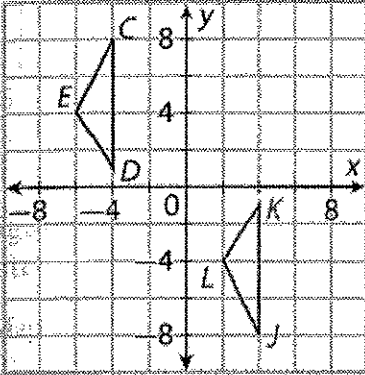


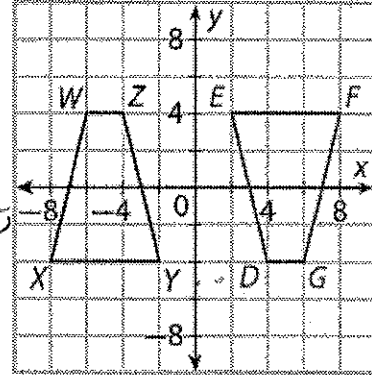
Use the definition of congruence to decide whether the two figures are congruent. Explain your answer. Give coordinate notation for the transformations you use.

1.



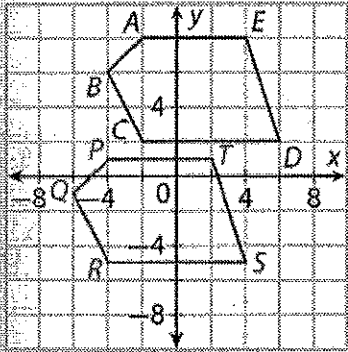
If I ...
 (1) reflect $\triangle CDE$ over the x -axis
 (2) then do a horizontal translation
 I get $\triangle CDE \cong \triangle KJL$.
 Refl: $(x, y) \rightarrow (x, -y)$
 Tran: $(x, y) \rightarrow (x+8, y)$

2.



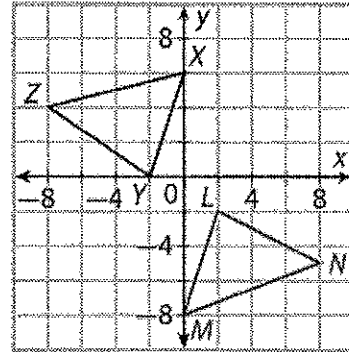
(1) Reflect across x -axis;
 (2) horizontal translation
 $WXYZ \cong DEFG$
 Refl: $(x, y) \rightarrow (x, -y)$
 Tran: $(x, y) \rightarrow (x-10, y)$

3.



$ABCDE \cong PQRST$
 (1) Use a translation
 $(x, y) \rightarrow (x-2, y-7)$

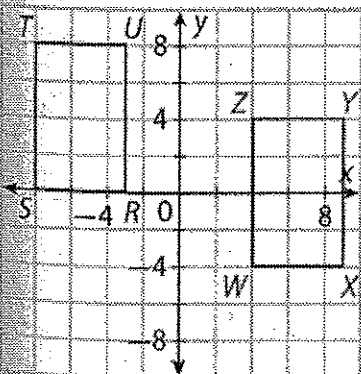
4.



No sequence of rigid transformations will map one \triangle onto the other. They are not \cong .

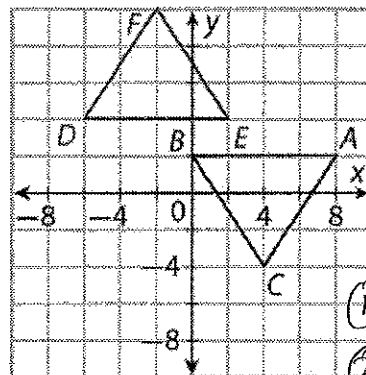
The figures shown are congruent. Find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.

6. $RSTU \cong WXYZ$



(1) Reflected across the y -axis,
 (2) a translation
 Refl: $(x, y) \rightarrow (-x, y)$
 Transl: $(x, y) \rightarrow (x+1, y-4)$

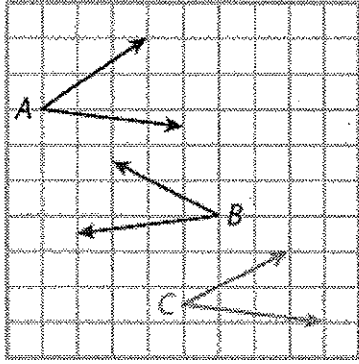
7. $\triangle ABC \cong \triangle DEF$



(1) rotation of 180° around the origin,
 (2) translation
 (1) $(x, y) \rightarrow (-x, -y)$
 (2) $(x, y) \rightarrow (x+2, y+6)$

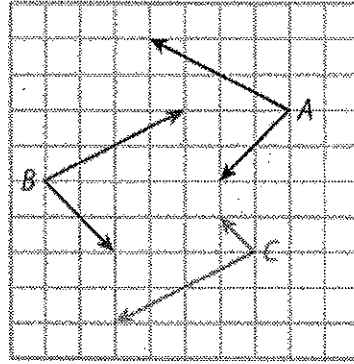
Determine which of the angles are congruent. Which transformations can be used to verify the congruence?

10.



None of the \angle s \cong .
No transformation will map one \angle onto another.

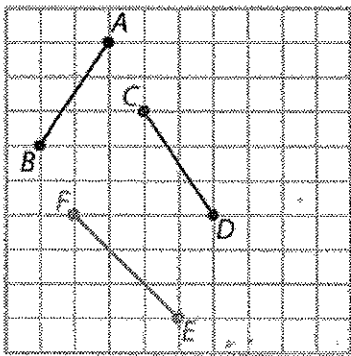
11.



