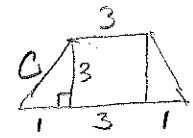
# Volume and Surface Area

Geometry (H)
Chapter review

Name: KEY

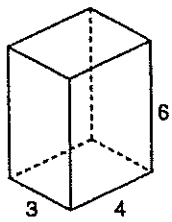
Find volume, L.A and T.A of each solid.

$1^2+3^2=c^2$
$\sqrt{10}=c$




Isosceles

1. Rectangular prism

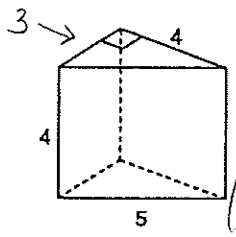



$V=Bh$
$=3(4)(6)$
$V=72$

$LA=ph$
$=14(6)$
$LA=84$

$TA=LA+2B$
$84+2(12)$
$84+24$
$TA=108$

2.* Right triangular prism

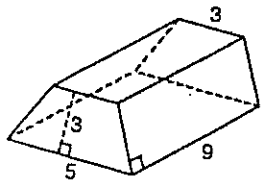



$V=\frac{1}{2}bh(H)$
$=\frac{1}{2}(4)(3)(4)$
$V=24$


$LA=ph$
$=12(4)$
$LA=48$

$TA=LA+2B$
$48+2(6)$
$TA=60$

3.* Trapezoidal prism

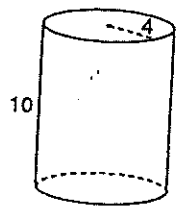



$V=Bh$
$\frac{1}{2}h(b_1+b_2)H$
$\frac{1}{2}(3)(3+5)(9)$
$V=108$

$LA=ph$
$=(8+2\sqrt{10})9$
$LA=72+18\sqrt{10}$

$TA=LA+2B$
$72+18\sqrt{10}+2(12)$
$TA=96+18\sqrt{10}$

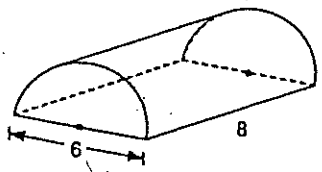
4. Cylinder




$V=Bh=\pi r^2h$
$\pi 4^2(10)$
$V=160\pi$

$TA=2\pi rh+2(\pi r^2)$
$=2\pi 4(10)+2(\pi 16)$
$80\pi+32\pi$
$TA=112\pi$

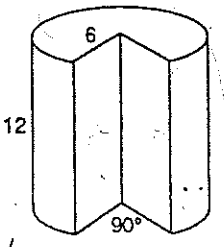
5.* Semicircular cylinder




$V=(\pi r^2h)\times\frac{1}{2}$
$\pi 3^2(8)\frac{1}{2}$
$V=36\pi$

$TA=(2\pi rh+2B)\frac{1}{2}+A_{rect.}$
$=\pi rh+B+LW$
$=\pi 3(8)+\pi 3^2+6(8)$
$=24\pi+9\pi+48$
$TA=33\pi+48$

6.* Cylinder with a 90° slice removed



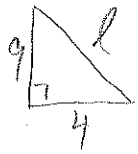

$V=Bh$
$=\frac{3}{4}(\pi r^2)(12)$
$=\frac{3}{4}\pi 6^2(12)$
$V=324\pi$

$TA=\frac{3}{4}(2\pi r)(h)+2(\frac{3}{4}\pi r^2)+A_{2rect}$
$\frac{3}{2}\pi(6)(12)+\frac{3}{2}\pi 36+2(6\cdot 12)$
$108\pi+54\pi+144$
$162\pi+144$


