Write a flow proof for the following.

Given: F is the midpoint of $\overline{BC}$

$\overline{DB} \cong \overline{EC}$; $\overline{DB} \perp \overline{DF}$; $\overline{EC} \perp \overline{EF}$

Prove: $\overline{AF} \perp \overline{BC}$

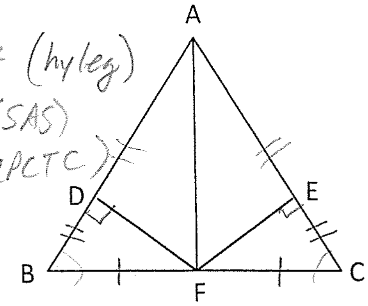
Plan

$\triangle DBF \cong \triangle ECF$ (hy leg)

$\triangle ABF \cong \triangle ACF$ (SAS)

$\angle AFB \cong \angle AFC$ (CPCTC)

$\overline{AF} \perp \overline{BC}$



① $\overline{DB} \perp \overline{DF}$ → ② $\angle BDF$ Rt. → ③ $\triangle BDF$ is Rt.

$\overline{EC} \perp \overline{EF}$ → $\angle CEF$ Rt. → $\triangle CEF$ is Rt.

④ F midpt. $\overline{BC}$ → ⑤ $\overline{BF} \cong \overline{FC}$

⑥ $\overline{DB} \cong \overline{EC}$

→ ⑦ $\triangle BDF \cong \triangle CEF$

⑧ $\angle B \cong \angle C$ → ⑨ $\overline{AB} \cong \overline{AC}$

⑧ $\angle B \cong \angle C$

⑤ $\overline{BF} \cong \overline{FC}$

→ ⑩ $\triangle ABF \cong \triangle ACF$ → ⑪ $\angle AFB \cong \angle AFC$

⑫ $\overline{AF} \perp \overline{BC}$

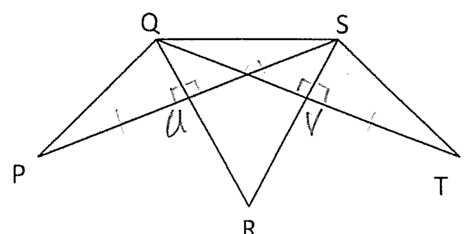
Given: $\overline{SR}$ is the perpendicular bisector of $\overline{QT}$

$\overline{QR}$ is the perpendicular bisector of $\overline{SP}$

Prove $\triangle PQU \cong \triangle SQU$ SAS

$\triangle QSV \cong \triangle TSV$ SAS

Prove: $\overline{PQ} \cong \overline{TS}$



① $\overline{SR}$ ⊥ bisector of $\overline{QT}$ → ② $\overline{QU} \perp \overline{PS}$ → ③ $\angle QUP$ & $\angle QUS$ Rt. $\angle$s → ④ $\angle QUP \cong \angle QUS$

⑤ $\overline{PU} \cong \overline{US}$

⑥ $\overline{QU} \cong \overline{QU}$

→ ⑦ $\triangle PQU \cong \triangle SQU$

⑧ $\overline{QR}$ ⊥ bisector of $\overline{SP}$ → ⑨ $\overline{QR} \perp \overline{SP}$ → ⑩ $\angle QVS$ & $\angle TVS$ Rt $\angle$s → ⑪ $\angle QVS \cong \angle TVS$

⑫ $\overline{SV} \cong \overline{SV}$

⑬ $\overline{QV} \cong \overline{VT}$

→ ⑭ $\triangle QSV \cong \triangle TSV$

⑮ $\overline{PQ} \cong \overline{QS}$

⑯ $\overline{QS} \cong \overline{ST}$

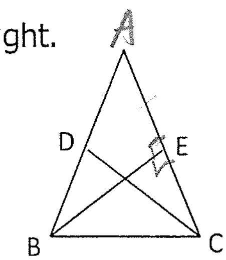
→ ⑰ $\overline{PQ} \cong \overline{TS}$

① Given
② Def of ⊥ bisector
③ Def ⊥ lines
④ All Rt. $\angle$s ≅.
⑤ Def. bisector
⑥ Reflexive Prop
⑦ SAS ≅ SAS
⑧ Given
⑨ Def of ⊥ bisector
⑩ Def ⊥ lines
⑪ All Rt $\angle$s ≅.
⑫ Reflexive Prop
⑬ Def of bisector
⑭ SAS ≅ SAS
⑮ CPCTC
⑯ CPCTC
⑰ Transitive Prop.

