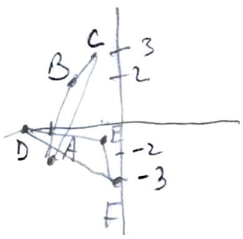
Coordinate Geom. of $\triangle$s. Textbook p. 452

⑩ Given: $A(-4,-2)$, $B(-3,2)$, $C(-1,3)$, $D(-5,0)$, $E(-1,-1)$, $F(0,-3)$

Prove: $\angle BCA \cong \angle EFD$



$AB = \sqrt{(-4+3)^2+(-2-2)^2} \rightarrow \sqrt{1+16} = \sqrt{17}$

$BC = \sqrt{(-3+1)^2+(2-3)^2} \rightarrow \sqrt{4+1} = \sqrt{5}$

$AC = \sqrt{(-4+1)^2+(-2-3)^2} \rightarrow \sqrt{9+25} = \sqrt{34}$

$DE = \sqrt{(-5+1)^2+(0+1)^2} \rightarrow \sqrt{16+1} = \sqrt{17}$

$EF = \sqrt{(-1)^2+(-1+3)^2} \rightarrow \sqrt{1+4} = \sqrt{5}$

$FD = \sqrt{(-5)^2+(0+3)^2} \rightarrow \sqrt{25+9} = \sqrt{34}$

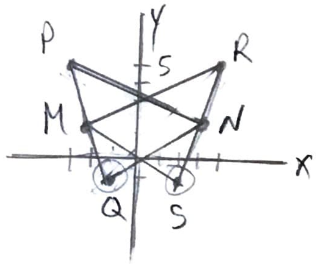
Since $AB = DE$, $BC = EF$, and $AC = FD$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{FD}$ b/c if segments are equal in measure, they are congruent. Since 3 pairs of sides are congruent, $\triangle ABC \cong \triangle DEF$ by the SSS $\triangle \cong$ Postulate. Therefore, we conclude $\angle BCA \cong \angle EFD$ b/c CPCTC.

⑪

(11) Given: $P(-3,5)$, $Q(-1,-1)$, $R(4,5)$, $S(2,-1)$

M midpt $\overline{PQ}$, N midpt $\overline{RS}$.

Prove: $\measuredangle PQN \cong \measuredangle RSM$ (Possible Plan: Show $\triangle PQN \cong \triangle RSM$)



$$M = \left(\frac{-3-1}{2}, \frac{5-1}{2}\right) \rightarrow M(-2, 2)$$

$$N = \left(\frac{4+2}{2}, \frac{5-1}{2}\right) \rightarrow N(3, 2)$$

$$PQ = \sqrt{(-3+1)^2+(5+1)^2} \rightarrow \sqrt{4+36} = \sqrt{40} \qquad RS = \sqrt{(4-2)^2+(5+1)^2} \Rightarrow \sqrt{4+36} = \sqrt{40}$$

$$QN = \sqrt{(-1-3)^2+(-1-2)^2} \rightarrow \sqrt{16+9} = 5 \qquad SM = \sqrt{(2+2)^2+(-1-2)^2} \rightarrow \sqrt{16+9} = 5$$

$$NP = \sqrt{(3+3)^2+(2-5)^2} \rightarrow \sqrt{36+9} = \sqrt{45} \qquad MR = \sqrt{(-2-4)^2+(2-5)^2} \rightarrow \sqrt{36+9} = \sqrt{45}$$

Since $PQ = RS$, $QN = SM$, $NP = MR$, $\overline{PQ} \cong \overline{RS}$, $\overline{QN} \cong \overline{SM}$, and $\overline{NP} \cong \overline{MR}$ b/c segments with equal measures are $\cong$. Consequently, $\triangle PQN \cong \triangle RSM$ by SSS $\triangle \cong$ Post. and $\measuredangle PQN \cong \measuredangle RSM$ by CPCTC.

Text p.453 (Coord. Geo of Δs)

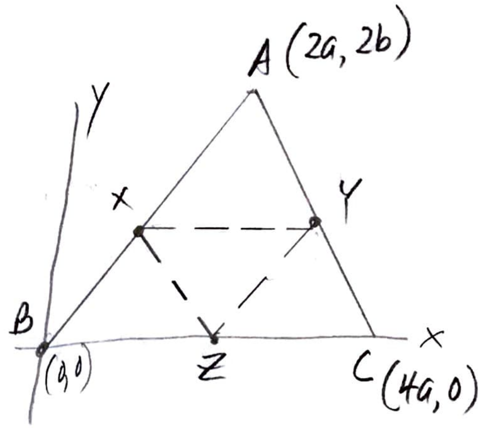
③ Given: ΔABC is isosceles

X midpt of $\overline{AB}$

Y midpt of $\overline{AC}$

Z is midpt of $\overline{BC}$

Prove: ΔXYZ isosceles.