$6^2+7^2=l^2$

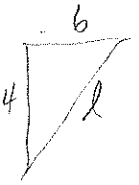
$85=l^2$

$\sqrt{85}=l$




$9^2+4^2=l^2$

$\sqrt{97}=l$

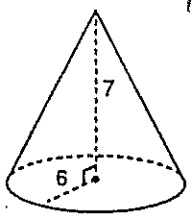



$l^2=14^2+36$

$l=\sqrt{232}$

$=2\sqrt{58}$

7. Right cone




$V=\frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi 6^2(7)$

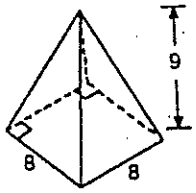
$\boxed{84\pi}$

$TA=\pi rl+\pi r^2$

$=\pi(6)\sqrt{85}+\pi 6^2$

$=\boxed{6\pi\sqrt{85}+36\pi}$

8. Square pyramid




$V=\frac{1}{3}e^2h$

$\frac{1}{3}8^2(9)$

$\boxed{V=192}$

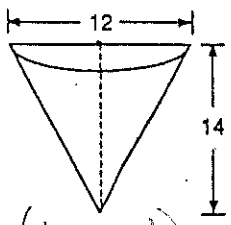
$TA=\frac{1}{2}pl+e^2$

$\frac{1}{2}32\sqrt{97}+8^2$

$TA=16\sqrt{97}+64$

$\boxed{TA=64+16\sqrt{97}}$

9. Semicircular cone




$V=\frac{1}{2}\left(\frac{1}{3}\pi r^2 h\right)$

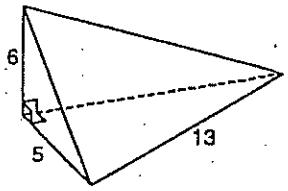
$=\frac{1}{6}\pi 36(14)=\boxed{84\pi}$

$TA=\frac{1}{2}(\pi rl+\pi r^2)+A_\Delta$

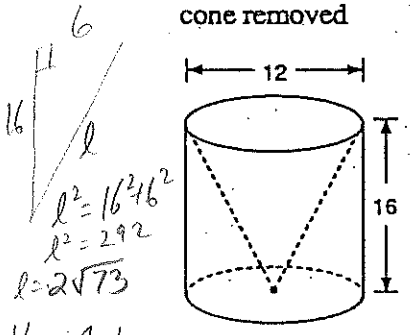
$\frac{1}{2}(\pi(6)2\sqrt{58}+\pi 36)+\frac{1}{2}(12)(14)$

$\boxed{TA=6\pi\sqrt{58}+18\pi+84}$

~~10.~~ Right triangular pyramid




11. Cylinder with cone removed




$l^2=16^2+6^2$

$l^2=292$

$l=2\sqrt{73}$

$V=Cyl.-Cone$

$\pi r^2h-\frac{1}{3}\pi r^2h$

$\pi 6^2(16)-\frac{1}{3}\pi 6^2(16)$

$576\pi-192\pi$

$\boxed{V=384\pi}$

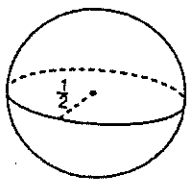
$TA=LA+B+LA_{cone}$

$2\pi rh+\pi r^2+\pi rl$

$2\pi 6(16)+\pi 36+\pi(6)(2\sqrt{73})$

$TA\ \boxed{228\pi+12\pi\sqrt{73}}$

12.




$V=\frac{4}{3}\pi r^3$

$\frac{4}{3}\pi\left(\frac{1}{2}\right)^3=\frac{4}{3}\cdot\frac{1}{8}\pi=\boxed{\frac{1}{6}\pi}$

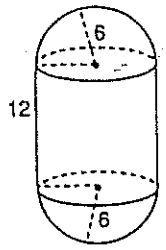
$TA=4\pi r^2$

$=4\pi\left(\frac{1}{2}\right)^2$

$=4\pi\frac{1}{4}$

$\boxed{TA=\pi}$

13.




$V = \pi r^2 h + \frac{4}{3}\pi r^3$

$\pi 6^2(12) + \frac{4}{3}\pi 6^3$

$432\pi + 288\pi$

$\boxed{V = 720\pi}$

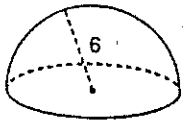
$TA = LA + \text{Sphere}$

$2\pi(6)(12) + 4\pi 6^2$

$144\pi + 144\pi$

$\boxed{TA = 288\pi}$

14.




$V = V_{\frac{1}{2}\text{sphere}}$

$= \left(\frac{4}{3}\pi 6^3\right)\frac{1}{2}$

$= \frac{2}{3}\pi(216)$

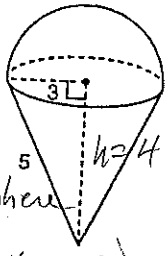
$\boxed{V = 144\pi}$

$TA = \frac{1}{2}(4\pi r^2) + 0$

$2\pi 36 + \pi 6^2$

$\boxed{TA = 108\pi}$

15.




$V = V_{\text{cone}} + V_{\frac{1}{2}\text{sphere}}$

$\frac{1}{3}\pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$\frac{1}{3}\pi 9(4) + \frac{2}{3}\pi 27$

$V = 12\pi + 18\pi = \boxed{30\pi}$

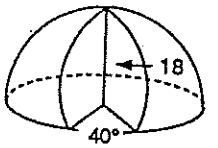
$TA = LA_{\text{cone}} + A_{\frac{1}{2}\text{sphere}}$

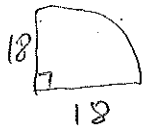
$= \pi(3)(5) + \left(4\pi 3^2\right)\frac{1}{2}$

$= 15\pi + 18\pi$

$\boxed{TA = 33\pi}$

16.




$V = \frac{320}{360}\left(\frac{1}{2}\cdot\frac{4}{3}\pi r^3\right)$

$= \frac{8}{9}\left(\frac{2}{3}\pi 18^3\right)$

$V = \frac{93312}{27}\pi = \boxed{3456\pi}$

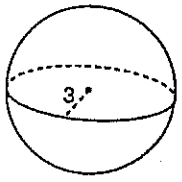
$TA = \frac{8}{9}\left(\frac{1}{2}(4\pi r^2)\right) + \frac{8}{9}(\pi r^2) + 2\left(\frac{1}{4}\cdot\pi 18^2\right)$

$= \frac{16}{9}\pi 18^2 + \frac{8}{9}\pi 18^2 + \frac{1}{2}\pi 18^2$

$= 864\pi + 162\pi$

$TA = \boxed{1026\pi}$

17.




