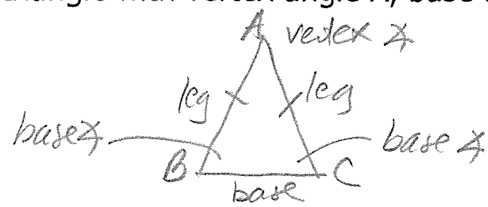
KEY

Geometry (h)
Section 4.4 – The Isosceles Triangle Theorems

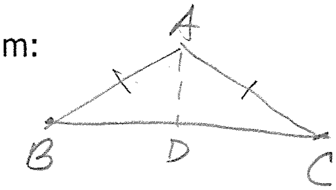
Draw an isosceles triangle with vertex angle A, base angles B and C. Label the legs and base of the triangle.



Isosceles Triangle Theorem –
If 2 sides of a △ are ≅, then the ∡s opposite those sides are ≅

Given: $\overline{AB} \cong \overline{AC}$

Diagram:



Prove: $\angle B \cong \angle C$

① Draw ∡ bisector $\overline{AD}$. → ② $\angle BAD \cong \angle CAD$, ③ $\overline{AB} \cong \overline{AC}$, ④ $\overline{AD} \cong \overline{AD}$ → ⑤ $\triangle BAD \cong \triangle CAD$ → ⑥ $\angle B \cong \angle C$

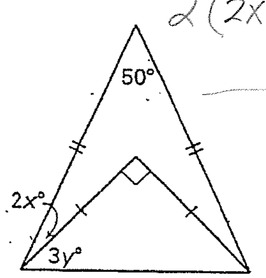
① Postulate: An ∡ has one & only one bisector.
② Def ∡ bisector
③ Given
④ Reflexive property
⑤ SAS ≅ SAS
⑥ CPCTC

The converse is also true:

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Ex 1: Find the value of x and y.

a.



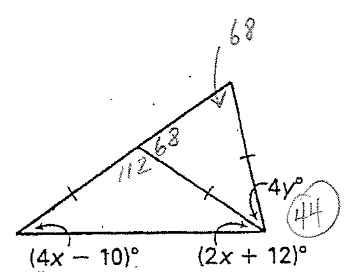
$2(2x+3y)+50=180$
$2(3y)=90$
$6y=90$
$y=15$

$4x+6y=130$
$4x+90=130$
$4x=40$
$x=10$

$2x=20$
$3y=45$

ck
$2(65)+50$
$130+50$

b.



34 34

$4X-10=2X+12$
$2X=22$
$X=11$

$4y+2(68)=180$
-136
$4y=44$
$y=11$

$180 - 68 = 112$

ck
$68 \cdot 2 = 136$, $+44$, 180 ✓
$68 + 34 + 78 = 180$ ✓

Corollary: Bisector of vertex ∡ of isos. △ bisects the base.

Corollary 1: An equilateral triangle is also equiangular.

Corollary: An equiangular △ is equilateral.

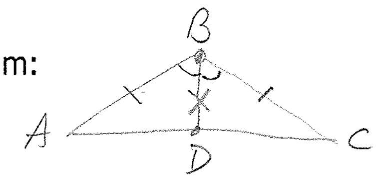
Corollary 2: An equilateral triangle has three 60° angles

Corollary 3: The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.

Let's prove the 3rd corollary!

Given: $\overline{BD}$ bisects ∡ABC, $\overline{AB} \cong \overline{CB}$

Diagram:



Prove: $\overline{BD} \perp \overline{AC}$, $\overline{AD} \cong \overline{DC}$

① $\overline{BD}$ bisects ∡ABC. → ② ∡ABD ≅ ∡CBD

③ $\overline{AB} \cong \overline{CB}$

④ $\overline{BD} \cong \overline{BD}$

→ ⑤ △ABD ≅ △CBD (SAS) → ⑥ ∡BDA ≅ ∡BDC

⑦ $\overline{BD} \perp \overline{AC}$

⑧ $\overline{AD} \cong \overline{DC}$

① Given

② Def ∡ bisector

③ Given

④ Reflexive property

⑤ △BAD ≅ △BCD

⑥ CPCTC

⑦ If ⊥ lines form ≅ adj. ∡s, → ⊥ lines

⑧ CPCTC

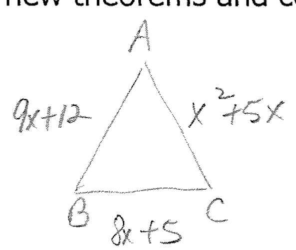
Let's try some more examples using the new theorems and corollary.

