KEY

Geometry(H)
Algebra Review with a Geometry Twist!

Let's review some formulas and concepts about lines from algebra.

from Pyth. thm $d^2 = (PR)^2 + (QR)^2$

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

(avg of 2 pts) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

If $a^2 + b^2 = c^2 \rightarrow$ right Δ
$a^2 + b^2 > c^2 \rightarrow$ acute Δ
$a^2 + b^2 < c^2 \rightarrow$ obtuse Δ

The slopes of parallel lines are equal.

The slopes of perpendicular lines are negative reciprocals. (product $= -1$)
m and $-\frac{1}{m}$

Let's try some review problems ...

For each problems use the points A(3,5) and B(-8,7)

1. Find the length of $\overline{AB}$.
$d = \sqrt{(3 - -8)^2 + (5 - 7)^2}$
$= \sqrt{11^2 + (-2)^2}$
$\sqrt{121 + 4} \rightarrow \sqrt{125} \rightarrow \boxed{d = 5\sqrt{5} \text{ units long}}$

2. Find the midpoint of $\overline{AB}$.
$\left(\frac{3 + -8}{2}, \frac{5+7}{2}\right)$
$\left(-\frac{5}{2}, 6\right)$

3. Find the equation of the line that passes through A and B.
$m = \frac{7-5}{-8-3} = \frac{2}{-11}$
$y = mx + b$
$5 = \frac{2}{-11}(3) + b$
$5 = -\frac{6}{11} + b$
$5\frac{6}{11} = b$
$\boxed{y = -\frac{2}{11}x + 5\frac{6}{11}}$

$(3,5)\ (-8,7)$

4. Find the equation of the perpendicular bisector of $\overline{AB}$.

$M_{AB} = -\frac{2}{11}$

$M_{\perp} = \frac{11}{2}$

midpt

$\left(\frac{3-8}{2}, \frac{5+7}{2}\right)$

$\left(-\frac{5}{2}, 6\right)$

$y = mx + b$

$6 = \frac{11}{2}\left(-\frac{5}{2}\right) + b$

$6 = -\frac{55}{4} + b$

$6 + 13\frac{3}{4} = b \rightarrow b = 19\frac{3}{4}$

$\boxed{y = \frac{11}{2}x + 19\frac{3}{4}}$

5. Find the equation of the line that is parallel to $\overline{AB}$ and passes through (-1,-2).

use $m = -\frac{2}{11}$

$-2 = -\frac{2}{11}(-1) + b$

$-2 - \frac{2}{11} = b$

$-2\frac{2}{11} = b$

$\boxed{y = -\frac{2}{11}x - 2\frac{2}{11}}$

6. A line with m = -1 contains the points (5,-2) and (x,-8). Solve for x.

$\frac{y-y}{x-x} = m$

$\frac{-2--8}{(5-x)} = -1$

$-1(5-x) = 6$

$5 - x = -6$

$11 = x$

ck

$\frac{-8--2}{11-5} = \frac{-6}{6} = -1$ ✓

7. Find the value of a, so that the line through (7,1) and (4,8) is parallel to the line through (2,a) and (a,-2).

$m = \frac{8-1}{4-7} = \frac{7}{-3}$

use →

$\frac{(a--2)}{(2-a)} = \frac{-7}{3}$

$3(a+2) = -7(2-a)$

$3a + 6 = -14 + 7a$

$20 = 4a$

$5 = a$

ck

$(2,5)(5,-2)$

$\frac{5--2}{2-5} = \frac{7}{-3}$ ✓

8. Points R(-5, -3), S(-1,-1) and T(5,x) are collinear. Find the value of x.

same slope

$M_{RS} = \frac{-3--1}{-5--1} = \frac{-2}{-4} = \frac{1}{2}$

$M_{ST} = \frac{y-y}{x-x}$

$\frac{1}{2} = \frac{(-1-x)}{(-1-5)}$

$2(-1-x) = -6$

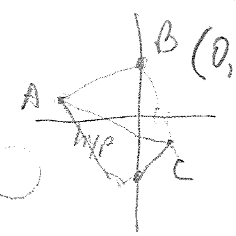
$-2 - 2x = -6$

$4 = 2x$

$2 = x$

ck $\frac{2--1}{5--1} = \frac{3}{6} = \frac{1}{2}$ ✓

9. $\triangle$ABC is a right triangle with coordinates A(-4,1) and C(2,-1). Point B is on the y-axis. Find the coordinates of B that would make m$\angle$B = 90°.



