

Chapter : Special Right Triangles

Lesson: Geometric Means

$$a:b=c:d \rightarrow \frac{a}{b} = \frac{c}{d}$$

means

Do Now: Set up a proportion to find the geometric mean, x , for each set of numbers.
(Think: geometric mean is the *mean* in a proportion.)

Example: For the geometric mean of 2 and 9, use: $\frac{x}{2} = \frac{9}{x}$ or $x = \sqrt{2 \cdot 9}$

1. The geometric mean of 4 and 25 is 10

$$\frac{x}{4} = \frac{25}{x} \quad x^2 = 100$$

$x = 10$ ← only positive value

2. The geometric mean of 9 and 20 is $6\sqrt{5}$ ← No decimals!

$$\frac{9}{x} = \frac{x}{20} \quad x = \sqrt{180} \rightarrow 6\sqrt{5}$$

DISCOVERY: Three right triangles on paper

Using the 2 smaller triangles, can you write a proportion to find the length of the altitude?

$$\frac{Alt}{y} = \frac{x}{Alt}$$

Using ΔI and the largest triangle, can you write a proportion to find the short leg, "a"?

$$\frac{a}{x} = \frac{c}{a}$$

LG → a (hyp) ← LG
sm → x (hyp) ← sm

Using ΔII and the largest triangle, can you write a proportion to find the long leg, "b"?

$$\frac{b}{y} = \frac{c}{b}$$

LG → b (hyp) ← LG
sm → y (hyp) ← sm

THEOREM: If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

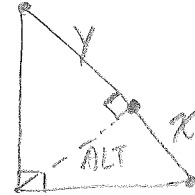
(summary: all 3 ds are ~)

TWO COROLLARIES:

1. When the altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean between the segments of the hypotenuse.

alt. $\rightarrow A$
part of hyp $\rightarrow x$

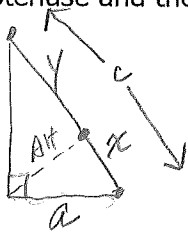
$$\frac{A}{x} = \frac{y}{A} \quad \text{or} \quad A = \sqrt{x \cdot y}$$



2. When the altitude is drawn to the hypotenuse of a right triangle, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is adjacent to that leg.

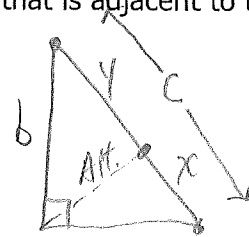
short leg $\rightarrow a$
hyp $\rightarrow c$

$$\frac{a}{c} = \frac{x}{a}$$

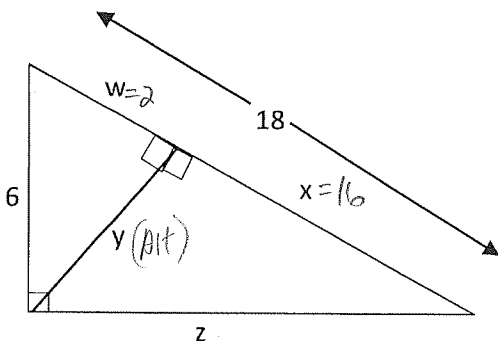


long leg $\rightarrow b$
hyp $\rightarrow c$

$$\frac{b}{c} = \frac{y}{b}$$



Example: Find the values of w , x , y , z .



$$\textcircled{1} \quad \frac{w}{18} = \frac{6}{16}$$

$$\boxed{w=2}$$

$$\textcircled{2} \quad 18 = w + x$$

$$18 = 2 + x$$

$$\boxed{16 = x}$$

$$\textcircled{4} \quad \frac{z}{18} = \frac{16}{z}$$

$$z = 12\sqrt{2}$$

$$\textcircled{3} \quad \frac{2}{y} = \frac{6}{16}$$

$$y^2 = 32$$

$$y = 4\sqrt{2}$$

No decimals!