Ex 2: △ABC isosceles with base $\overline{BC}$.

AC = $x^2 + 5x$

AB = $9x + 12$

BC = $8x + 5$



Find the perimeter of △ABC.

$9x + 12 = x^2 + 5x$

$0 = x^2 - 4x - 12$

$(x-6)(x+2) = 0$

$x = 6 \quad x = -2$

OMIT

$x = 6$

$AC = 36 + 30 = 66$

$AB = 54 + 12 = 66$

$BC = 48 + 5 = 53$

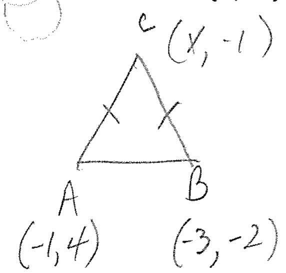
$P = 132 + 53 = \boxed{185 \text{ units}}$

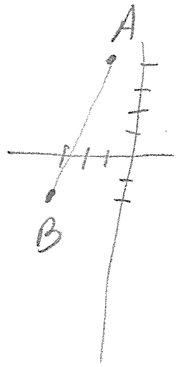
$x = -2$ OMIT

$AC = 4 - 10 = -6$

impossible

Ex 3: $\triangle ABC$ is isosceles with vertex angle C. The coordinates $\triangle ABC$ are A(-1,4), B(-3,-2) and C(x,-1). Find x.





$AC = BC$

$\sqrt{(x+1)^2+(-1-4)^2} = \sqrt{(x+3)^2+(-1+2)^2}$

$x^2+2x+1+25 = x^2+6x+9+1$

$2x+26 = 6x+10$

$16 = 4x$

$4 = x$

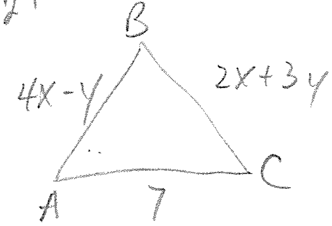
ck

$\sqrt{50} \stackrel{\checkmark}{=} \sqrt{50}$

$AC = BC$

Text p139/27

Ex 4: In equilateral triangle ABC, AB = 4x – y, BC = 2x + 3y, AC = 7. Find x and y.

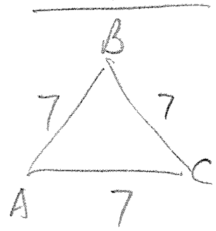


$\boxed{\begin{aligned} 4x-y &= 2x+3y \\ 4x-y &= 7 \end{aligned}}$ or $\boxed{\begin{aligned} 4x-y &= 7 \\ 2x+3y &= 7 \end{aligned}}$

$12x-3y=21$
$2x+3y=7$
$14x=28$
$x=2$

$4x-y=7$
$8-y=7$
$1=y$

ck

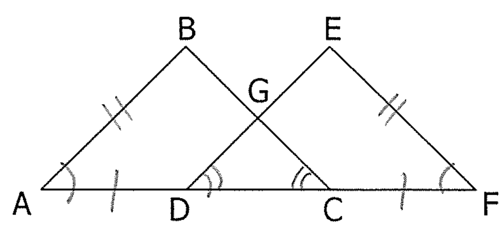


✓

Ex 5: For each of the following problems write a flow proof.

a. Given: $\overline{AD} \cong \overline{CF}$
$\overline{AB} \cong \overline{EF}$
$\angle A \cong \angle F$

Prove: $\triangle DGC$ isosceles



① $\overline{AD} \cong \overline{CF}$ → ② $\overline{AC} \cong \overline{DF}$
③ $\overline{AB} \cong \overline{EF}$
④ $\angle A \cong \angle F$
→ ⑤ $\triangle ABC \cong \triangle FED$ → ⑥ $\angle C \cong \angle D$ → ⑦ $\overline{DG} \cong \overline{CG}$ → ⑧ $\triangle DGC$ isosceles

① Given
② Common Segment Thm
③ Given
④ Given

⑤ SAS ≅ SAS
⑥ CPCTC
⑦ If 2 ∠s ≅, then sides opposite ≅.

