Lesson 67

Name Notes

Advanced Algebra (H) - Exponential Function

Every two seconds, nine babies are born and three people die. The net increase of three people each second results in a growth in world population of 10,600 per hour, 254,000 per day, 1.8 million per week, 7.7 million per month, and 93 million per year. It is estimated that by the year 2000 it will be 98 million. Social scientists who study population often use exponential functions to model the growth.

If you assume a constant growth rate, you can see that millions more people will soon be living in India. Enter the data into your calculator table and find an exponential function that models this data.

$y = ab^x$

Year	Population (in Millions)
1991	835
1992	883.6
1993	900.3
1994	917.4
1995	934.8
1996	952.6

FOR TI-85'S:
PRESS [STAT]
GO TO EDIT [F2]
PRESS [ENTER] TWICE
INSERT DATA
PRESS [EXIT] ONCE
GO TO CALC [F1]
PRESS [ENTER] TWICE
GO TO EXPR [F4]

Remember: To get the equation

FOR TI-82'S:
[STAT] → CALC
[ALPHA] [A]

To get the graph
1. [y =] [clear]
2. [VARS] [5] → EQ [7] [GRAPH]

FOR. TI-83'S: TI-84

[STAT] → CALC →

PUSH "0" FOR EXP REG →

TYPE L1, L2, Y1 AFTER EXPReg. Hit [enter] fill after calculat

[ENTER]

[GRAPH]

Vars y-vars #1 enter

Write the equation of the model: $y = 829.82(1.0245)^x$

Sketch the graph below and find the approximate population of India in the year 2000. Teach Table Setup

X=0 is 1990 2000 → X=10 y = 1056.9 million people

Problems:

1. The population (in millions) of the People's Republic of China was as follows:

Year	Population (In Millions)
1991	1151
1992	1168
1993	1186
1994	1204
1995	1222
1996	1240

Find an equation that models this data: $y = 1133.89(1.015)^x$

Use the model population equation from India and determine the year (and population) when the populations of the two countries will be about equal.

Year 2023 : $x =$ $y = 1870.99$ million people
or 2024

2. A lad by the name of Jack Fum made a shrewd trade of an undernourished bovine for a start in a new experimental crop, leguman magicous. With the help of his mother he planted the bean just outside his kitchen window. It immediately sprouted 2.56 cm above the ground. Contrary to popular legend, it did not reach its full height in one night. Being a student of mathematics and the sciences, Jack kept a careful log of the growth of the sprout. On the first day at 8:00 am, 24 hours after planting, he found the plant to be 6.4 cm tall. At 8:00 am on the second day, the growing bean sprout was 16.0 cm in height. At 8:00 am on the third day, he recorded 40.0 cm. At the same time on the fourth day, he found it to be 1 m (100 cm) tall.

Time in days	Initially	After 1 day	After 2 days	After 3 days	After 4 days
Height	2.56 cm	6.4 cm	16.0 cm	40.0 cm	1m or 100 cm

a. Find a function that models this growth. If the pattern were to continue, what would be the heights on the fifth and sixth days?

$y = 2.56(2.5^x)$

$x = 5 \rightarrow ht = 250$ cm

$x = 6 \rightarrow ht = 625$ cm

b. Jack's younger brothers Phee and Fy measured the plant at 8:00 pm on the third day and found it to be 63.25 cm tall. Show how this value can be found mathematically. You may need to experiment with your calculator.

Use $x = 3.5$ $ht = 63.245$

$\frac{4}{24}$

c. Find the height that his youngest brother Foe tried to measure at 12:00 noon on the sixth day.

$x = 6\frac{1}{6}$ $\quad$ $ht = 728.12$

d. Experiment with the equation to find the day and time (to the nearest hour) when the stalk reached its final height of one kilometer (1000 m or 100,000 cm).

$ht = 100{,}000$

$$100{,}000 = 2.56\,(2.5^x)$$

$$\left(\frac{100{,}000}{2.56}\right) = 2.5^x$$

$$\log(\quad) = x \log 2.5$$

$$\frac{\log(\quad)}{\log 2.5} = x$$

$x \approx 11.538$

$\downarrow$

day 11, 8 PM

3. Given that f(x) is an exponential function and the f(4) = 1229 and f(5) = 3442, give your best guess for the value of f(4.5). Justify your answer.

$$\boxed{y = 19.976\,(2.800^x)}$$

Calculator

$(x = 4.5,\ y = 2056.749)$

Algebraically:

System: $1229 = a \cdot b^4$ $\rightarrow$ $\frac{1229}{b^4} = \frac{3442}{b^5}$

$3442 = a \cdot b^5$

$$\frac{b^5}{b^4} = \frac{3442}{1229}$$

$$b = 2.801$$

$$1229 = a\,(2.801)^4$$

$$a = 19.966$$

$$\boxed{y = 19.966\,(2.801)^x}$$