hexagon with a side length of 3 cm?



$\frac{(6-n)180}{6} = 120$

$A = \frac{1}{2}\left(\frac{3}{2}\sqrt{3}\right)(18)$

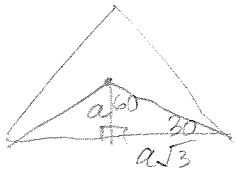
$= \frac{27\sqrt{3}}{2}$

OR $A = \frac{1}{2}\left(\frac{3}{2}\sqrt{3}\right)(3)$

$= \frac{9\sqrt{3}}{4}$

$A = 6\left(\frac{9\sqrt{3}}{4}\right) = \frac{27\sqrt{3}}{2}$

5. An equilateral triangle has an area of $24\sqrt{3}$ cm^2. Find the length of each side.



$\frac{1}{2}ap = 24\sqrt{3}$

$\frac{1}{2}a(6a\sqrt{3}) = 24\sqrt{3}$

$3a^2 = 24$

$a^2 = 8$

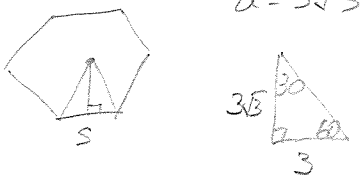
$a = 2\sqrt{2}$

Side $= 4\sqrt{6}$

Side $= 2a\sqrt{3}$

Side $= 2a\sqrt{3} = 2 \cdot 2\sqrt{2}\sqrt{3}$

6. The apothem of a regular hexagon is $3\sqrt{3}$. What is the length of the side?



$a = 3\sqrt{3}$

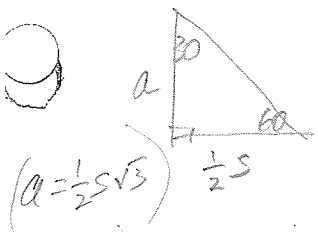
each $\angle = \frac{(6-2)180}{6}$

$= \frac{4}{6}(180)$

$= 120$

each side $= 6$ units

7. The area of a regular hexagon is $72\sqrt{3}$. What is its side length?



$72\sqrt{3} = \frac{1}{2}ap$

$= \frac{1}{2}\left(\frac{\sqrt{3}}{2}s\right)(6s)$

$= \frac{3\sqrt{3}s^2}{2}$

$\left(a = \frac{1}{2}s\sqrt{3}\right)$

$\frac{72\sqrt{3}}{1}\left(\frac{2}{3\sqrt{3}}\right) = s^2$

$48 = s^2$

$4\sqrt{3} = s$

$a = \frac{1}{2}4\sqrt{3}\sqrt{3} = 6$

Ck

$72\sqrt{3} = \frac{1}{2}ap$

$= \frac{1}{2}(6)(24\sqrt{3})$

$= 72\sqrt{3}$ ✓