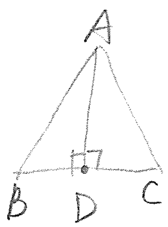
Geometry (H)
Chapter 4 review

Name: KEY

1. $\triangle ABC$ is isosceles with base $\overline{BC}$ and altitude $\overline{AD}$. BD = 3x – 10, DC = 5y + 7 and m∠ADB = x + 2y. Find BC.



$\boxed{\begin{array}{c} 3x-10=5y+7 \\ x+2y=90 \end{array}}$

$3x-5y=17$
$-3x-6y=-270$
$-11y=-253$
$y=23$

$x+2(23)=90$
$x=44$

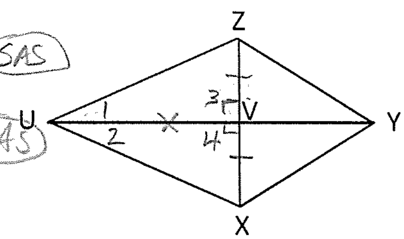
$BD=132-10=122$
$DC=115+7=122$
$BC=244$

CK: $m\angle ADB=44+46=90$ ✓

2. Write a flow proof for each of the following.

a. Given: V is the midpoint of $\overline{XZ}$
$\overline{UY} \perp \overline{XZ}$

$\triangle UVZ \cong \triangle UVX$ (SAS)
$\triangle UZY \cong \triangle UXY$ (SAS)



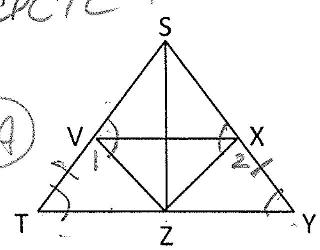
Prove: ∠UXY ≅ ∠UZY

① V midpt of $\overline{XZ}$ → ② $\overline{ZV} \cong \overline{VX}$
③ $\overline{UY} \perp \overline{XZ}$ → ④ ∠3 & ∠4 Rt. ∠s → ⑤ ∠3 ≅ ∠4
⑥ $\overline{UV} \cong \overline{UV}$
→ ⑦ $\triangle UVZ \cong \triangle UVX$ → ⑧ $\overline{UZ} \cong \overline{UX}$
→ ⑨ ∠1 ≅ ∠2
⑩ $\overline{UY} \cong \overline{UY}$
→ ⑪ $\triangle UZY \cong \triangle UXY$ → ⑫ ∠UXY ≅ ∠UZY

Reasons
① Given
② Def midpt
③ Given
④ Def. ⊥ lines.
⑤ All rt ∠s ≅.
⑥ Reflexive Prop
⑦ SAS ≅ SAS
⑧ CPCTC
⑨ CPCTC
⑩ Reflexive Prop
⑪ SAS ≅ SAS
⑫ CPCTC

b. Given: ∠T ≅ ∠Y; ∠SVZ ≅ ∠SXZ; $\overline{TV} \cong \overline{YX}$

$\triangle TVZ \cong \triangle YXZ$ (ASA)



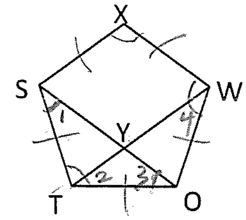
Prove: $\overline{SZ}$ is the median to $\overline{TY}$

① ∠1 & ∠SVZ linear pair. → ② ∠1 supp ∠SVZ
∠2 & ∠SXZ linear pair → ∠2 supp ∠SXZ
③ ∠SVZ ≅ ∠SXZ
→ ④ ∠1 ≅ ∠2
⑤ $\overline{TV} \cong \overline{YX}$
⑥ ∠T ≅ ∠Y
→ ⑦ $\triangle TVZ \cong \triangle YXZ$
→ ⑧ $\overline{TZ} \cong \overline{ZY}$ → ⑨ Z midpt of $\overline{TY}$
→ ⑩ $\overline{SZ}$ median to $\overline{TY}$

