AA2H

Chapter: Exponential Growth and Dxecay

Name__________

date___________

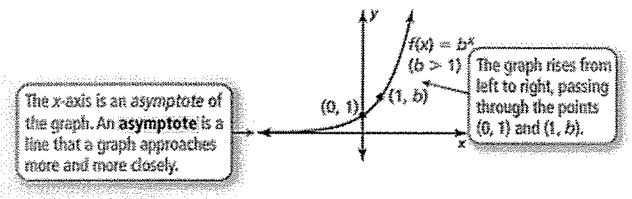
Exponential Function: $y = ab^x$, where $a \neq 0$ and the base b is a positive real number other than 1.

- If $a > 0$ and $b > 1$, then $y = ab^x$ is an **exponential growth function**, and b is called the **growth factor.** The simplest type of exponential growth function has the form $y = b^x$.
- If $a > 0$ and $0 < b < 1$, then $y = ab^x$ is an **exponential decay function** and b is called the **decay factor.**

Core Concept

Parent Function for Exponential Growth Functions

The function $f(x) = b^x$, where $b > 1$, is the parent function for the family of exponential growth functions with base b. The graph shows the general shape of an exponential growth function.

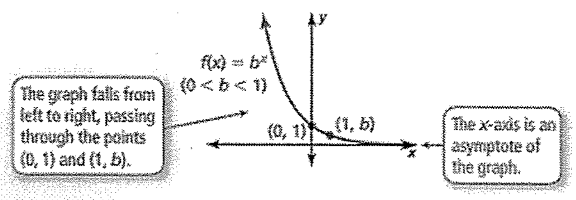


The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

Core Concept

Parent Function for Exponential Decay Functions

The function $f(x) = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base b. The graph shows the general shape of an exponential decay function.

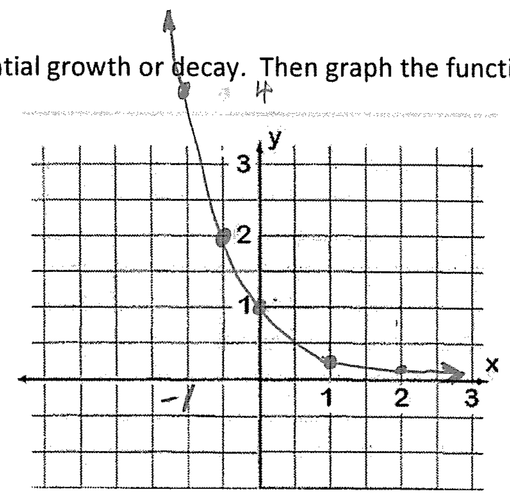


The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

Ex 1: Graphing exponential growth and decay functions

Tell whether the function $f(x) = (0.25)^x$ represents exponential growth or decay. Then graph the function. Be sure to label a few points.

x	f(x)
0	1
1	0.25
2	0.0625
3	0.015625
-1	4
$-\frac{1}{2}$	2



Section II: Exponential Models

Some real-life quantities increase or decrease by a fixed percent each year (or some other time period). The amount y of such a quantity after *t* years can be modeled by one of these equations.

Exponential Growth Model

$y = a(1 + r)^t$

Exponential Decay Model

$y = a(1 - r)^t$

- *a* is the initial amount
- *r* is the percent increase or decrease written as a decimal
- *1+r* is the growth factor.
- *1-r* is the decay factor.

Again, what makes up the growth factor and the decay factor? growth factor = 1 + % increase
decay factor = 1 - % decrease

Growth factor is 1+ % increase. Decay factor is 1 − % decrease.

Which part of the models represents the percent increase and percent decrease? r

Example 2: Depreciation

The value of a car y (in thousands of dollars) can be approximated by the model $y = 25(0.85)^t$, where t is the number of years since the car was new.

a. Does the model represent exponential growth or exponential decay? decay

b. What is the annual percent increase or decrease in the value of the car? 15% decrease $(1-0.85)$

c. When will the car have a value of \$8000? Use the TRACE feature of your graphing calculator and enter the function in Y_1. in year 7

Example 3: Population

initial amount

In 2000, the world population was about 6.09 billion. During the next 13 years, the world population increased by about 1.18% each year. 2000 = year 0

r 0.0118

a. Write an exponential growth model giving the population y (in billions) t years after 2000. Estimate the world population in 2005.