$\angle A \cong \angle B \cong \angle C$
① reflection
② translation

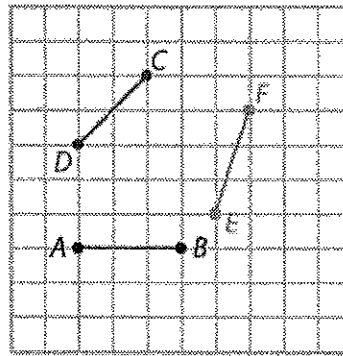
Determine which of the segments are congruent. Which transformations can be used to verify the congruence?

12.



$\overline{AB} \cong \overline{CD}$
① reflection
② Translation

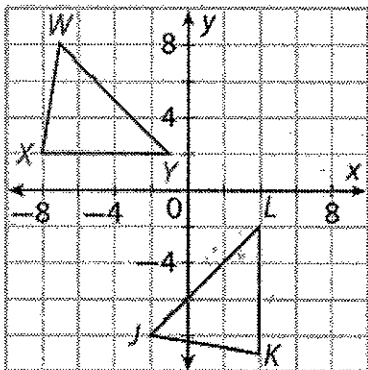
13.



None are \cong .
No transformation will map one onto another.

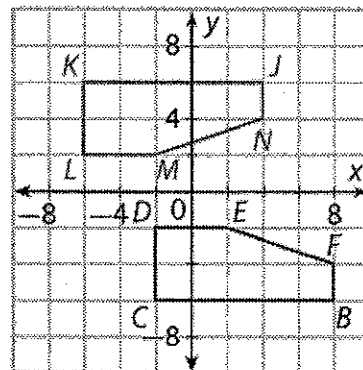
Use the definition of congruence to decide whether the two figures are congruent. Explain your answer. Give the coordinate notation for the transformation you use.

14.



Yes, $\triangle JKL \cong \triangle WXY$
① rotate 90° clockwise around the origin
 $(x, y) \rightarrow (y, -x)$
② translation $(x, y) \rightarrow (x+1, y+6)$

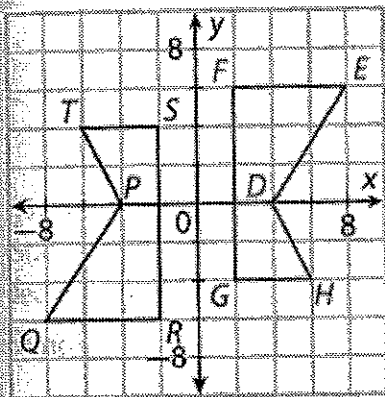
15.



Yes, $BCDEF \cong JKLMN$
① reflect across x-axis
 $(x, y) \rightarrow (x, -y)$
② horiz. translation $(x, y) \rightarrow (x+4, y)$

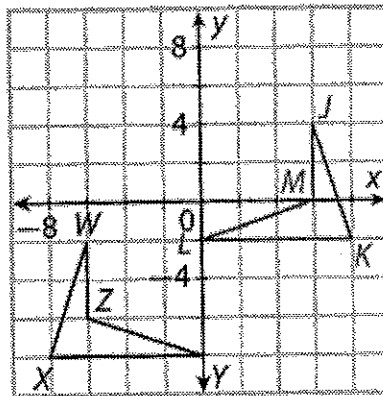
The figures shown are congruent. Find a sequence of transformations for the indicated mapping. Give coordinate notation for the transformations you use.

18. Map $DEFGH$ to $PQRST$.



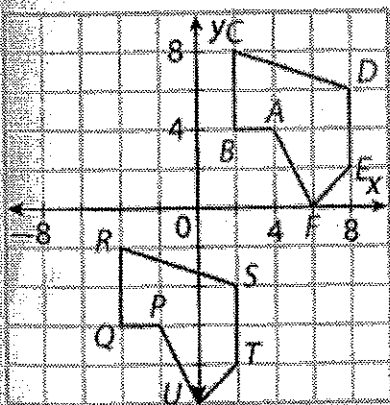
① Rotate 180° around origin,
 $(x,y) \rightarrow (-x,-y)$

19. Map $JKLM$ to $WXYZ$.



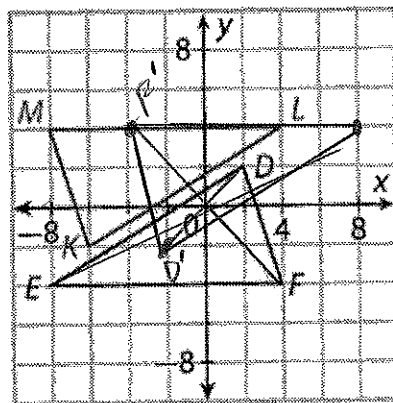
Quad $JKLM \cong$ Quad $WXYZ$
 ① Reflected across y -axis $(x,y) \rightarrow (-x,y)$
 ② Vertical Translation $(x,y) \rightarrow (x,y-6)$

20. Map $ABCDEF$ to $PQRSTU$.



A Translation
 $(x,y) \rightarrow (x-6, y-10)$

21. Map $\triangle DEF$ to $\triangle KLM$.



① Rotation 180° around the origin
 ② Horizontal translation
 Rotation $(x,y) \rightarrow (-x,-y)$
 Translation $(x,y) \rightarrow (x-4, y)$

Section II: Complete the following on graph paper.

Draw the preimage and image of each triangle under the given translation.