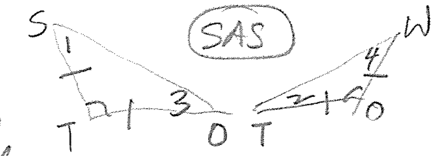
① Def linear pair
② Linear pair postulate
③ Given
④ ≅ supplements Thm
⑤ Given
⑥ Given
⑦ ASA ≅ ASA
⑧ CPCTC
⑨ Def of midpoint
⑩ A median is a seg. connecting midpt of seg to opp vertex

c. Given: XSTOW is a regular pentagon

Prove: ΔTYO is isosceles



① XSTOW is regular pentagon → ② $\overline{ST} \cong \overline{TO} \cong \overline{OW}$
③ $\measuredangle STO \cong \measuredangle WOT$

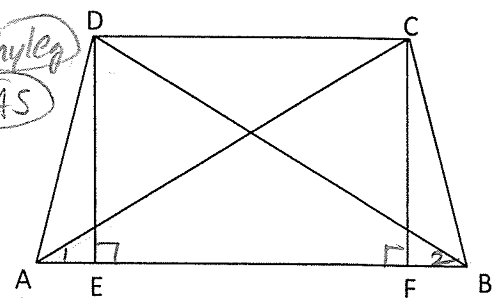


④ $\Delta STO \cong \Delta WOT$ → ⑤ $\measuredangle 2 \cong \measuredangle 3$ → ⑥ $\overline{TY} \cong \overline{OY}$ → ⑦ ΔTYO isosceles.

① Given
② In a reg pentagon, all sides ≅.
③ In a reg. pentagon, all ∡s ≅
④ SAS ≅ SAS
⑤ CPCTC
⑥ If 2 ∡s ≅, sides opp ≅.
⑦ Def of isosceles Δ.

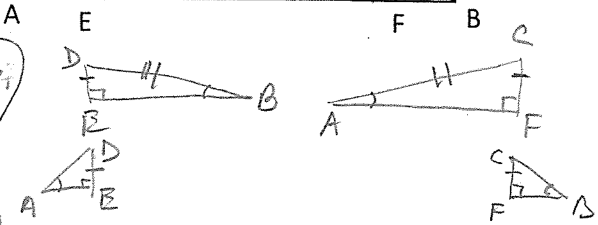
d. Given: $\overline{DE}$ altitude in ΔADB
$\overline{CF}$ altitude is ΔBCA
$\overline{DE} \cong \overline{CF}$; $\overline{AC} \cong \overline{BD}$

ΔDEB ≅ ΔCFA (hyleg)
ΔDEA ≅ ΔCFB (AAS)



Prove: $\overline{AD} \cong \overline{BC}$

① $\overline{DE}$ altit. ΔADB → ② $\overline{DE} \perp \overline{AB}$ → (2a) ∡DEB Rt∡
$\overline{CF}$ altit. ΔBCA → $\overline{CF} \perp \overline{AB}$ → ∡CFA Rt∡
→ ③ ΔDEB & ΔCFA Rt Δs
④ $\overline{DE} \cong \overline{CF}$
⑤ $\overline{AC} \cong \overline{BD}$



⑥ ΔDEB ≅ ΔCFA → ⑦ ∡1 ≅ ∡2
⑧ ∡DEA is Rt.
⑨ ∡CFB is Rt.
→ ⑩ ∡DEA ≅ ∡CFB
⑪ $\overline{DE} \cong \overline{CF}$
→ ⑫ ΔADE ≅ ΔBCF → ⑬ $\overline{AD} \cong \overline{BC}$

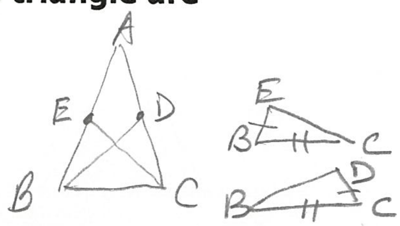
① Given
② Def of altitude
(2a) ⊥ lines form Rt ∡s.
③ Def Rt. Δs.
④ Given
⑤ Given
⑥ hyleg ≅ hyleg
⑦ CPCTC
⑧ Def ⊥ lines.
⑨ Def ⊥ lines
⑩ All Rt ∡s ≅.
⑪ Given
⑫ AAS ≅ AAS
⑬ CPCTC

3. Prove the following statement. Provide the given, prove, diagram and flow proof.

The medians drawn from the base angles of an isosceles triangle are congruent.

