Name Notes Date

2.1 Transformations of Quadratic Functions

For use with Exploration 2.1

Essential Question How do the constants *a*, *h*, and *k* affect the graph of the quadratic function $g(x) = a(x-h)^2 + k$?

1 EXPLORATION: Identifying Graphs of Quadratic Functions

Work with a partner. Match each quadratic function with its graph. Explain your reasoning. Then use a graphing calculator to verify that your answer is correct.

a. $g(x) = -(x-2)^2$ D

b. $g(x) = (x-2)^2 + 2$ C

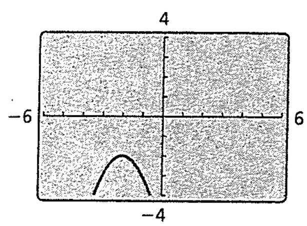
c. $g(x) = -(x+2)^2 - 2$ A

d. $g(x) = 0.5(x-2)^2 - 2$ F

e. $g(x) = 2(x-2)^2$ B

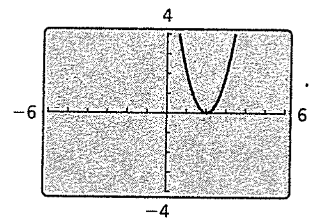
f. $g(x) = -(x+2)^2 + 2$ E

A.



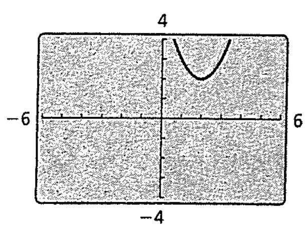
$g(x) = -(x+2)^2 - 2$

B.



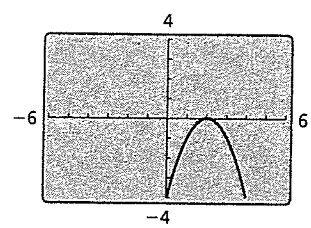
$g(x) = 2(x-2)^2$

C.



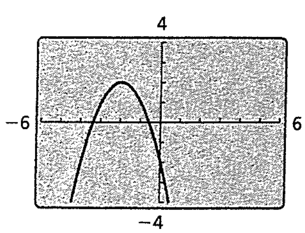
$g(x) = (x-2)^2 + 2$

D.



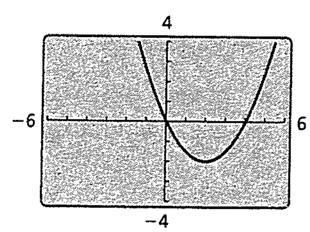
$g(x) = -(x-2)^2$

E.



$g(x) = -(x+2)^2 + 2$

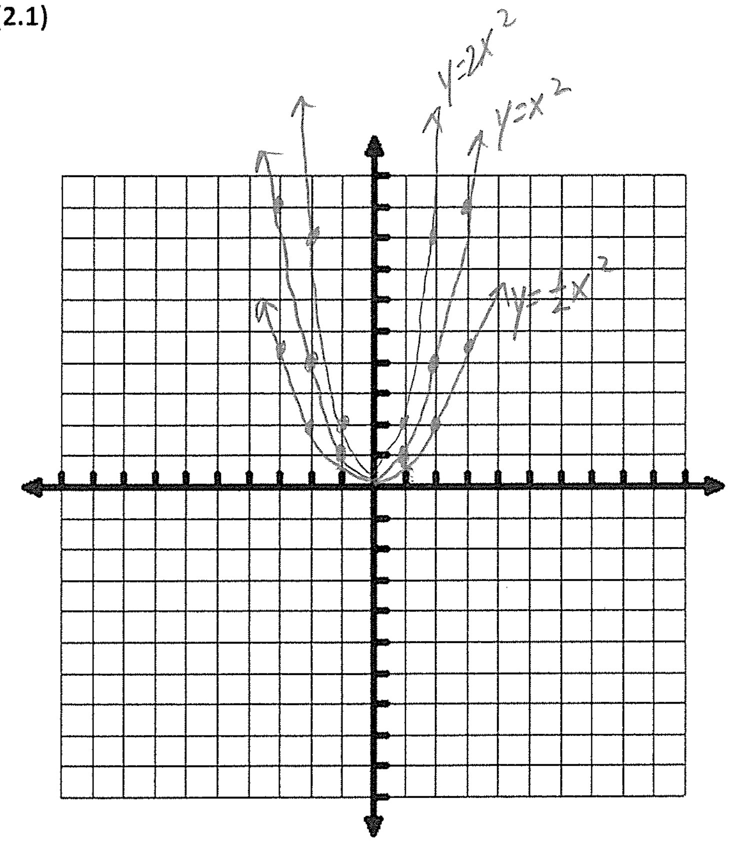
F.