$$X = \left(\frac{0+2a}{2}, \frac{0+2b}{2}\right) \Rightarrow (a, b)$$

$$Z = \left(\frac{0+4a}{2}, \frac{0+0}{2}\right) \rightarrow (2a, 0)$$

$$Y = \left(\frac{2a+4a}{2}, \frac{2b+0}{2}\right) \Rightarrow (3a, b)$$

$$XY = \sqrt{(a-3a)^2 + (b-b)^2} \Rightarrow \sqrt{(-2a)^2} \Rightarrow \sqrt{4a^2} = 2a$$

$$XZ = \sqrt{(a-2a)^2 + (b-0)^2} \Rightarrow \sqrt{a^2+b^2}$$

$$YZ = \sqrt{(3a-2a)^2 + (b-0)^2} \Rightarrow \sqrt{a^2+b^2}$$

Since $XZ = YZ$, $\overline{XZ} \cong \overline{YZ}$ b/c segments with equal measures are congruent. Therefore, ΔXYZ is an isosceles triangle b/c an isosceles Δ has at least 2 congruent sides.

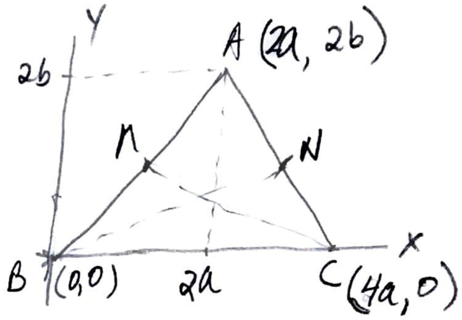
⑤ Given: $\triangle ABC$ is isosceles.

M midpt of $\overline{AB}$.

N midpt of $\overline{AC}$.

$\overline{AB} \cong \overline{AC}$

Prove: $\overline{MC} \cong \overline{NB}$



$$M = \left(\frac{2a+0}{2}, \frac{2b+0}{2}\right) \Rightarrow (a, b)$$

$$N = \left(\frac{2a+4a}{2}, \frac{2b+0}{2}\right) \Rightarrow (3a, b)$$

$$MC = \sqrt{(4a-a)^2+(0-b)^2} \Rightarrow \sqrt{9a^2+b^2}$$

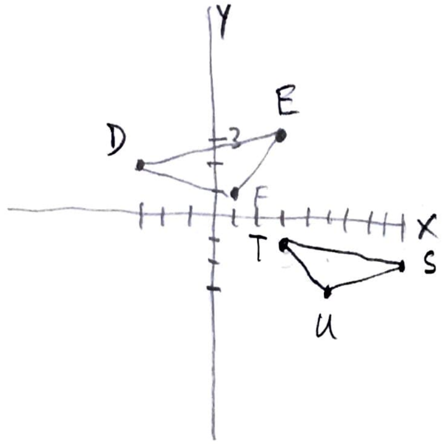
$$NB = \sqrt{(3a-0)^2+(b-0)^2} \Rightarrow \sqrt{9a^2+b^2}$$

Since $MC = NB$, $\overline{MC} \cong \overline{NB}$ b/c segments with equal measures are congruent.

p. 454

(9) Given: $D(-3,2)$, $E(3,3)$, $F(1,1)$, $S(9,-2)$, $T(3,-1)$, $U(5,-3)$

Prove: $\angle FDE \cong \angle UST$



$DE = \sqrt{(-3-3)^2+(2-3)^2} \Rightarrow \sqrt{36+1} = \sqrt{37}$

$EF = \sqrt{(3-1)^2+(3-1)^2} \Rightarrow \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$DF = \sqrt{(1+3)^2+(1-2)^2} \Rightarrow \sqrt{16+1} = \sqrt{17}$

$TU = \sqrt{(5-3)^2+(-3+1)^2} \Rightarrow \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$TS = \sqrt{(9-3)^2+(-2+1)^2} \Rightarrow \sqrt{36+1} = \sqrt{37}$

$SU = \sqrt{(9-5)^2+(-2+3)^2} \Rightarrow \sqrt{16+1} = \sqrt{17}$

It has been shown that $DE = TS$, $EF = TU$, and $DF = SU$. Therefore, $\overline{DE} \cong \overline{TS}$, $\overline{EF} \cong \overline{TU}$, and $\overline{DF} \cong \overline{SU}$ since segments with equal measures are congruent. We can conclude that $\triangle DEF \cong \triangle STU$ by the SSS $\triangle \cong$ Thm and as a result, $\angle FDE \cong \angle UST$ b/c CPCTC.