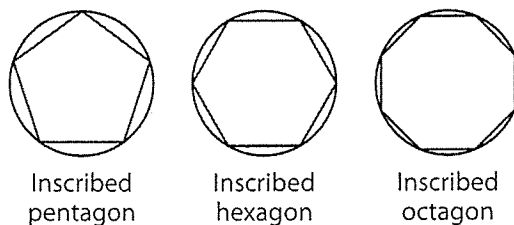


# 16.1 Justifying Circumference and Area of a Circle

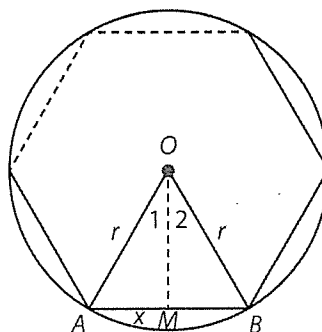
**Essential Question:** How can you justify and use the formulas for the circumference and area of a circle?

## Explore Justifying the Circumference Formula

To find the circumference of a given circle, consider a regular polygon that is inscribed in the circle. As you increase the number of sides of the polygon, the perimeter of the polygon gets closer to the circumference of the circle.



Let circle  $O$  be a circle with center  $O$  and radius  $r$ . Inscribe a regular  $n$ -gon in circle  $O$  and draw radii from  $O$  to the vertices of the  $n$ -gon.



Let  $\overline{AB}$  be one side of the  $n$ -gon. Draw  $\overline{OM}$ , the segment from  $O$  to the midpoint of  $\overline{AB}$ .

(A) Then  $\triangle AOM \cong \triangle BOM$  by \_\_\_\_\_.

(B) So,  $\angle 1 \cong \angle 2$  by \_\_\_\_\_.

(C) There are  $n$  triangles, all congruent to  $\triangle AOB$ , that surround point  $O$  and fill the  $n$ -gon.

Therefore,  $m\angle AOB = \boxed{\phantom{000}}$  and  $m\angle 1 = \boxed{\phantom{000}}$ .

- Ⓓ Since  $\angle OMA \cong \angle OMB$  by CPCTC, and  $\angle OMA$  and  $\angle OMB$  form a linear pair, these angles are supplementary and must have measures of  $90^\circ$ . So  $\triangle AOM$  and  $\triangle BOM$  are right triangles.

$$\text{In } \triangle AOM, \sin \angle 1 = \frac{\text{length of opposite leg}}{\text{length of hypotenuse}} = \frac{x}{r}.$$

So,  $x = r \sin \angle 1$  and substituting the expression for  $m\angle 1$  from above gives

$$x = r \sin \frac{180^\circ}{n}.$$

- Ⓔ Now express the perimeter of the  $n$ -gon in terms of  $x$ .

The length of  $\overline{AB}$  is  $2x$ , because \_\_\_\_\_.

This means the perimeter of the  $n$ -gon is \_\_\_\_\_.

Substitute the expression for  $x$  in Step D.

The perimeter of the  $n$ -gon in terms of  $x$  is \_\_\_\_\_.

- Ⓕ Your expression for the perimeter of the  $n$ -gon should include the factor  $n \sin \left( \frac{180^\circ}{n} \right)$ . What happens to this factor as  $n$  gets larger?

What happens to the value of  $x \sin \left( \frac{180^\circ}{x} \right)$  as  $x$  gets larger?

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- Ⓖ Look at the expression you wrote for the perimeter of the  $n$ -gon. What happens to the value of this expression, as  $n$  gets larger?

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### Reflect

1. When  $n$  is very large, does the perimeter of the  $n$ -gon ever equal the circumference of the circle? Why or why not?

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2. How does the above argument justify the formula  $C = 2\pi r$ ?

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