$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi 3^3$

$V = \boxed{36\pi}$

$TA = 4\pi r^2$

$4\pi 3^2$

$\boxed{TA = 36\pi}$

## Some examples of good problems from Chapter 12 ...

1. An equilateral triangular-based prism is 12 cm high. It has LA = 288 $cm^2$. Find the SA and the V of the prism.

$h=12$

$LA = ph$

$288 = 3e(12)$

$8 = e$

$B = \frac{s^2\sqrt{3}}{4} = \frac{8^2\sqrt{3}}{4} = 16\sqrt{3}$

$SA = 288 + 32\sqrt{3}$

$V = 16\sqrt{3}(12)$

$= \boxed{192\sqrt{3}}$

2. The surface area of a regular square pyramid is 48 $cm^2$. If the slant height is equal to the base-edge length, find the area of the base.

$l = e$

$SA = \frac{1}{2}pl + e^2$

$48 = \frac{1}{2}4l \cdot l + l^2$

$48 = 3l^2$

$16 = l^2 \rightarrow l = 4$

$\boxed{A_{base} = 16}$

3. A prism with a regular hexagonal base and a height of 3" has a volume of $288\sqrt{3}$ cu in. Find the length of each edge of the base.

$a = \frac{e\sqrt{3}}{2}$ $\frac{e}{2}$

$V = \frac{1}{2}aph$

$288\sqrt{3} = \frac{1}{2}\frac{e\sqrt{3}}{2}6e(3)$

$288\sqrt{3} = \frac{9e^2\sqrt{3}}{2}$

$64 = e^2$

$\boxed{e = 8}$

4. Find the volume of a pyramid whose height is 14 cm and whose base is a rhombus with diagonals 6 and 8 cm.

$V = \frac{1}{3}\frac{1}{2}d_1d_2h$

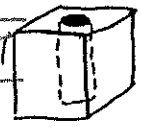
$= \frac{1}{6}48(14)$

$\boxed{V = 112}$

5. A cylindrical hole with diameter 6" is cut through a cube 8" on a side. Find the surface area of this solid.

$SA = LA + 2B_{cube} - 2B_{cyl} + LA_{cylinder}$

$= 4(8)(8) + 2(8^2) - 2\pi 3^2 + 2\pi(3)8 = \boxed{384 + 30\pi}$



6. Find the volume of the solid in # 5.

$V = Bh - V_{cylinder}$

$V = 8^3 - \pi 3^2(8)$

$\boxed{512 - 72\pi}$

7. Find the volume of a right cone if the circumference of the base is 6 $\pi$ and the altitude is 7.

$C = 2\pi r$

$6\pi = 2\pi r$

$3 = r$

$V = \frac{1}{3}(\pi \cdot 3^2)(7)$

$\boxed{V = 21\pi}$

8. Find the surface area of a sphere if its volume is 36 $\pi$.

$36\pi = \frac{4}{3}\pi r^3$

$27 = r^3 \rightarrow r = 3$

$SA = 4\pi 3^2 = \boxed{36\pi}$

9. Two cylinders have heights in a ratio of 3 to 4.

Cyl #1 : Cyl. 2

a. Find the ratio of the circumferences of their bases.
b. Find the ratio of their lateral surface areas.
c. Find the ratio of the diameters of their bases.
d. Find the ratio of their volumes.

} SKIP

$r = 4$

10. A cylinder has a height of 15" and a base radius of 4". If paint costs 86 cents per square foot, how much would it cost to paint the ENTIRE outside of the cylinder?

$TA = 2\pi(4)(15) + 2\pi 4^2$

$120\pi + 32\pi = 152\pi$

$\frac{477.28 \text{ sq in}}{144} \approx 3.31 \text{ sq ft}$ $\times .86 = \boxed{\$2.85}$

11. Find the surface area and volume of a right cylinder that is 22" in diameter and 6' high.

$r = 11$ $h = 72$

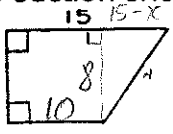
$TA = 2\pi 11(72) + 2\pi 11^2$

$1584\pi + 242\pi = \boxed{1826\pi \text{ sq in.}}$

$V = \pi 11^2(72) = \boxed{8712\pi \text{ cu. in.}}$

12. A pool with a cross section shown is 65 feet long. How many cubic yards of water are necessary to fill the pool?

1 cu yd = 27 cu. ft.




$B = \frac{1}{2}(8)(25)$

$B = 100$

$V = 100(65) = 6500 \text{ ft}^3$

$V = \frac{6500}{27} \text{ yds}^3$ water