$g(x) = 0.5(x-2)^2 - 2$

Transformations of Quadratic Functions (2.1)

A. On this same quadrant, graph:



$y = x^2$

outside → function

$y = \frac{1}{2}x^2$

"a" value

$y = 2x^2$ ← function

"a" value

What is the difference between $y = \frac{1}{2}x^2$ and $y = 2x^2$?

x^2 gives y values.
"$\frac{1}{2}$" decreases each y value.
The final y values are growing slower → vertical shrink.

x^2 gives y values.
"2" doubles each resulting y value.
So final y values are growing faster → vertical stretch

CONCLUSION: Vertical Stretch & Shrink

$a > 1$ $\quad$ $0 < a < 1$

B. On this same quadrant, graph:

$y = x^2$

inside

$y = (\frac{1}{2}x)^2$

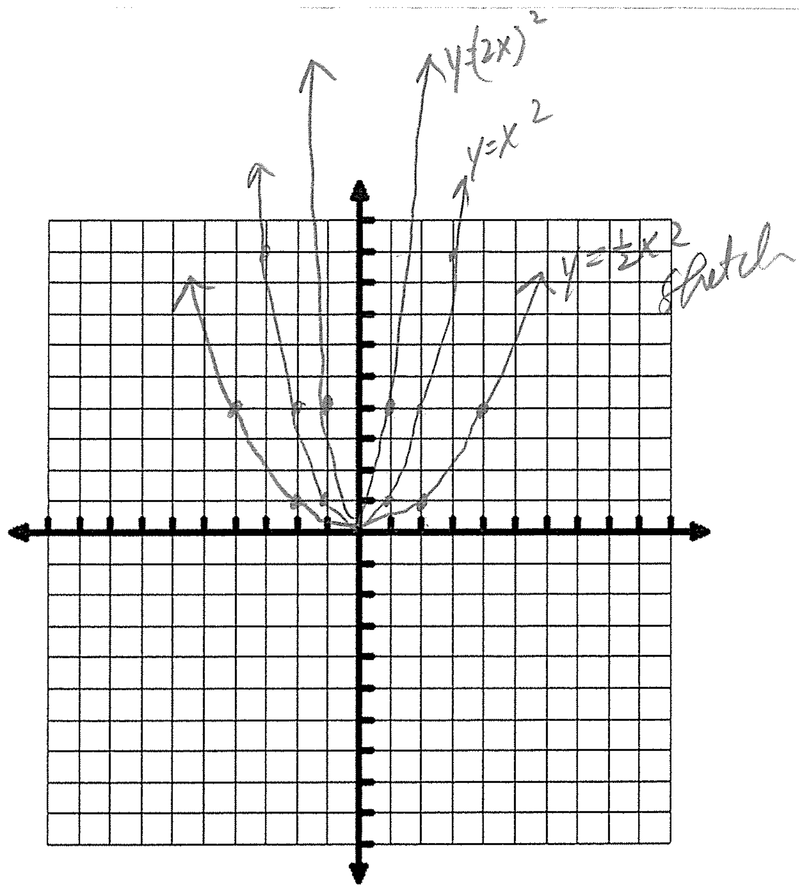
x	y
2	1
4	4

Horizontal stretch

$y = (2x)^2$

x	y
1	4
2	16

Horizontal shrink



What is the difference between $y = (\frac{1}{2}x)^2$ and $y = (2x)^2$?

B/c x values are being halved and then squared, y values growing slower, making arms of parabola more spread out → horizontal stretch

B/c x is doubled and then the resulting value is squared, the y values grow very fast. Viewed another way, the arms of the parabola move away from x-axis → horizontal shrink.

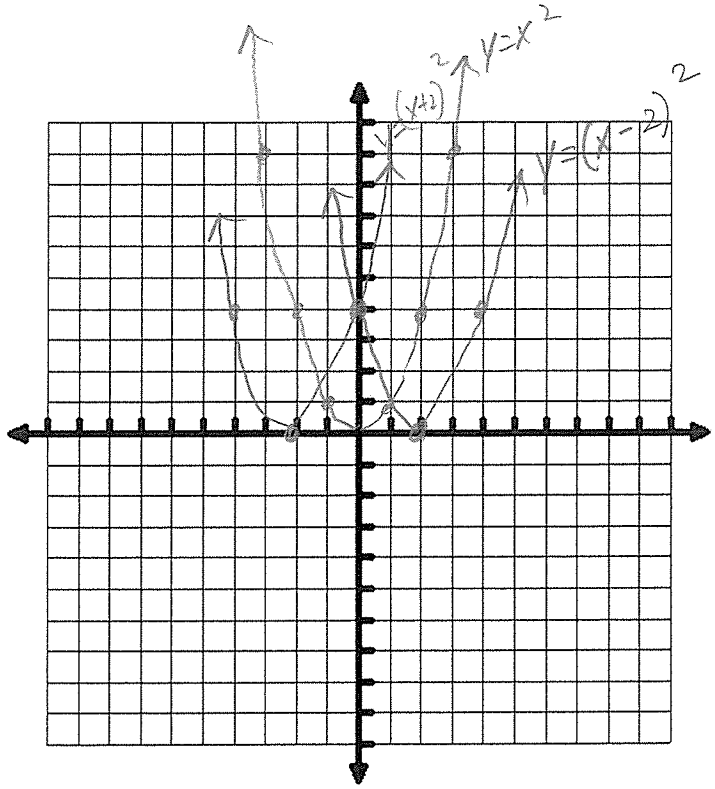
CONCLUSION:

Horizontal stretch & shrink

↓ $0<a<1$

↓ $a>1$

C. On this same quadrant, graph:



$y = x^2$

$y = (x-2)^2$

X	Y
0	4
2	0
4	4

$y = (x+2)^2$

X	Y
-2	0
0	4
2	16

What is the difference between $y = (x-2)^2$ and $y = (x+2)^2$?

Shifts right 2 units.

Shifts left 2 units

CONCLUSION: Compare to $y = a(x-h)^2 + K$

$$y = (x+2)^2$$

$$\downarrow$$

$$y = (x - \underbrace{-2}_{\uparrow})^2$$

why shift left.

So: $y = a(x-h)^2 + K$ ← shift vertically

← shift horizontally

reflection across x-axis

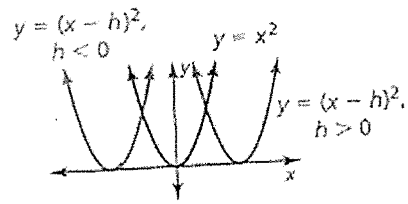
vertical stretch or shrink

Core Concept

Horizontal Translations

$f(x) = x^2$

$f(x - h) = (x - h)^2$

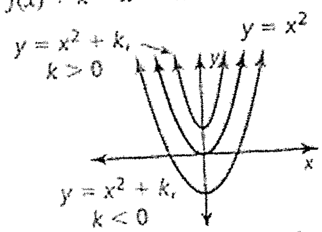


- shifts left when $h < 0$
- shifts right when $h > 0$

Vertical Translations

$f(x) = x^2$

$f(x) + k = x^2 + k$



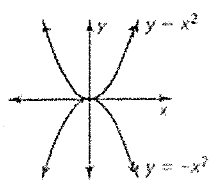
- shifts down when $k < 0$
- shifts up when $k > 0$

Core Concept

Reflections in the x-Axis

$f(x) = x^2$

$-f(x) = -(x^2) = -x^2$

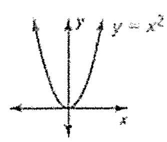


flips over the x-axis

Reflections in the y-Axis

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

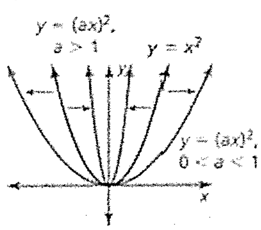


$y = x^2$ is its own reflection in the y-axis.

Horizontal Stretches and Shrinks

$f(x) = x^2$

$f(ax) = (ax)^2$

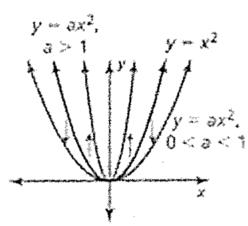


- horizontal stretch (away from y-axis) when $0 < a < 1$
- horizontal shrink (toward y-axis) when $a > 1$

Vertical Stretches and Shrinks

$f(x) = x^2$

$a \cdot f(x) = ax^2$



- vertical stretch (away from x-axis) when $a > 1$
- vertical shrink (toward x-axis) when $0 < a < 1$

Examples:

1. Describe ~~the~~ each transformations of $y=x^2$ represented by

   a. $y = (x+2)^2 - 1$ $\rightarrow$ $y=(x--2)^2-1$

   $y=x^2$ has been translated 2 units left and 1 unit down

   b. $y = -\frac{1}{2}x^2$

   $y=x^2$ has been reflected across the x-axis and underwent a vertical shrink by a factor of $\frac{1}{2}$.

2. Write an equation for each transformation:

   a. Let the graph of g be a vertical stretch by a factor of 2 and a reflection in the x-axis, followed by a translation 3 units down of the graph of $y = x^2$. Write a rule for g and identify the vertex.

   outside

   $K=-3$

   $\left.\begin{array}{l}\text{vertical stretch by } 2\\ \text{reflection in x-axis}\end{array}\right\}\ a=-2$

   $y=a(x-h)^2+k$

   $y=-2(x-0)^2-3$

   $y=-2x^2-3$