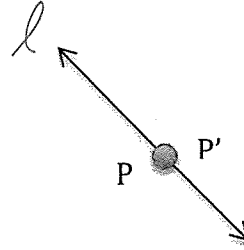
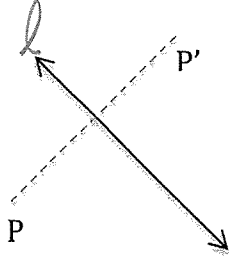


A **reflection** is a transformation which maps the figure over a line.
This line is called the line of reflection.

A reflection across line l maps a point P to its image P' .

- If P is not on line l , then line l is the perpendicular bisector of $\overline{PP'}$.
- If P is on line l , then $P = P'$.



Example 1: Explore a reflection across the x-axis.

$\triangle ABC$ is being reflected over the x-axis.
Draw and label the image $\triangle A'B'C'$.

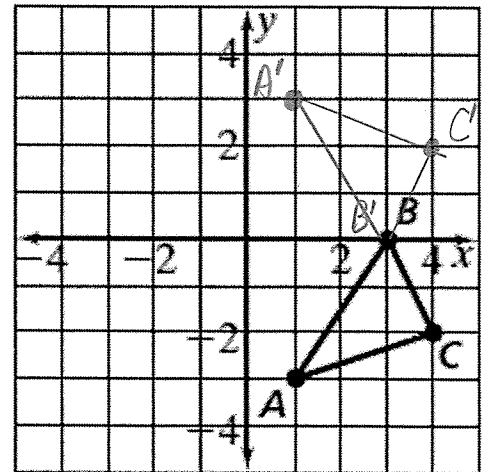
Be sure to satisfy the two conditions:

- (1) the x-axis must be the line of reflection between A and A' , between B and B' , and between C and C' .
- (2) the segments $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$.

\perp bisector to

We can use an arrow to describe this reflection.

$$\triangle ABC \rightarrow \triangle A'B'C'$$



Write the coordinates of:

Preimage	Image
$A(1, -3) \rightarrow$	$A'(1, 3)$
$B(3, 0) \rightarrow$	$B'(3, 0)$
$C(4, -2) \rightarrow$	$C'(4, 2)$

Look for a pattern in the coordinates.
The x-coordinate of each image point equals the x-coordinate of its preimage.

The y-coordinate of each image point is opposite of the y-coordinate of its preimage.

Use your coordinates above to write a general rule for an x-axis reflection (reflection across the x-axis):

$$(x, y) \rightarrow (x, -y)$$

Another way of writing this line reflection: $r_{x\text{-axis}}(x, y) = (x, -y)$

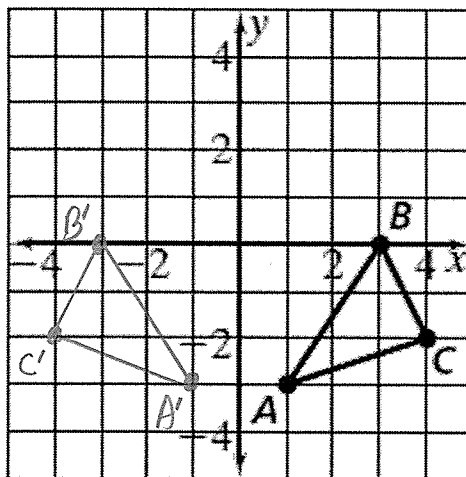
Example 2: Explore a reflection across the y-axis. $\triangle ABC$ is reflected over the y-axis.Draw the image $\triangle A'B'C'$.

What are the coordinates of:

$$A \underline{(1, -3)} \rightarrow A' \underline{(-1, -3)}$$

$$B \underline{(3, 0)} \rightarrow B' \underline{(-3, 0)}$$

$$C \underline{(4, -2)} \rightarrow C' \underline{(-4, -2)}$$

Use the coordinates above to write a general rule for a y-axis reflection:

$$(x, y) \rightarrow (\underline{-x}, \underline{y}).$$

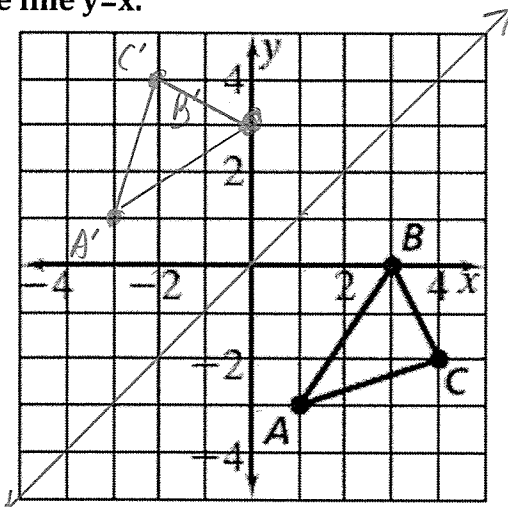
Another way of writing this line reflection: $r_{y\text{-axis}}(x, y) = (-x, y)$ **Example 3: Explore a reflection across the line $y=x$.** $\triangle ABC$ is reflected over the line $y = x$.Draw the image $\triangle A'B'C'$.

What are the coordinates of:

$$A \underline{(1, -3)} \rightarrow A' \underline{(-3, 1)}$$

$$B \underline{(3, 0)} \rightarrow B' \underline{(0, 3)}$$

$$C \underline{(4, -2)} \rightarrow C' \underline{(-2, 4)}$$

Write a general rule for a reflection across the line $y = x$:

$$(x, y) \rightarrow (\underline{y}, \underline{x}).$$

Another way of writing this line reflection: $r_{y=x}(x, y) = (y, x)$

Example 4: Explore a reflection across the line $y = -x$.