$B\ (0,y)$

$M_{\overline{AB}} = \frac{y-1}{4}$

$M_{\overline{BC}} = \frac{y+1}{-2}$

$(M_{\overline{AB}})(M_{\overline{BC}}) = -1$

$\frac{(y-1)}{4}\frac{(y+1)}{-2} = -1$

$\frac{y^2-1}{-8} = -1$

$y^2 - 1 = 8$

$y^2 = 9$

$y = \pm 3$

$B(0,3)$

$B(0,-3)$

10. Find each value of k for which the lines $y = 9kx - 1$ and $kx + 4y = 12$ are perpendicular.

$Kx + 4y = 12$

$4y = -Kx + 12$

$y = \frac{-Kx}{4} + 3$

$(9K)\left(-\frac{K}{4}\right) = -1$

$-\frac{9}{4}K^2 = -1$

$K^2 = \frac{4}{9}$

$K = \pm\frac{2}{3}$

$K = \frac{2}{3}$

$y = 9\left(\frac{2}{3}\right)x - 1$

$y = 6x - 1$

$y = -\frac{\frac{2}{3}}{4}x + 3$

$y = -\frac{1}{6}x + 3$

$K = -\frac{2}{3}$

$y = 9\left(-\frac{2}{3}\right)x - 1$

$= -6x - 1$

$y = -\frac{-\frac{2}{3}}{4}x + 3$

$y = \frac{1}{6}x + 3$

11. The distance between points (1,2) and (x,8) is 10. Find x.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$10 = \sqrt{(1-x)^2 + (2-8)^2}$

$100 = 1 - 2x + x^2 + 36$

$0 = x^2 - 2x - 63$

$0 = (x-9)(x+7)$

$x = 9$ $\quad x = -7$

ck

$10 = \sqrt{(1-9)^2 + (2-8)^2}$

$10 = \sqrt{64 + 36}$

$10 \stackrel{\checkmark}{=} 10$

$10 = \sqrt{(1- -7)^2 + 36}$

$10 = \sqrt{64 + 36}$

$10 \stackrel{\checkmark}{=} 10$

12. ΔABC had coordinates A(-1,-6), B(5,2) and C(-3,-2). Classify this triangle by its sides and angles.

$d_{\overline{AB}} = \sqrt{(-1-5)^2 + (-6-2)^2}$

$= \sqrt{36 + 64}$

$d_{\overline{AB}} = 10$

$d_{\overline{BC}} = \sqrt{(5 - -3)^2 + (2 - -2)^2}$

$= \sqrt{64 + 16}$

$= \sqrt{80}$

$d_{\overline{BC}} = 4\sqrt{5}$

$d_{\overline{AC}} = \sqrt{(-1 - -3)^2 + (-6 - -2)^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

$d_{\overline{AC}} = 2\sqrt{5}$

$a^2 + b^2 = c^2$

$(4\sqrt{5})^2 + (2\sqrt{5})^2 = 10^2$

$16(5) + 4(5) = 100$

$80 + 20 \stackrel{\checkmark}{=} 100$

Scalene

right triangle

13. ΔSRT had coordinates A(-1,-1), B(2,0) and C(0,10). Classify this triangle by its sides and angles.

$d_{\overline{AB}} = \sqrt{(-1-2)^2 + (-1-0)^2}$

$= \sqrt{9 + 1}$

$= \sqrt{10}$

≈ 3

$d_{\overline{BC}} = \sqrt{(2-0)^2 + (0-10)^2}$

$= \sqrt{4 + 100}$

$= \sqrt{104}$

$= 2\sqrt{26}$

≈ 10.2

$d_{\overline{AC}} = \sqrt{(-1-0)^2 + (-1-10)^2}$

$= \sqrt{1 + 121}$

$= \sqrt{122}$

≈ 11.05

$a^2 + b^2 = c^2$

$(\sqrt{10})^2 + (2\sqrt{26})^2 = \sqrt{122}^2$

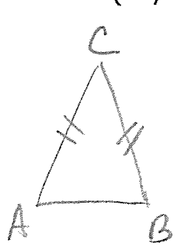
$10 + 4(26) = 122$

$114 < 122$

Scalene

acute triangle

14. Given isosceles triangle ABC with AC = BC and vertices with coordinates A(-1,4), B(-3,-2) and C(x,-1). Find x.



$AC = BC$

$\sqrt{(-1-x)^2 + (4 - -1)^2} = \sqrt{(-3-x)^2 + (-2 - -1)^2}$

$1 + 2x + x^2 + 25 = 9 + 6x + x^2 + 1$

$x^2 + 2x + 26 = x^2 + 6x + 10$

$16 = 4x$

$4 = x$

ck $\quad$ checks!

$\sqrt{(-1-4)^2 + (4 - -1)^2} = d_{\overline{AC}}$

$\sqrt{25 + 25} = d_{\overline{AC}}$

$5\sqrt{2} = d_{\overline{AC}}$

$\sqrt{(-3-4)^2 + (-2 - -1)^2} = d_{\overline{BC}}$

$\sqrt{49 + 1}$

$\sqrt{50} = 5\sqrt{2} = d_{\overline{BC}}$