Geometry (H)
Section 4.5 & 4.6 – Notes & Problems

Key

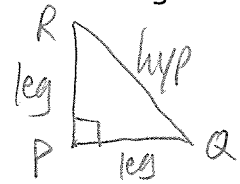
Today we will continue our discussion of proving triangles congruent. We will introduce one more theorem for proving triangles congruent and then look at different types of problems that would require proving triangles congruent to solve.

List all the ways to prove triangles congruent.

1. SSS
2. ASA
3. SAS
4. AAS
5. ~~~~

There is one other method that applies to right triangles only …

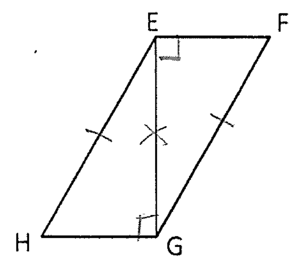
Draw $\triangle PRQ$, with right $\angle P$. Label the legs and hypotenuse.



HL (Hypotenuse-Leg) Theorem – If the hypotenuse & leg of one right $\triangle$ are $\cong$ to the corresp. parts of another $\triangle$, then the $\triangle$s are $\cong$. (hy leg $\cong$ hy leg) (Must establish right $\triangle$s)

Ex 1: Given: $\overline{EF} \perp \overline{EG}$; $\overline{HG} \perp \overline{EG}$
$\overline{EH} \cong \overline{GF}$

Prove: $\overline{EF} \cong \overline{GH}$

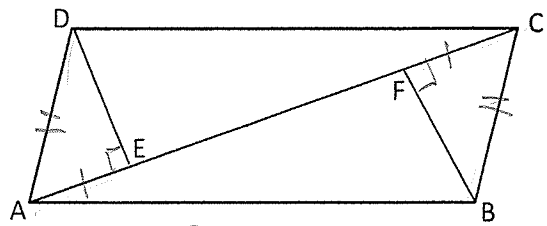


HL Theorem

① $\overline{EF} \perp \overline{EG}$ → ② $\angle FEG$ is right → ③ $\triangle FEG$ is Rt $\triangle$.
④ $\overline{HG} \perp \overline{EG}$ → ⑤ $\angle EGH$ is right → ⑥ $\triangle EGH$ is Rt $\triangle$.
⑦ $\overline{EH} \cong \overline{GF}$
⑧ $\overline{EG} \cong \overline{EG}$
→ ⑨ $\triangle FEG \cong \triangle EGH$
⑩ $\overline{EF} \cong \overline{GH}$

① Given
② $\perp$ lines form Rt. $\angle$s.
③ Def of Rt. $\triangle$
④ Given
⑤ $\perp$ lines form Rt $\angle$s.
⑥ Def of Rt. $\triangle$.
⑦ Given
⑧ Reflexive Prop
⑨ Hy Leg $\cong$ Hy Leg
⑩ CPCTC

Ex: 2: Given: $\overline{AF} \cong \overline{EC}$; $\overline{DA} \cong \overline{BC}$
$\overline{DE} \perp \overline{AC}$; $\overline{BF} \perp \overline{AC}$



Prove: $\overline{DA} \parallel \overline{BC}$

① $\overline{AF} \cong \overline{EC}$ → ② $\overline{AE} \cong \overline{FC}$
③ $\overline{DA} \cong \overline{BC}$

④ $\overline{DE} \perp \overline{AC}$ → ⑤ $\measuredangle DEA$ is Rt. → ⑧ $\triangle DEA$ is Rt
⑥ $\overline{BF} \perp \overline{AC}$ → ⑦ $\measuredangle BFC$ is Rt. → ⑨ $\triangle BFC$ is Rt.