Homework

Geometry (H)
Altitude, Median &The Equidistant Theorems – Problems

For example 1 and 2 using the diagram at the right.



1. Supposed that $\overline{BE}$ and $\overline{CD}$ are medians. If AD = 2x + 3, BD = 3x – 2, AE = 4x – 7, explain why AB = AC.

AD = BD
2x+3 = 3x − 2
5 = x

AD = 13
BD = 13

AE = 13

AB = 26.
Since AE = 13, EC also is 13.
AE + EC = 26. Since AB = 26 and AC = 26, AB = AC.

2. Suppose that $\overline{BE}$ is an altitude. m∠AEB = 4x + 10, AD = 2x + 5, BD = 3x – 15, is $\overline{CD}$ a median?

90

m∠AEB = 90
4x + 10 = 90
4x = 80
x = 20

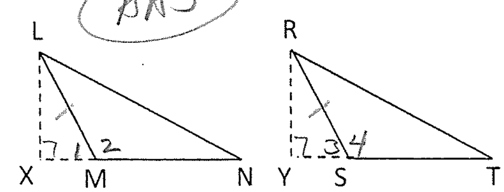
AD = 2(20) + 5 = 45
BD = 3(20) − 15 = 45
Since AD = BD, $\overline{CD}$ is a median.

3. Write a flow proof.

Given: ΔLMN ≅ ΔRST; $\overline{LX}$ and $\overline{RY}$ are altitudes

Prove: $\overline{LX}$ ≅ $\overline{RY}$

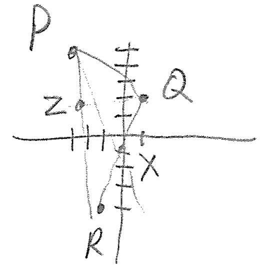
AAS



① ∠1 & ∠2 linear pair → ② ∠1 supp ∠2
∠3 & ∠4 linear pair → ∠3 supp ∠4 → ⑤ ∠1 ≅ ∠3
③ ΔLMN ≅ ΔRST → ④ ∠2 ≅ ∠4
→ ⑥ $\overline{LM}$ ≅ $\overline{RS}$
⑦ $\overline{LX}$ & $\overline{RY}$ altitudes. → ⑦a $\overline{LX} \perp \overline{RY}$ → ⑧ ∠X & ∠Y are Rt ∠s. → ⑨ ∠X ≅ ∠Y
→ ⑩ ΔLXM ≅ ΔRYS → ⑪ $\overline{LX}$ ≅ $\overline{RY}$

① Def. linear pair
② linear pair postulate
③ Given
④ CPCTC
⑤ CPCTC
⑦ Given
⑦a Def. of altitude
⑧ ⊥ lines form Rt ∠s.
⑨ All Rt. ∠s ≅.
⑩ AAS ≅ AAS
⑪ CPCTC

4. In ΔPQR, $P(-3,5)$, $Q(1,2)$ and $R(-1,-4)$



a. Find the equation of the altitude $\overline{QZ}$.

slope of $\overline{PR}$

$m_{RP} = \frac{5+4}{-3+1} = \frac{9}{-2}$

$*m_\perp = \frac{2}{9}$

$Q(1,2)$

y-int

$y = mx+b$

$2 = \frac{2}{9}(1)+b$

$\frac{16}{9} = b$

altitude $\overline{QZ}$

$$\boxed{y = \frac{2}{9}m + \frac{16}{9}}$$

b. Find the equation of the median $\overline{PX}$.

midpoint of $\overline{QR}$

$\left(\frac{1-1}{2}, \frac{2-4}{2}\right)$

$X(0,-1)$

slope of $\overline{PX}$

$m_{\overline{PX}} = \frac{5+1}{-3-0} = \frac{6}{-3} = -2$

y-int

$-1 = (-2)(0)+b$

$-1 = b$

median $\overline{PX}$

$$\boxed{y = -2X - 1}$$

c. Find the equation of the perpendicular bisector of $\overline{PQ}$. (does not necessarily go thru the vertex)

slope of $\overline{PQ}$

$m_{\overline{PQ}} = \frac{5-2}{-3-1} = \frac{3}{-4}$

$*\ m_\perp = \frac{4}{3}$

midpoint of $\overline{PQ}$

$\left(\frac{-3+1}{2}, \frac{5+2}{2}\right)$

$\left(-1, \frac{7}{2}\right)$

y-int

$m = \frac{4}{3}, \left(-1, \frac{7}{2}\right)$

$\frac{7}{2} = \frac{4}{3}(-1)+b$

$\frac{21}{6} + \frac{8}{6} = b$

$\frac{29}{6} = b$

$\perp$ bisector

$$\boxed{y = \frac{4}{3}x + \frac{29}{6}}$$

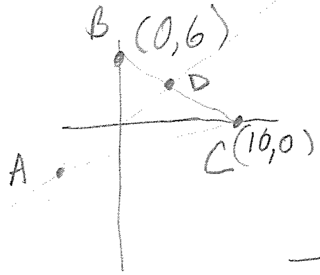
Problem (6) Reasons.