5. Triangle: $A(-3, -1)$;
 $B(-2, 2)$; $C(0, -1)$;
 Vector: $(3, 2)$

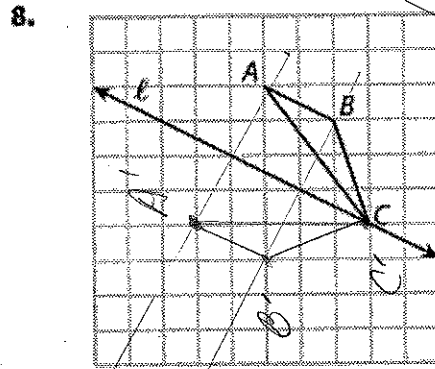
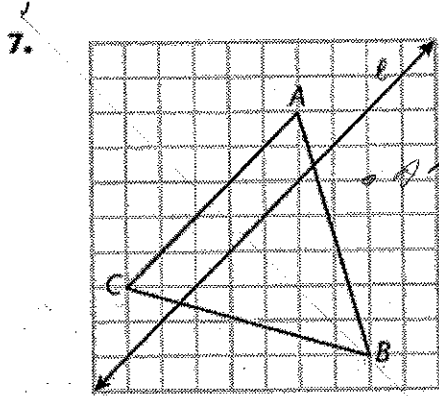
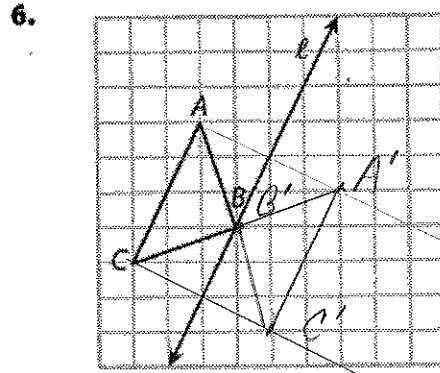
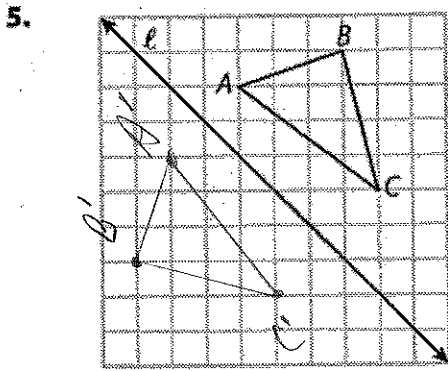
6. Triangle: $P(1, -3)$;
 $Q(3, -1)$; $R(4, -3)$;
 Vector: $(-1, 3)$

7. Triangle: $X(0, 3)$;
 $Y(-1, 1)$; $Z(-3, 4)$;
 Vector: $(4, -2)$

See separate graph paper.

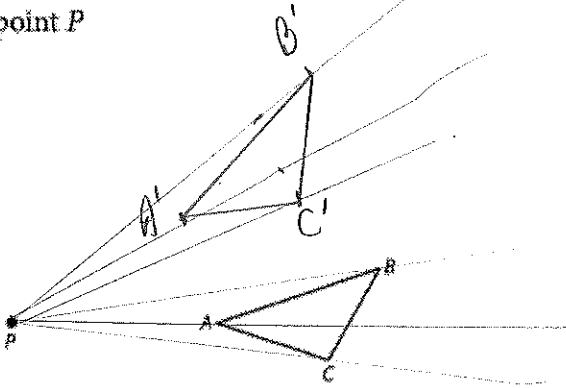
Section III

Draw the image of $\triangle ABC$ after a reflection across line l .

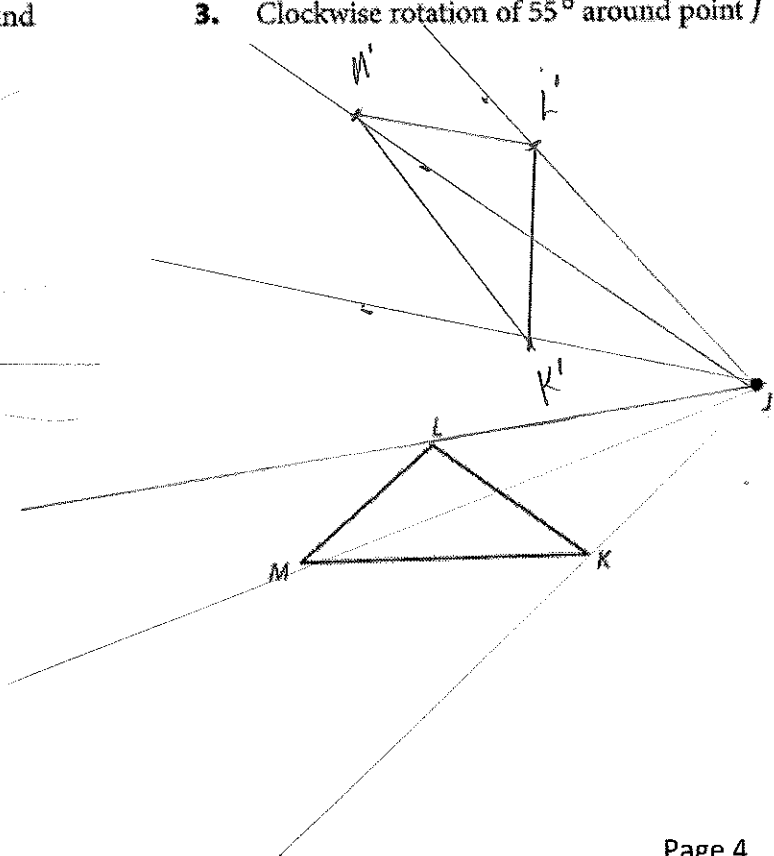


Section IV : Draw the image of the triangle after the given rotation.

2. Counterclockwise rotation of 30° around point P



3. Clockwise rotation of 55° around point J



4.

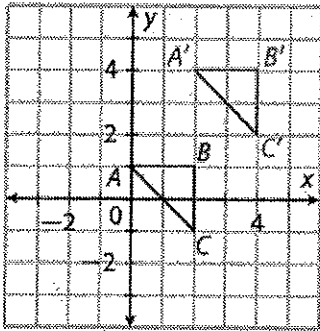
Find the coordinates of the image under the transformation $\langle 6, -11 \rangle$.