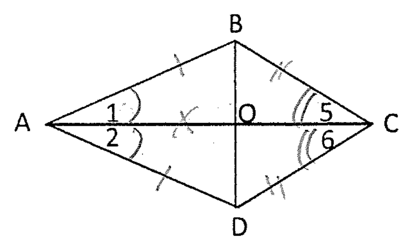
→ ⑩ $\triangle DEA \cong \triangle BFC$

⑪ $\measuredangle DAE \cong \measuredangle BCF$ → ⑫ $\overline{DA} \parallel \overline{BC}$

① Given
② Common Segmt Thm
③ Given
④ Given
⑤ ⊥ lines form Rt. ∡
⑥ Given
⑦ Def ⊥ lines
⑧ Def Rt. △
⑨ Def Rt △
⑩ hy leg ≅ hy leg
⑪ CPCTC
⑫ 2 line alt. int ∡s ≅ → ∥ line

Sometimes two triangles that you want to prove congruent have common parts with two other triangles that you can easily prove congruent. You may then be able to use corresponding parts of these other triangles to prove the original triangles congruent.

Ex 3: Given: $\angle 1 \cong \angle 2$; $\angle 5 \cong \angle 6$



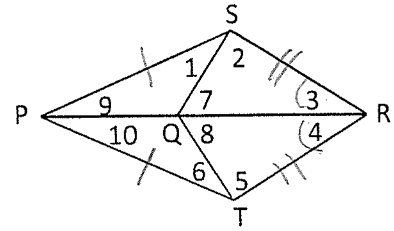
Prove: $\overline{AC} \perp \overline{BD}$

① $\measuredangle 1 \cong \measuredangle 2$, $\measuredangle 5 \cong \measuredangle 6$
② $\overline{AC} \cong \overline{AC}$
→ ③ $\triangle ABC \cong \triangle ADC$ (ASA)
→ ④ $\overline{AB} \cong \overline{AD}$ (CPCTC)
⑤ $\overline{AO} \cong \overline{AO}$
⑥ $\measuredangle 1 \cong \measuredangle 2$
→ ⑦ $\triangle ABO \cong \triangle ADO$ (SAS) → ⑧ $\measuredangle AOB \cong \measuredangle AOD$

⑨ $\overline{AC} \perp \overline{BD}$

① Given
② Reflexive property
③ ASA ≅ ASA
④ CPCTC
⑤ Reflexive property
⑥ Given
⑦ SAS ≅ SAS
⑧ CPCTC
⑨ 2 lines & ≅ adj. ∡s → ⊥ lines.

Ex 4: Given: $\overline{PS} \cong \overline{PT}$; $\overline{SR} \cong \overline{TR}$



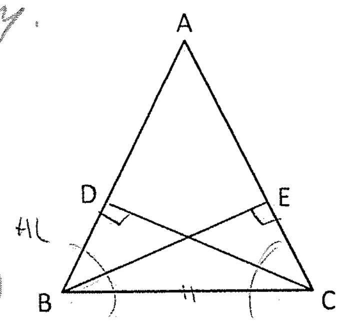
Prove: $\angle 2 \cong \angle 5$

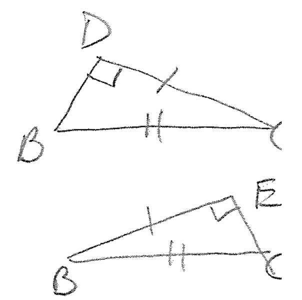
① $\overline{PS} \cong \overline{PT}$, $\overline{SR} \cong \overline{TR}$, ② $\overline{PR} \cong \overline{PR}$ → ③ $\triangle PSR \cong \triangle PTR$ (SSS) → ④ $\angle 3 \cong \angle 4$, ⑤ $\overline{QR} \cong \overline{QR}$, ⑥ $\overline{SR} \cong \overline{TR}$ → ⑦ $\triangle SRQ \cong \triangle TRQ$ (SAS) → ⑧ $\angle 2 \cong \angle 5$

① Given
② Reflexive prop
③ SSS ≅ SSS
④ CPCTC
⑤ Reflexive Prop
⑥ Given
⑦ SAS ≅ SAS
⑧ CPCTC

Sometimes the triangles that we need to prove congruent are a little harder to find. They may be overlapping. * Draw △s separately.