Given: $\triangle ABC$ isosceles w/ vertex A
$\overline{BD}$ & $\overline{CE}$ are medians.

Diagram:



Prove: $\overline{BD} \cong \overline{CE}$

① $\triangle ABC$ isos. → ② $\overline{AB} \cong \overline{AC}$ → ③ $AB = AC$ → ④ $\frac{1}{2}AB = \frac{1}{2}AC$
⑤ $\overline{BD}$ median $\overline{AC}$ → ⑥ D midpt of $\overline{AC}$ → ⑦ $DC = \frac{1}{2}AC$
⑧ $\overline{CE}$ median $\overline{AB}$ → ⑨ E midpt of $\overline{AB}$ → ⑩ $EB = \frac{1}{2}AB$
→ ⑪ $DC = EB$ → ⑫ $\overline{DC} \cong \overline{EB}$
$\angle BC \cong \angle DCB$ ⑮ $\triangle BEC \cong \triangle CD$
$\overline{CE}$

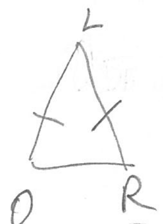
① Given
② Def isos. $\triangle$
③ Def $\cong$ seg.
④ Division prop.
⑤ Given
⑥ Def median
⑦ Midpt Theorem
⑧ Given
⑨ Def median
⑩ Midpt Thm
⑪
⑫
⑬
⑭

#5?

$x = \frac{337}{73}$

$y = \frac{579}{219}$

4. $\triangle LOR$ is isosceles with vertex $\angle L$. LO
Find LO and LR.



$LO = LR$

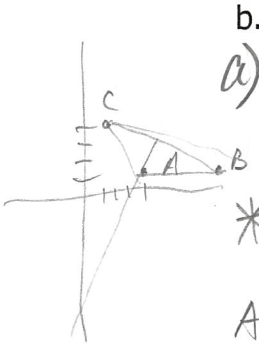
$\boxed{x^2 + 5 = 4x + 2}$

$x^2 - 4x +$

$(x-3)(x-$

$x = 3$ x

5. $\triangle ABC$ has vertices A(4,1), B(9,1) and C(1,4).
   a. Write the equation of the line containing the altitude from vertex A to $\overline{BC}$.
   b. Find the length of the altitude.



a) $m_{\overline{BC}} = \frac{1-4}{9-1} = \frac{-3}{8}$

$* \; m_{ALT} = \frac{8}{3}$

A(4,1)

$y = mx + b$

$1 = \frac{8}{3}(4) + b$

$1 - \frac{32}{3} = b$

$-\frac{29}{3} = b$

Altitude

$\boxed{a)\ y = \frac{8}{3}x - \frac{29}{3}}$

B(9,1)

$1 = -\frac{3}{8}(9) + b$

$1 + \frac{27}{8} = b$

$\frac{35}{8} = b$

EQ of $\overline{BC}$

$\boxed{y = \frac{-3}{8}x + \frac{35}{8}}$

$\frac{8}{3}x - \frac{29}{3} = -\frac{3}{8}x + \frac{35}{8}$

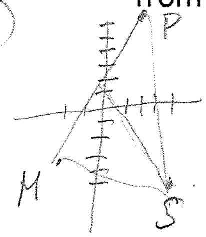
$\frac{64}{24}x + \frac{9}{24}x = \frac{232}{24} + \frac{105}{24}$

$\frac{73}{24}x = \frac{337}{24} \cdot \frac{24}{73}$

$x =$

$\boxed{b)\ \text{length} = \frac{\sqrt{16{,}801}}{73}}$

6. ΔMSP has vertices M(-2,-3), S(4,-5) and P(2,9). Write the equation of the median from vertex S to $\overline{MP}$.