$(x, y) \rightarrow \underline{\quad? \quad} (x+6, y-11)$ $(2, -3) \rightarrow \underline{\quad? \quad} (8, -14)$

$(3, 1) \rightarrow \underline{\quad? \quad} (9, -10)$ $(4, -3) \rightarrow \underline{\quad? \quad} (10, -14)$

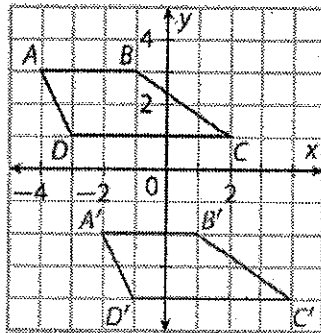
Specify the component form of the vector that maps each figure to its image.

12.



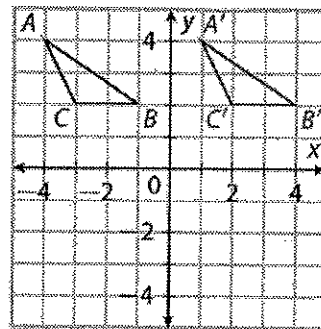
$\langle 2, 3 \rangle$

13.



$\langle 2, -5 \rangle$

14.



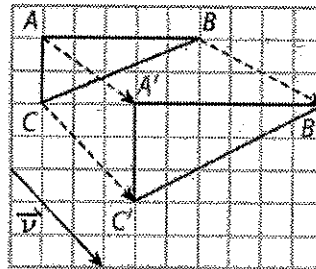
$\langle 5, 0 \rangle$

15. **Explain the Error** Andrew is using vector \vec{v} to draw a copy of $\triangle ABC$. Explain his error.

The vectors from A to A' and B to B' are incorrect.

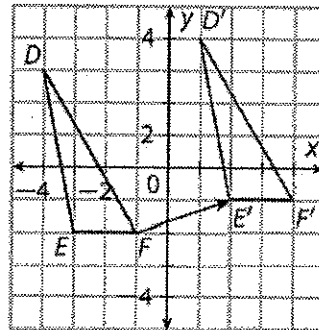
Correct vector is 3 right, 3 down.

From A to A', he went down 2; From B to B', he moved right 4 and down 2.

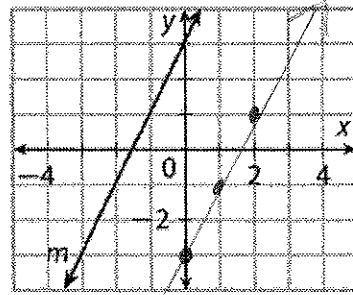


16. **Explain the Error** Marcus was asked to identify the vector that maps $\triangle DEF$ to $\triangle D'E'F'$. He drew a vector as shown and determined that the component form of the vector is $\langle 3, 1 \rangle$. Explain his error.

His error was going from F to E'. He should've gone from E to E'.



- 17. Algebra** A cartographer is making a city map. Line m represents Murphy Street. The cartographer translates points on line m along the vector $\langle 2, -2 \rangle$ to draw Nolan Street. Copy the graph. Draw the line for Nolan Street on the coordinate plane and write its equation. What is the image of the point $(0, 3)$ in this situation?



Equation: $y = 2x - 3$
 $(0, 3) \rightarrow (2, 1)$

$y = 2x - 3$

Section V

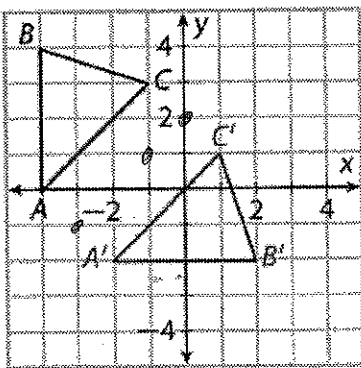
Complete on separate graph paper.

Reflect the figure with the given vertices across the given line.

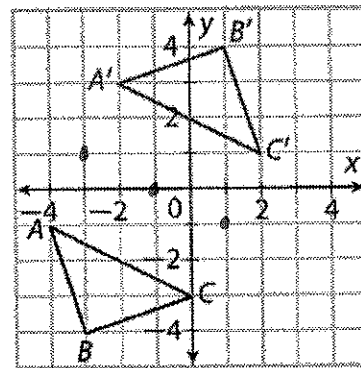
9. $P(-2, 3), Q(4, 3), R(-1, 0), S(-4, 1)$; x-axis
 10. $A(-3, -3), B(1, 3), C(3, -1)$; y-axis
 11. $J(-1, 2), K(2, 4), L(4, -1)$; $y = -x$
 12. $D(-1, 1), E(3, 2), F(4, -1), G(-1, -3)$; $y = x$

$\Delta A'B'C'$ is the image of ΔABC under a reflection. Draw the line of reflection.

13.

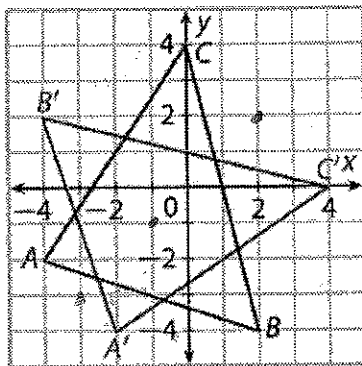


14.
 $\text{midpt } \overline{AA'} = (-3, -1)$
 $\text{midpt } \overline{BB'} = (-1, 1)$
 $\text{midpt } \overline{CC'} = (0, 2)$

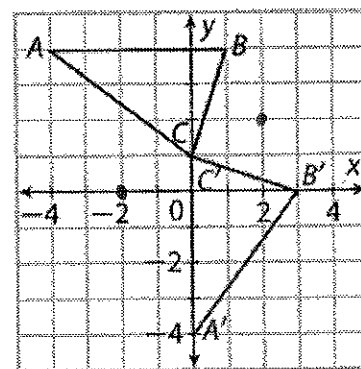


$\text{midpt } \overline{AA'} = (-3, 1)$
 $\text{midpt } \overline{BB'} = (-1, 0)$
 $\text{midpt } \overline{CC'} = (1, -1)$

15.



