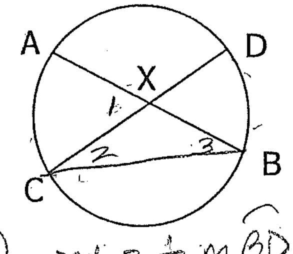
Thm 9.9 – The measure of an angle formed by two chords intersecting inside a circle is one half the <u>sum</u> of the intercepted arc.

Given: chords $\overline{AB}$ and $\overline{CD}$ intersecting at point X.

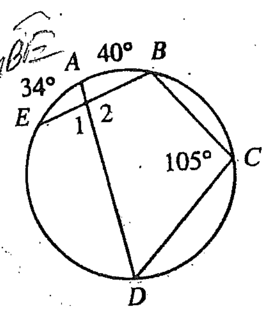
Prove: m∠AXC = $\frac{1}{2}(mAC + mBD)$



① Draw $\overline{BC}$.
② $m\angle 1 = m\angle 2 + m\angle 3$
③ $m\angle 2 = \frac{1}{2} m\widehat{BD}$ → ④ $m\angle 1 = \frac{1}{2} m\widehat{BD} + m\angle 3$
⑤ $m\angle 3 = \frac{1}{2} m\widehat{AC}$ → ⑥ $m\angle 1 = \frac{1}{2} m\widehat{BD} + \frac{1}{2} m\widehat{AC}$
⑦ $m\angle AXC = \frac{1}{2}(m\widehat{BD} + m\widehat{AC})$

① 2 pts determine a line.
② Exterior ∠ Thm
③ Inscribed ∠ = ½ intercept arc
④ Substitution
⑤ Inscribed ∠ = ½ intercepted arc
⑥ Substitution
⑦ distributive prop.

Example 1: Find m∠1

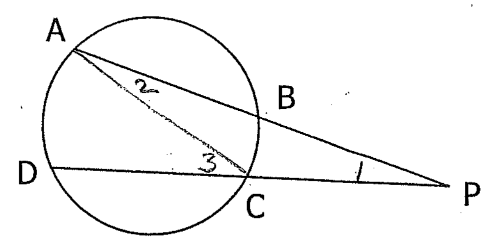


$m\angle 1 = \frac{1}{2}(m\widehat{DE} + 40)$
$= \frac{1}{2}(136 + 40)$
$m\angle 1 = 88$

$m\widehat{DE} = m\widehat{BD} - m\widehat{BE}$
$= 210 - 74$
$= 136$

Thm 9.10 – The measure of an angle formed by two secants, two tangents or a secant and a tangent drawn from a point in the exterior of a circle is equal to half the <u>difference</u> of the measures of the intercepted arc.

Let's prove the case of two secant lines.



Given: $\overrightarrow{PA}$ and $\overrightarrow{PD}$ secants

Prove: m∠P = $\frac{1}{2}(mAD - mBC)$

How?

$m\angle P = \frac{1}{2} m\widehat{AD} - \frac{1}{2} m\widehat{BC}$

$m\angle P + \frac{1}{2} m\widehat{BC} = \frac{1}{2} \widehat{AD}$

① Draw $\overline{AC}$.
② $m\angle 3 = m\angle 1 + m\angle 2$
③ $m\angle 3 = \frac{1}{2} m\widehat{AD}$ → ④ $\frac{1}{2} m\widehat{AD} = m\angle 1 + m\angle 2$
⑤ $m\angle 2 = \frac{1}{2} m\widehat{BC}$ → ⑥ $\frac{1}{2} m\widehat{AD} = m\angle 1 + \frac{1}{2} m\widehat{BC}$
⑦ $\frac{1}{2} m\widehat{AD} - \frac{1}{2} m\widehat{BC} = m\angle 1$
⑧ $\frac{1}{2}(m\widehat{AD} - m\widehat{BC}) = m\angle P$

① 2 pts determine a line
② Ext. ∠ Thm
③ Inscribed ∠ = ½ intercepted arc
⑤ Inscribed ∠ = ½ int. arc