midpoint $\overline{MP}$

$(0, 3)$

slope of median

$m = \frac{-5-3}{4-0} = \frac{-8}{4} = -2$

EQ of median

$\boxed{y = -2x + 3}$

ck

$S(4,-5)$

$-5 \stackrel{?}{=} -2(4)+3$

$-5 = -5$ ✓

7. Find the equation of the perpendicular bisector of $\overline{BC}$. B(-3,6) and C(1,9).

$m_{\overline{BC}} = \frac{9-6}{1+3} = \frac{3}{4}$

$* \; m_{\perp} = -\frac{4}{3}$

midpt of $\overline{BC}$

$\left(\frac{-3+1}{2}, \frac{6+9}{2}\right)$

$* \left(-1, \frac{15}{2}\right)$

$y = mx + b$

$\frac{15}{2} = -\frac{4}{3}(-1) + b$

$\frac{15}{2} - \frac{4}{3} = b$

$\frac{45}{6} - \frac{8}{6} = \frac{37}{6} = b$

$\boxed{y = -\frac{4}{3}x + \frac{37}{6}}$

$\perp$ bisector

8. Is $\Delta ZUM \cong \Delta CAR$? Show all your work including a written explanation.

Z(3,6), U(-2,5), M(-1,0) and C(7,-3) A(6,-8), R(1,-7)

$d_{\overline{UZ}} = \sqrt{(3+2)^2 + (6-5)^2}$
$= \sqrt{25+1} = \sqrt{26}$

$d_{\overline{MZ}} = \sqrt{(3+1)^2 + (6-0)^2}$
$\sqrt{16+36} = \sqrt{52}$

$d_{\overline{UM}} = \sqrt{(-2+1)^2 + (5-0)^2}$
$\sqrt{1+25} = \sqrt{26}$

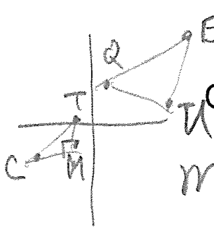
$d_{\overline{RC}} = \sqrt{(7-1)^2 + (-3+7)^2}$
$\sqrt{36+16} = \sqrt{52}$

$d_{\overline{RA}} = \sqrt{(6-1)^2 + (-8+7)^2}$
$\sqrt{25+1} = \sqrt{26}$

$d_{\overline{AC}} = \sqrt{(7-6)^2 + (-3+8)^2}$
$\sqrt{1+25} = \sqrt{26}$

Yes, $\Delta ZUM \cong \Delta CAR$
Since $MZ = RC \rightarrow \overline{MZ} \cong \overline{RC}$,
and $UZ = RA, \rightarrow \overline{UZ} \cong \overline{RA}$,
and $UM = AC \rightarrow \overline{UM} \cong \overline{AC}$,
therefore $\Delta ZUM \cong \Delta CAR$
by SSS $\cong$ SSS.

9. Show that the following right triangles ΔCMT and ΔQUE are congruent. You may not use SSS. You must use another postulate or theorem.



C(-9,-4), M(-3,-6), T(-1,0) and Q(1,5), U(7,3), E(9,9)

$m_{\overline{TM}} = \frac{-6-0}{-3+1} = \frac{-6}{-2} = \frac{3}{1}$

$m_{\overline{CM}} = \frac{-4+6}{-9+3} = \frac{2}{-6} = -\frac{1}{3}$

$\rightarrow \overline{TM} \perp \overline{CM}$

$d_{\overline{TM}} = \sqrt{(-3+1)^2 + (-6-0)^2} = \sqrt{4+36} = \sqrt{40}$
$= 2\sqrt{10}$

$d_{\overline{CM}} = \sqrt{(-9+3)^2 + (-4+6)^2} = \sqrt{36+4} = \sqrt{40}$
$= 2\sqrt{10}$

$m_{\overline{QU}} = \frac{5-3}{1-7} = \frac{2}{-6} = -\frac{1}{3}$

$m_{\overline{EU}} = \frac{9-3}{9-7} = \frac{6}{2} = \frac{3}{1}$

$\rightarrow \overline{QU} \perp \overline{EU}$

$d_{\overline{QU}} = \sqrt{(1-7)^2 + (5-3)^2} = \sqrt{36+4} = 2\sqrt{10}$

$d_{\overline{EU}} = \sqrt{(9-7)^2 + (9-3)^2} = \sqrt{4+36} = 2\sqrt{10}$

Since both $\angle$s are Rt, $\angle$s $\cong$, and
& $\overline{TM} \cong \overline{EU}$ & $\overline{CM} \cong \overline{QU}$; therefore, $\Delta CMT \cong \Delta QUE$ by SAS