16.
 $\text{midpt } \overline{AA'} = (-3, -3)$
 $\text{midpt } \overline{BB'} = (-1, -1)$
 $\text{midpt } \overline{CC'} = (2, 2)$

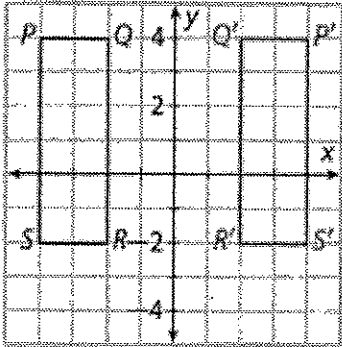


$\text{midpt } \overline{AA'} = (-2, 0)$
 $\text{midpt } \overline{BB'} = (2, 2)$

$\text{Midpt } \overline{AA'} = \left(\frac{-4-2}{2}, \frac{-2-4}{2} \right)$
 $(-3, -3)$

23. Communicate Mathematical Ideas

The figure shows rectangle PQRS and its image after a reflection across the y-axis. A student said that PQRS could also be mapped to its image using the translation $(x, y) \rightarrow (x + 6, y)$. Do you agree? Explain why or why not.

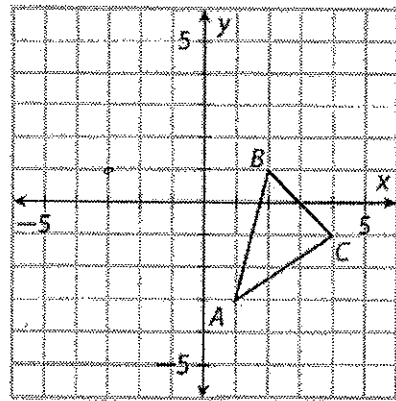


No, I disagree.
 That translation would move
 P to Q', Q to P', S to R',
 and R to S'.
 That is incorrect.

24.

Which of the following transformations map $\triangle ABC$ to a triangle that intersects the x-axis? Write all that apply.

- A. $(x, y) \rightarrow (-x, y)$ D. $(x, y) \rightarrow (-y, -x)$
 B. $(x, y) \rightarrow (x, -y)$ E. $(x, y) \rightarrow (x, y + 1)$
 C. $(x, y) \rightarrow (y, x)$



A. $A'(-1, -3)$ $B'(-2, 1)$ $C'(-4, -1)$
 $\overline{A'B'}$ intersects x-axis.

B. $A'(1, 3)$ $B'(2, -1)$ $C'(4, 1)$
 $\overline{A'B'}$ intersects x-axis.

C. No side intersects x-axis.

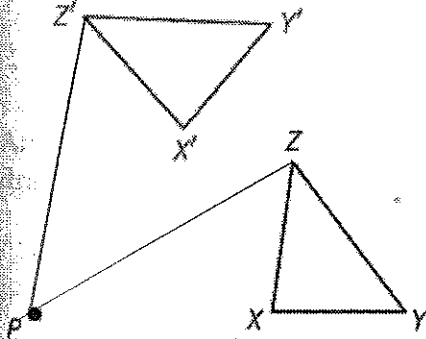
D. No side " "

E. $A'(1, -2)$ $B'(2, 2)$ $C'(4, 0)$ $\overline{A'B'}$ intersects the x-axis.

Section VI

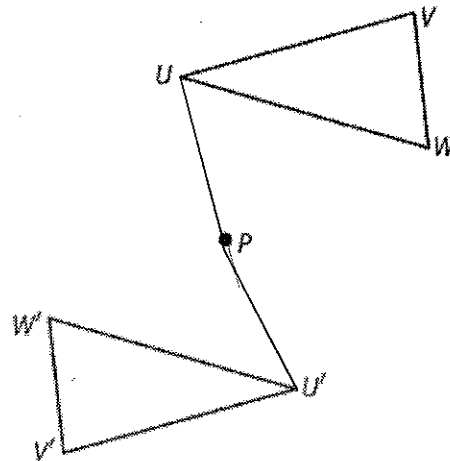
Find the angle of rotation and direction of rotation in the given figure. Point P is the center of rotation.

9.



50° counterclockwise

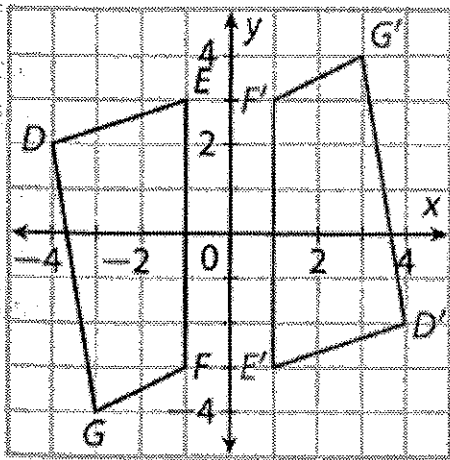
10.



190° counterclockwise

Write an algebraic rule for the rotation shown. Then describe the transformation in words.

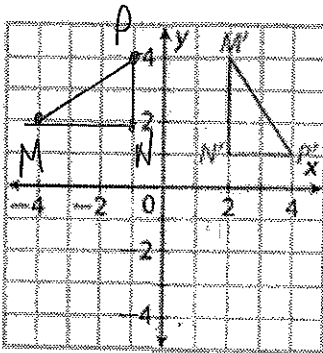
11.