1. Given
2. Def. of $\perp$ bisector
3. Def of $\perp$ bisector
4. $\perp$ lines form Rt. $\angle$s.
5. All Rt. $\angle$s $\cong$.
6. Reflexive Prop.
7. SAS $\cong$ SAS
8. Given
9. If exterior sides form acute adj. $\angle$s are $\perp$ $\rightarrow$ complement. $\angle$s.
10. $\perp$ lines form Rt $\angle$s.
11. Def of Rt. $\triangle$
12. In a Rt $\triangle$, acute $\angle$s complementary.
13. CPCTC
14. $\cong$ complements Thm
15. If $\angle$s $\cong$, then sides opp. $\cong$.
16. CPCTC
17. Transitive Prop.
18. Def of midpoint
19. A median is a seg. that connects a midpt of a segment & the opp. vertex.

lies on $\perp$ bisector

5. Find the coordinates of the point that is equidistant from the points (0,6) and (10,0) and lies on the line $y = x$.

intersects with $y=x$



$M_{\overline{BC}} = \frac{6-0}{0-10} = \frac{3}{-5}$

$* \; M_{\perp} = \frac{5}{3}$

midpt of $\overline{BC}$

$\left(\frac{0+10}{2}, \frac{6+0}{2}\right)$

D(5, 3)

y-int. of $\perp$ bisector

$b = -\frac{16}{3}$

EQ of $\perp$ bisector

$y = \frac{5}{3}x - \frac{16}{3}$

Solve system:

$y = x$
$y = \frac{5}{3}x - \frac{16}{3}$

$x = \frac{5}{3}x - \frac{16}{3}$

$8 = x$

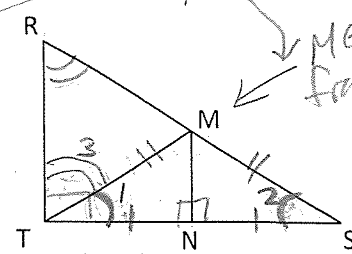
$y = 8$

$\boxed{\text{point is } (8, 8)}$

6. Write a flow proof for each of the following.

Given: $\overline{RT} \perp \overline{TS}$
$\overline{MN}$ is the perpendicular bisector of $\overline{TS}$

Prove: $\overline{TM}$ is a median
(Hint: use parallel lines)



M equidistant from T & S

Plan: ① Prove $\triangle TMN \cong \triangle SMN$ (SAS) or (ASA)
② Show $\angle 2$ comp. $\angle 3$, $\angle 2$ comp. $\angle R$ $\} \rightarrow \angle 3 \cong \angle R$
③ $\overline{TM} \cong \overline{RM} \cong \overline{MS} \rightarrow \overline{TM}$ median

$\angle 3$ comp. $\angle 2$
$\angle R$ comp $\angle 2$

① $\overline{MN}$ $\perp$ bisector of $\overline{TS}$. → ② $\overline{TN} \cong \overline{NS}$

③ $\overline{MN} \perp \overline{TS}$ → ④ $\angle TNM$ & $\angle SNM$ are Rt $\angle$s. → ⑤ $\angle TNM \cong \angle SNM$

⑥ $\overline{MN} \cong \overline{MN}$

→ ⑦ $\triangle TMN \cong \triangle SMN$

⑧ $\overline{RT} \perp \overline{TS}$

→ ⑩ $\angle RTN$ is Rt. → ⑪ $\triangle RTS$ is Rt. →

⑨ $\angle 3$ comp $\angle 1$
⑫ $\angle R$ comp $\angle 2$
⑬ $\angle 1 \cong \angle 2$

→ ⑭ $\angle 3 \cong \angle R$

⑮ $\overline{TM} \cong \overline{RM}$
⑯ $\overline{TM} \cong \overline{MS}$

→ ⑰ $\overline{RM} \cong \overline{MS}$ → ⑱ M is the midpoint $\overline{RS}$

⑲ $\overline{TM}$ is median.