$y = a(1+r)^t \rightarrow y = 6.09(1+0.0118)^t$

$\boxed{y = 6.09(1.0118)^t}$

Year 2005 → $t = 5$

$y = 6.09(1.0118)^5$

$y \approx 6.457 \rightarrow \boxed{6.46 \text{ billion}}$

b. Estimate the year when the world population was 7 billion. Use the TABLE feature on your graphing calculator and enter the function in Y_1. year 2012

According to UN, in 2018, pop'n = 7.6 bill

Example 4: Decay

The amount y (in grams) of the radioactive isotope chromium-51 remaining after t days is $y = a(0.5)^{\frac{t}{28}}$, where a is the initial amount in grams. What percent of the chromium-51 decays each day?

$y = a\left[(0.5)^{\frac{1}{28}}\right]^t$ power of a power prop.

$a(0.9755)^t$

$1 - r = 0.9755 \rightarrow r = 0.02445 \rightarrow \boxed{2.45\% \text{ decays each day}}$

Example 5: Compound Interest

You deposit \$9000 in an account that pays 1.46% annual interest. Find the balance after 3 years when the interest is compounded quarterly.

Core Concept

Compound Interest

Consider an initial principal P deposited in an account that pays interest at an annual rate r (expressed as a decimal), compounded n times per year. The amount A in the account after t years is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$= 9000\left(1 + \frac{0.0146}{4}\right)^{4 \cdot 3}$

$= 9402.2106$

$\boxed{\text{Balance} = \$9402.21}$

Practice Exponential Growth and Decay

1. The value of a car y (in thousands of dollars) can be approximated by the model $y = 31(0.92)^t$, where t is the number of years since the car was new.
   a. Does the model represent exponential growth or exponential decay? decay
   b. What is the annual percent increase or decrease in the value of the car? 8% decrease
   c. When will the car have a value of \$9600? After 14 years
   (May have to use TABLE)

2. In 2000, the world population was about 1.04 million. During the next 14 years, the world population increased by about 2.05% each year.
   $r = 0.0205$ $\quad y = a(1+r)^t$
   a. Write an exponential growth model giving the population y (in millions) t years after 2000. Estimate the world population in 2008.
   $y = 1.04(1+0.0205)^t \rightarrow \boxed{y = 1.04(1.0205)^t}$
   $y = 1.04(1.0205)^8 \Rightarrow 1.223 \rightarrow \boxed{1.22 \text{ million}}$
   b. Estimate the year when the city's population was 1.3 million. 2011
   $y = 11 \rightarrow$

3. The amount y (in grams) of the radioactive isotope barium-140 remaining after t days is $y = a(0.5)^{\frac{t}{13}}$, where a is the initial amount in grams. What percent of the barium-140 decays each day?
   $y = a(0.5)^{\frac{t}{13}}$
   $y = a(0.5^{\frac{1}{13}})^t \rightarrow y = a(0.948)^t$
   $1 - r = 0.948$
   $r \approx 0.0519 \rightarrow \boxed{5.19\% \text{ per day}}$

4. You deposit \$8600 in an account that pays 1.32% annual interest. Find the balance after 4 years when the interest is compounded quarterly.
   P $\quad r = 0.0132$ $\quad t = 4$ $\quad n = 4$
   $A = P(1 + \frac{r}{n})^{nt}$
   $A = 8600(1 + \frac{0.0132}{4})^{4 \cdot 4} \rightarrow \boxed{\$9065.49}$

5. The amount y (in grams) of the radioactive isotope iodine-123 remaining after t ~~days~~ hours is $y = a(0.5)^{\frac{t}{13}}$, where a is the initial amount in grams. What percent of the iodine-123 decays each hour?
   5.19% each hr.

6. You deposit \$9000 in an account that pays 1.46% annual interest. Find the balance after 3 years when the interest is compounded daily.
   P $\quad n = 365$ $\quad r = 0.0146$ $\quad t = 3$
   $A = P(1 + \frac{r}{n})^{nt}$
   $= 9000(1 + \frac{0.0146}{365})^{365 \cdot 3} \Rightarrow \$9402.952 \rightarrow \approx \$9402.95$