Rule: $(x, y) \rightarrow (-x, -y)$

Transformation is a rotation of 180° .

12. **Critical Thinking** The figure shows the image of $\triangle MNP$ after a counterclockwise rotation of 270° . On a coordinate grid, draw and label $\triangle MNP$.



13. Determine whether each statement about the rotation $(x, y) \rightarrow (y, -x)$ is true or false.

a. Every point in Quadrant I is mapped to a point in Quadrant II.

b. Points on the x -axis are mapped to points on the y -axis. $(4, 0) \rightarrow (0, -4)$

c. The origin is a fixed point under the rotation.

$(2, 1) \rightarrow (1, -2)$

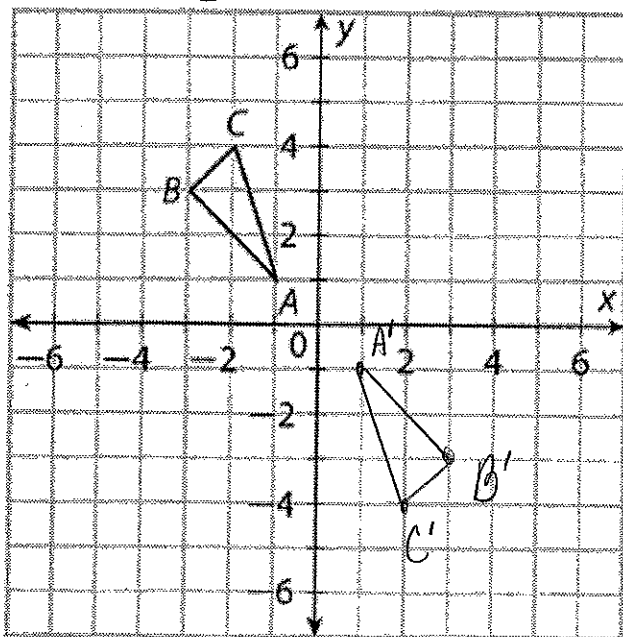
d. The rotation has the same effect as a 90° clockwise rotation. $(2, 1) \rightarrow (1, -2)$

e. The angle of rotation is 180° .

f. A point on the line $y = x$ is mapped to another point on the line $y = x$.

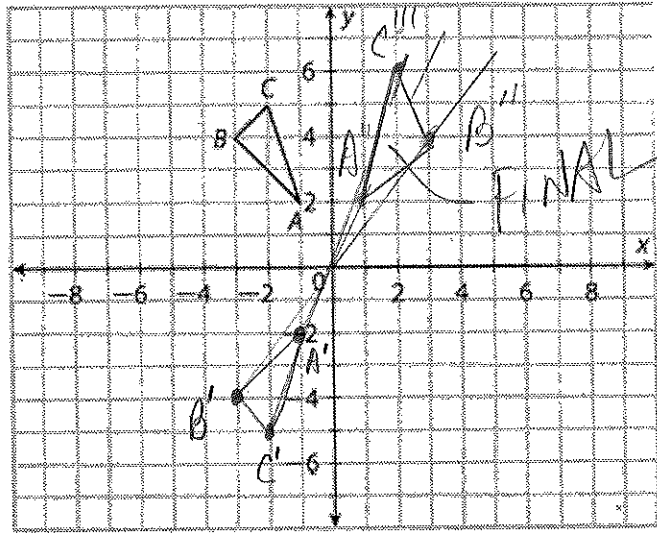
$(2, 2) \rightarrow (2, -2)$

14. Rotate $\triangle ABC$ 90 degrees clockwise about the origin and then reflect over the x -axis.



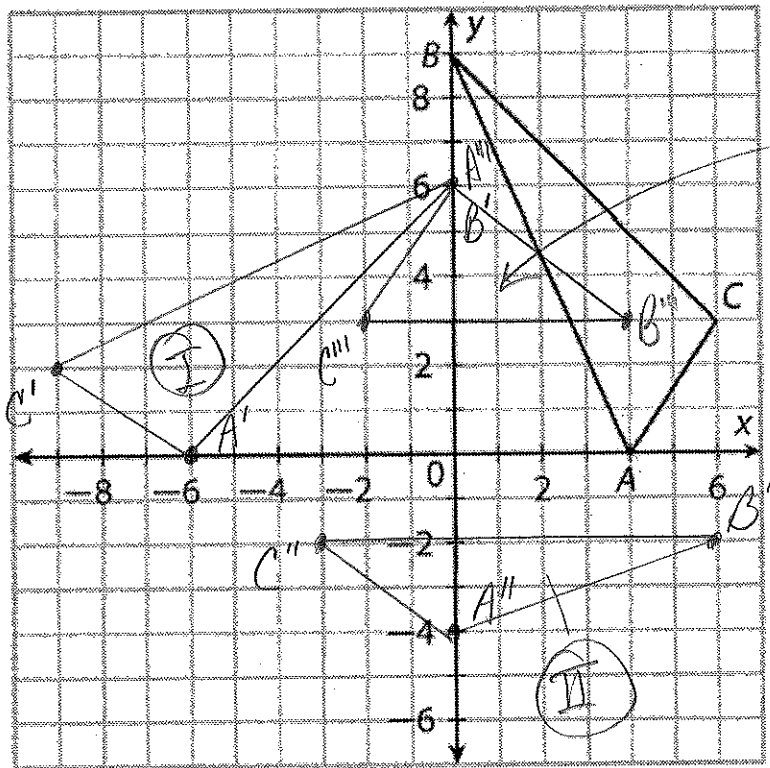
15.