Ex 5: Given: $\overline{CD} \perp \overline{AB}$; $\overline{BE} \perp \overline{AC}$
$\overline{BE} \cong \overline{CD}$





Prove: △ABC is isosceles

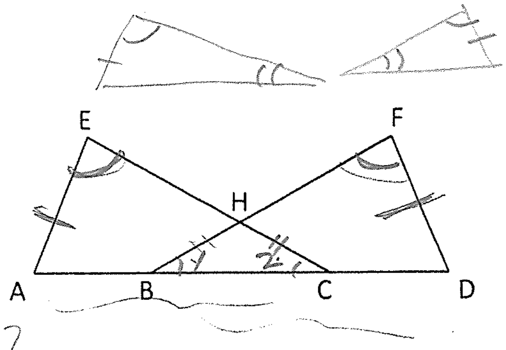
① $\overline{CD} \perp \overline{AB}$, $\overline{BE} \perp \overline{AC}$ → ② $\angle BDC$ is Rt. → ④ $\triangle BDC$ Rt.; ③ $\angle CEB$ is Rt. → ⑤ $\triangle CEB$ Rt.; ⑥ $\overline{BE} \cong \overline{CD}$; ⑦ $\overline{BC} \cong \overline{BC}$ → ⑧ $\triangle BDC \cong \triangle CEB$ → ⑨ $\angle DBC \cong \angle EC$ → ⑩ $\overline{AB} \cong \overline{AC}$ → ⑪ $\triangle ABC$ is isosceles

① Given
② ③ ⊥ lines form Rt ∡s.
④ Def. Rt. △
⑤ Def Rt △
⑥ Given
⑦ Reflexive prop
⑧ hyleg ≅ hyleg
⑨ CPCTC
⑩ If 2 ∡s ≅, sides opp ≅.
⑪ Def of isosceles △.

Ex 6: Given: $\overline{AE} \cong \overline{DF}$; $\angle E \cong \angle F$

$\triangle BCH$ isosceles with vertex $\angle H$

Prove: $\overline{AB} \cong \overline{CD}$



① $\triangle BCH$ isosceles. → ② $\overline{BH} \cong \overline{HC}$ → ③ $\measuredangle 1 \cong \measuredangle 2$

④ $\measuredangle E \cong \measuredangle F$

⑤ $\overline{AE} \cong \overline{DF}$ → ⑥ $\triangle ABC \cong \triangle DFB$

⑦ $\overline{AC} \cong \overline{BD}$ → ⑧ $\overline{AB} \cong \overline{CD}$

① Given

② Def. of isosceles △

③ Isosceles Triangle Thm: If 2 sides ≅, ∡s opp ≅.

④ Given

⑤ Given

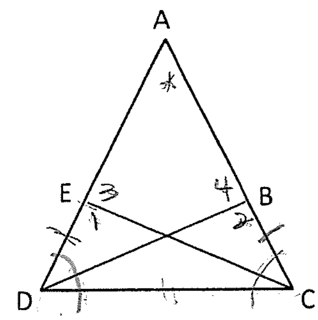
⑥ AAS ≅ AAS (use diagram to avoid mistake)

⑦ CPCTC

⑧ Common segment thm

Ex 7: Given: $\overline{BC} \cong \overline{ED}$; $\angle ADC \cong \angle ACD$

Prove: $\overline{AB} \cong \overline{AE}$



① $\measuredangle ADC \cong \measuredangle ACD$ → ② $\overline{AD} \cong \overline{AC}$ → ③ $AD = AC$

④ $\overline{BC} \cong \overline{ED}$ → ⑤ $BC = ED$

⑥ $AD - ED = AC - BC$ → ⑦ $AB = AE$ → ⑧ $\overline{AB} \cong \overline{AE}$

$\triangle EDC \cong \triangle BCD \to \angle 1 \cong \angle 2$

① Given

② If 2 ∡s of a △ are ≅, then sides opposite ≅.

③ def of ≅ segments

④ Given

⑤ def of ≅ segments

⑥ Subtraction Property

⑦ Subtraction Property

$\to \angle 3 \cong \angle 4$

$\angle A \cong \angle A$

$\overline{BD} \cong \overline{EC}$