$\triangle ABC$ is reflected over the line $y = -x$.

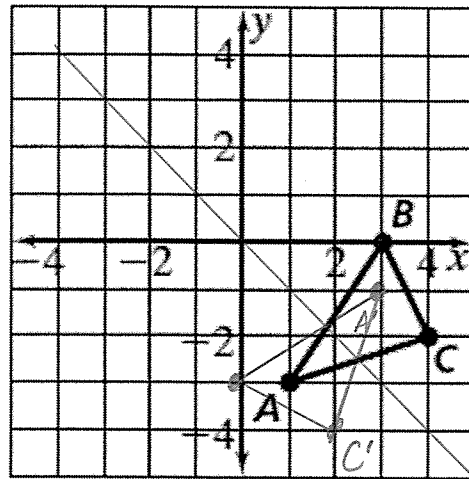
Draw the image $\triangle A'B'C'$.

What are the coordinates of:

$$A (1, -3) \rightarrow A' (3, -1)$$

$$B (3, 0) \rightarrow B' (0, -3)$$

$$C (4, -2) \rightarrow C' (2, -4)$$



Write a general rule for a reflection across the line $y = -x$:

$$(x, y) \rightarrow (-y, -x).$$

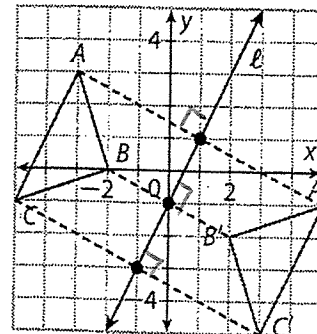
Another way of writing this line reflection: $r_{y=-x}(x, y) = (-y, -x)$

Example 5: Find the line of reflection.

Given that $\triangle A'B'C'$ is the image of $\triangle ABC$ under a reflection, draw the line of reflection and write its equation.

In the graph to the right, the line of reflection is drawn in. If it is not drawn in, you would do the following to find line of reflection.

1. Use the midpoint formula to find the midpoints of $\overline{AA'}$, of $\overline{BB'}$, and of $\overline{CC'}$.
2. Plot the three points.
3. Draw a line through the three points.
4. Write the equation of the line of reflection.

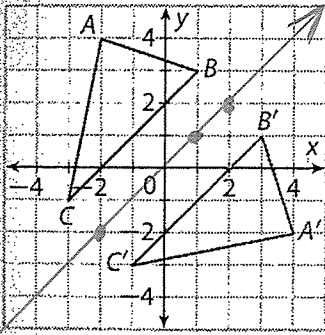


↳ $y = 2x - 1$

Your Turn

$\triangle A'B'C'$ is the image of $\triangle ABC$ under a reflection. On a coordinate grid, draw $\triangle ABC$, $\triangle A'B'C'$, and the line of reflection.

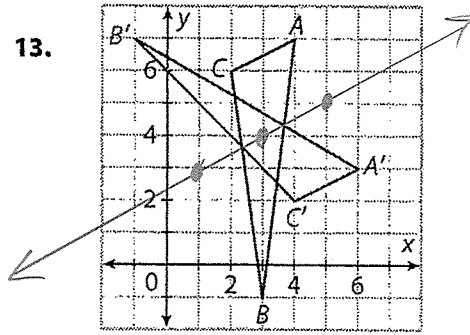
12.



Write its equation:

$$y = x$$

13.



Write its equation:

$$m = \frac{1}{2} \quad 5 = \frac{1}{2}(5) + b$$

$$2\frac{1}{2} = b$$

$$y = \frac{1}{2}x + 2\frac{1}{2}$$

Show your work with the midpoint formula below.

12) $A(-2, 4) \quad B(1, 3) \quad C(-3, -1)$
 $A'(4, -2) \quad B'(3, 1) \quad C'(-1, -3)$

$$MP \text{ of } \overline{AA'} = (1, 1)$$

$$MP \text{ of } \overline{BB'} = (2, 2)$$

$$MP \text{ of } \overline{CC'} = (-2, -2)$$

13) $A(4, 7) \quad B(3, -1) \quad C(2, 6)$
 $A'(6, 3) \quad B'(-1, 7) \quad C'(4, 2)$

$$MP_{\overline{AA'}} = \left(\frac{4+6}{2}, \frac{7+3}{2} \right) \rightarrow (5, 5)$$

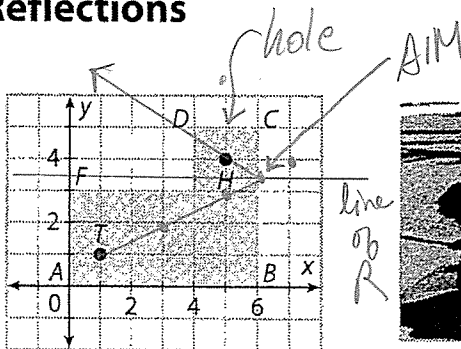
$$MP_{\overline{BB'}} = \left(\frac{3-1}{2}, \frac{-1+7}{2} \right) \rightarrow (1, 3)$$

$$MP_{\overline{CC'}} = \left(\frac{2+4}{2}, \frac{6+2}{2} \right) \rightarrow (3, 4)$$

Explain 4 Applying Reflections

Example 4

The figure shows one hole of a miniature golf course. It is not possible to hit the ball in a straight line from the tee T to the hole H. At what point should a player aim in order to make a hole in one?



$$\frac{1}{2} \rightarrow \frac{5}{7}$$