Reflect $\triangle ABC$ over the x -axis, translate by $(-3, -1)$, and rotate 180 degrees around the origin.



16. Draw and label the final image of $\triangle ABC$ after the given sequence of transformations. Show all work.

$$(x, y) \xrightarrow{A'} \left(-\frac{3}{2}x, \frac{2}{3}y\right) \xrightarrow{A''} (x+6, y-4) \xrightarrow{A'''} \left(\frac{2}{3}x, -\frac{3}{2}y\right)$$



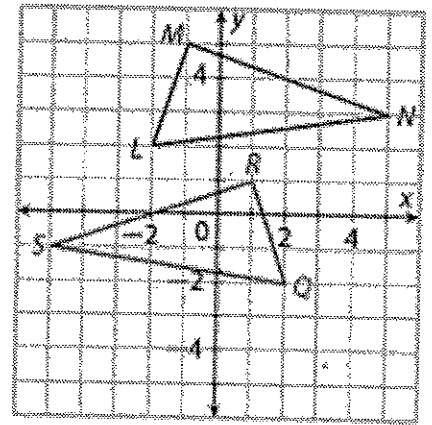
$$\begin{aligned} A(4, 0) &\rightarrow A'(-6, 0) \rightarrow A''(0, -4) \rightarrow A'''(0, 6) \\ B(0, 9) &\rightarrow B'(0, 6) \rightarrow B''(6, -2) \rightarrow B'''(4, -3) \\ C(6, 3) &\rightarrow C'(-9, 2) \rightarrow C''(-3, -2) \rightarrow C'''(-2, 3) \end{aligned}$$

Determine whether each statement is always, sometimes, or never true.

17. A double rotation can always be written as a single rotation.
18. A sequence of a reflection across the x-axis and then a reflection across the y-axis results in a 180° rotation of the preimage. always
19. A sequence of rigid transformations will ? result in an image that is the same size and orientation as the preimage. sometimes
20. A sequence of a rotation and a dilation will ? result in an image that is the same size and orientation as the preimage. sometimes

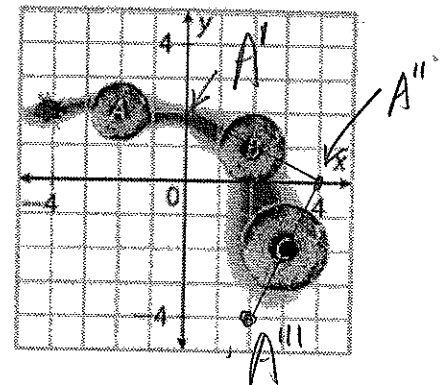
21. $\triangle QRS$ is the image of $\triangle LMN$ under a sequence of transformations. Tell whether each of the following sequences can be used to create the image, $\triangle QRS$, from the preimage, $\triangle LMN$.

- a. Reflect across the y-axis and then translate along the vector $\langle 0, -4 \rangle$. Yes
- b. Translate along the vector $\langle 0, -4 \rangle$ and then reflect across the y-axis. Yes
- c. Rotate 90° clockwise about the origin, reflect across the x-axis, and then rotate 90° counterclockwise about the origin. NO
- d. Rotate 180° about the origin, reflect across the x-axis, and then translate along the vector $\langle 0, -4 \rangle$. Yes



22. An animator is drawing a scene in which a ladybug moves around three mushrooms. The figure shows the starting position of the ladybug. The animator rotates the ladybug 180° around mushroom A, then 180° around mushroom B, and finally 180° around mushroom C. What are the final coordinates of the ladybug? (2, -4)

1st rotation puts LB at (0, 2).
 2nd " " " " (4, 0).
 3rd " " " " (2, -4)



23. **Represent Real-World Problems** A builder is trying to level out some ground with a front-end loader. He picks up some excess dirt at $(9, 16)$ and then maneuvers through the job site along the vectors $\langle -6, 0 \rangle$, $\langle 2, 5 \rangle$, $\langle 8, 10 \rangle$ to get to the spot to unload the dirt. Find the coordinates of the unloading point. Find a single vector from the loading point to the unloading point.

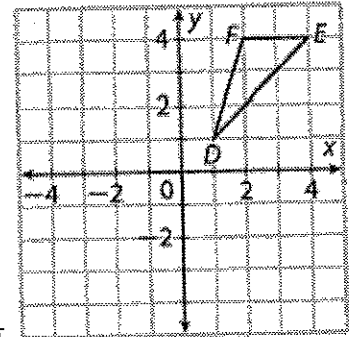
Unloading point $(13, 31)$ $\left\{ \begin{array}{l} (9, 16) \rightarrow (3, 16) \rightarrow (5, 21) \\ (13, 31) \end{array} \right.$
 Vector $\langle 4, 15 \rangle$

24. **Communicate Mathematical Ideas** Suppose you are given a figure and a center of rotation P . Describe two different ways you can use a ruler and protractor to draw the image of the figure after a 210° counterclockwise rotation around P .

1st way: Rotate 150° clockwise around P .

2nd way: Reflect 180° ; then rotate counterclockwise 30° .

25. **Look for a Pattern** Isaiah uses software to draw $\triangle DEF$ as shown. Each time he presses the left arrow key, the software rotates the figure on the screen 90° counterclockwise. Explain how Isaiah can determine which quadrant the triangle will lie in if he presses the left arrow key n times.