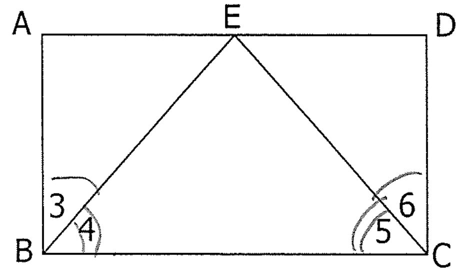
⑧ Def of isosceles △
If 2 sides of △ ≅, then isosceles △.

b. Given: $\angle 3 \cong \angle 6$

$\overline{AB} \perp \overline{BC}$

$\overline{DC} \perp \overline{BC}$

Prove: $\triangle$EBC isosceles



① $\overline{AB} \perp \overline{BC}$ → ② ∡3 comp' ∡4

③ $\overline{DC} \perp \overline{BC}$ → ④ ∡6 comp' ∡5

⑤ ∡3 ≅ ∡6

→ ⑥ ∡4 ≅ ∡5 → ⑦ $\overline{EB} \cong \overline{EC}$ → ⑧ $\triangle$EBC isosceles

① Given

② ⊥ lines form ≅ adj. ∡s → ∡s complementary

③ Given

④ ⊥ lines form ≅ adj. ∡s → ∡s complementary

⑤ Given

⑥ ≅ complements Thr

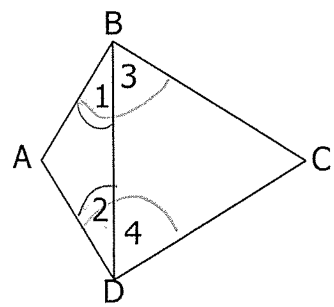
⑦ If 2 ∡s ≅, then sides opposite ≅.

⑧ Def isosceles $\triangle$

c. Given: $\angle ABC \cong \angle ADC$

$\angle 1 \cong \angle 2$

Prove: $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$



① ∡1 ≅ ∡2 → ② $\overline{AB} \cong \overline{AD}$

③ ∡ABC ≅ ∡ADC → ④ m∡ABC = m∡ADC

⑤ m∡ABC = m∡1 + m∡3

m∡ADC = m∡2 + m∡4

→ ⑥ m∡1 + m∡3 = m∡2 + m∡4

⑦ ∡1 ≅ ∡2 → ⑧ m∡1 = m∡2

⑨ m∡3 = m∡4 → ⑩ ∡3 ≅ ∡4 → ⑪ $\overline{BC} \cong \overline{DC}$

① Given

② If 2 ∡s ≅, sides opposite ≅.

③ Given

④ def of ≅ ∡s

⑤ Angle Addition Postulate

⑥ Substitution

⑦ Given

⑧ Def of ≅ ∡s

⑨ Subtraction Prop.

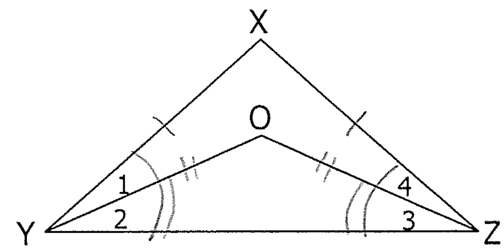
⑩ Def of ≅ ∡s

⑪ If 2 ∡s ≅, sides opposite ≅.

p138 #17

d. Given: $\overline{XY} \cong \overline{XZ}$
$\overline{OY} \cong \overline{OZ}$

Prove: m∠1 = m∠4



(1) $\overline{XY} \cong \overline{XZ}$ → (2) $\angle XYZ \cong \angle XZY$ → (3) $m\angle XYZ = m\angle XZY$

(4) $m\angle XYZ = m\angle 1 + m\angle 2$
$m\angle XZY = m\angle 4 + m\angle 3$

(5) $m\angle 1 + m\angle 2 = m\angle 4 + m\angle 3$

(6) $\overline{OY} \cong \overline{OZ}$ → (7) $\angle 2 \cong \angle 3$ → (8) $m\angle 2 = m\angle 3$

(9) $m\angle 1 = m\angle 4$

(1) Given
(2) If 2 sides ≅, ∠s opp. are ≅.
(3) Def of ≅ ∠s
(4) Angle Addition Postulate
(5) Substitution
(6) Given
(7) If 2 sides ≅, ∠s opp are ≅.
(8) Def of ≅ ∠s
(9) Subtraction Property