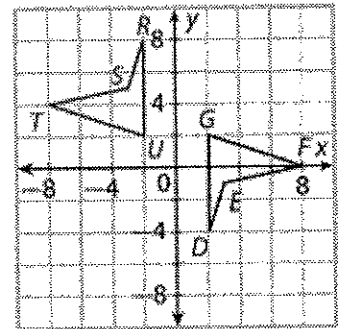
Make a table. Look for a pattern.

$$\frac{n}{4} = \# \text{ and remainder}$$

The remainder tells the following:

$$R=0 \rightarrow \text{Q I}; R=1 \rightarrow \text{Q II}; R=2 \rightarrow \text{Q III}; R=3 \rightarrow \text{Q IV}$$

26. **Justify Reasoning** Two students are trying to show that the two figures are congruent. The first student decides to map $DEFG$ to $RSTU$ using a rotation of 180° about the origin, followed by the vertical translation $(x, y) \rightarrow (x, y + 4)$. The second student uses a reflection across the x -axis, followed by the vertical translation $(x, y) \rightarrow (x, y + 4)$, followed by a reflection across the y -axis. Are both students correct, is only one student correct, or is neither student correct?



Both are correct. Either sequence will map $DEFG$ to $RSTU$.

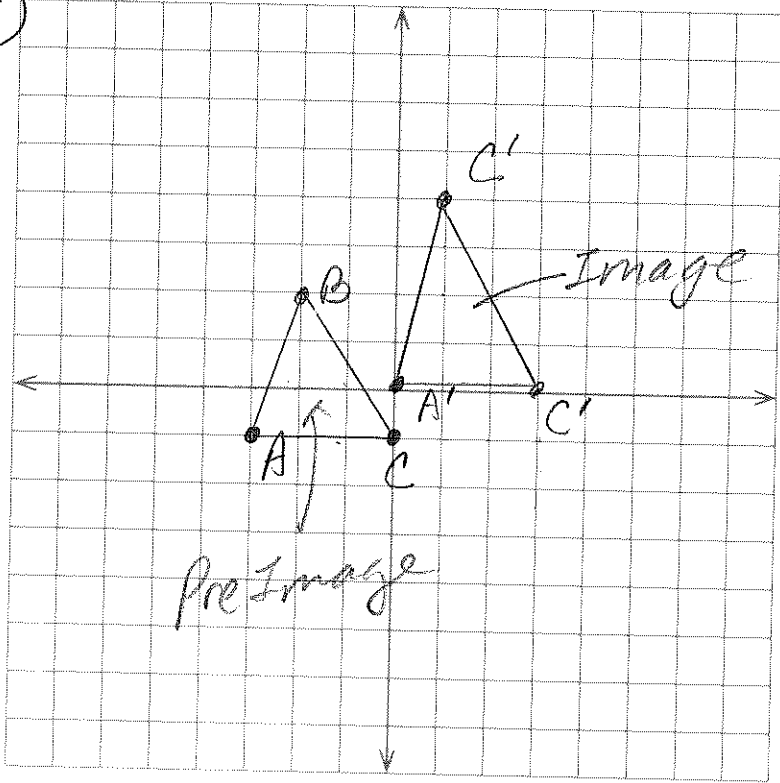
* A rotation of 180° around the origin is the same as a reflection across both axes.

27. **Analyze Relationships** Suppose you know that $\triangle ABC$ is congruent to $\triangle DEF$ and that $\triangle DEF$ is congruent to $\triangle GHJ$. Can you conclude that $\triangle ABC$ is congruent to $\triangle GHJ$? Explain.

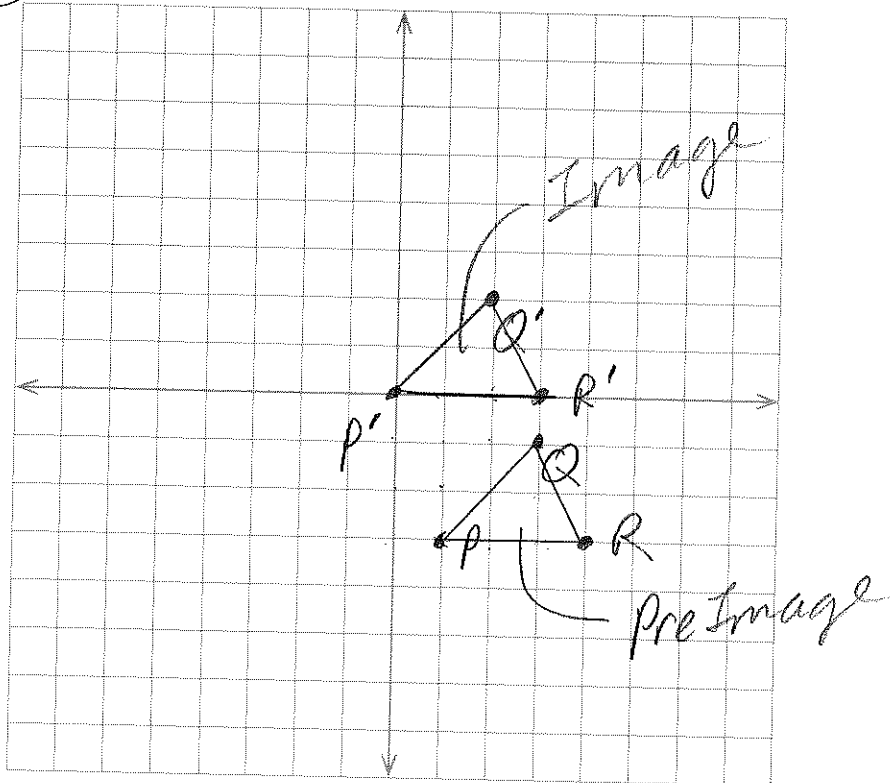
Yes, $\triangle ABC \cong \triangle GHJ$. If $\triangle ABC \cong \triangle DEF$, this means there is a sequence of rigid motion(s) that will map $\triangle ABC$ to $\triangle DEF$. Same applies to $\triangle DEF \cong \triangle GHJ$. Therefore, there must be a sequence of rigid motions that will map $\triangle ABC$ to $\triangle GHJ$.

Section II

#5



#6



#7

