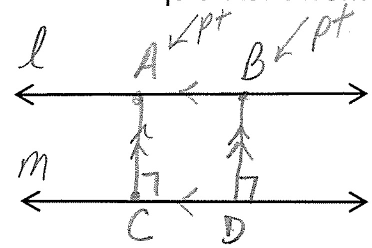
Geometry (H) date_______________

Section ~~5-2~~ 5-3

NOTES

Special Theorems Involving Parallel Lines

(1) Theorem: If 2 lines are parallel, then all points on one line are equidistant from the points on the other line.



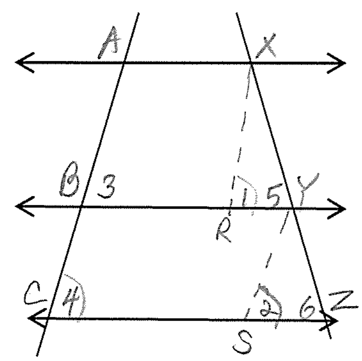
given: l ∥ m, $\overline{AC} \perp m$; $\overline{BD} \perp m$

Prove: AC = BD

① $\overline{AC} \perp m$, $\overline{BD} \perp m$ } → ② $\overline{AC} \parallel \overline{BD}$, ③ $l \parallel m$ } → ④ ABCD is ▱ → ⑤ AC = BD

① Given
② If 2 lines ⊥ to same line → ∥
③ Given
④ Def of ▱
⑤ ▱ → opp sides ≅.

(2) Theorem: If 3 parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.



Given: $\overleftrightarrow{AX} \parallel \overleftrightarrow{BY} \parallel \overleftrightarrow{CZ}$

$\overline{AB} \cong \overline{BC}$

Prove: $\overline{XY} \cong \overline{YZ}$

(I) Draw $\overline{XR}$ ∥ to $\overline{AB}$. Draw $\overline{YS}$ ∥ to $\overline{BC}$.
(II) Show ▱ABRX, ▱BCSY.
(III) Show $\overline{XR} \cong \overline{YS}$ and ∠1 ≅ ∠2.
(IV) Show △XRY ≅ △YSZ by AAS.
(V) By CPCTC, $\overline{XY} \cong \overline{YZ}$.

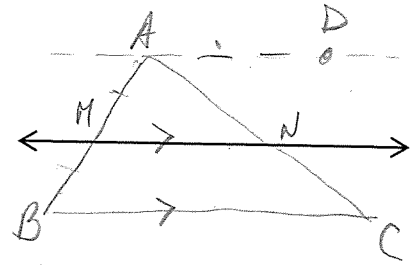
① Draw $\overline{XR} \parallel \overline{AB}$ → ② ∠1 ≅ ∠3 } → ⑤ ∠1 ≅ ∠4
③ $\overline{BY} \parallel \overline{CZ}$ → ④ ∠3 ≅ ∠4 }
⑥ Draw $\overline{YS} \parallel \overline{BC}$ → ⑦ ∠2 ≅ ∠4
⑤, ⑦ → ⑧ ∠1 ≅ ∠2
⑨ $\overline{BY} \parallel \overline{CZ}$ → ⑩ ∠5 ≅ ∠6
⑪ $\overline{AX} \parallel \overline{BR}$, ⑫ $\overline{AB} \parallel \overline{XR}$ } → ⑬ ▱ABRX → ⑭ $\overline{AB} \cong \overline{XR}$
⑮ $\overline{AB} \cong \overline{BC}$ (given)
⑭, ⑮ → ⑯ $\overline{XR} \cong \overline{BC}$
⑰ $\overline{BY} \parallel \overline{CS}$, ⑱ $\overline{BC} \parallel \overline{YS}$ } → ⑲ ▱BCSY → ⑳ $\overline{BC} \cong \overline{YS}$
⑯, ⑳ → ㉑ $\overline{XR} \cong \overline{YS}$
⑧, ⑩, ㉑ → ㉒ △XRY ≅ △YSZ (AAS)
㉓ $\overline{XY} \cong \overline{YZ}$

① Thru a pt outside a line, there is exactly 1 line ∥ to given line.

Reasons to proof for theorem #2

1. Thru a pt outside a line, there is exactly 1 line ∥ to given line.
2. 2 ∥ lines → corresp. ∠s ≅.
3. Given
4. 2 ∥ lines → corresp. ∠s ≅.
5. Transitive prop.
6. same as #1
7. 2 ∥ lines → corresp. ∠s ≅.
8. Transitive prop.
9. Given
10. 2 ∥ lines → corresp. ∠s ≅
11. Given
12. See #1
13. Opp sides ∥ → ▱
14. ▱ → opp sides ≅.
15. Given
16. Transitive prop.
17. Given
18. See #1
19. Both prs. opp sides ∥ → ▱
20. ▱ → opp sides ≅.
21. Transitive prop.
22. AAS ≅ AAS
23. CPCTC

(3) Theorem: A line that contains the midpoint of one side of a triangle and is parallel to another side passes through the midpoint of the third side.



Given: M is the midpoint of $\overline{AB}$

$\overleftrightarrow{MN} \parallel \overline{BC}$

Prove: N is the midpoint of $\overline{AC}$

* Use previous thm to prove this. See 3 ∥ lines.

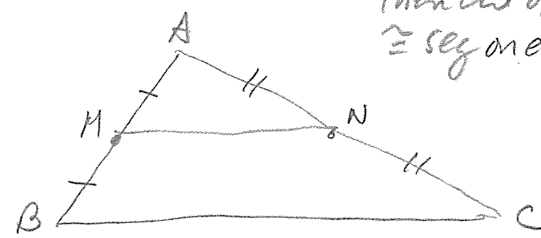
① Draw $\overline{AD} \parallel \overline{MN}$. ② $\overline{MN} \parallel \overline{BC}$ → ③ $\overline{AD} \parallel \overline{BC}$

④ M midpt $\overline{AB}$ → ⑤ $\overline{AM} \cong \overline{MB}$, ⑥ $\overline{AD} \parallel \overline{MN}$, ⑦ $\overline{MN} \parallel \overline{BC}$

→ ⑧ $\overline{AN} \cong \overline{NC}$ → ⑨ N midpt of $\overline{AC}$.

① Thru a pt outside a line, there is exactly 1 line ∥ to give line.
② Given
③ If 2 lines ∥ same line → ∥ each other.
④ Given
⑤ Def of midpt.
⑥ See #1
⑦ Given
⑧ If 3 ∥ lines cut off ≅ seg on 1 trans, then cut off ≅ seg on every trans.
⑨ Def of midpt.

(4) Theorem: The segment that joins the midpoints of two sides of a triangle
(1) is parallel to the third side;
(2) is half as long as the third side.



Given: M is the midpoint of $\overline{AB}$

~~$\overleftrightarrow{MN} \parallel \overline{BC}$~~ N midpt $\overline{AC}$

Prove: ~~N is the midpoint of $\overline{AC}$~~

Part ① $\overline{MN} \parallel \overline{BC}$

② $MN = \frac{1}{2} BC$

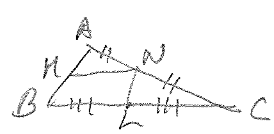
Part #1

① M midpt $\overline{AB}$ → ② $\overline{AM} \cong \overline{MB}$

N midpt $\overline{AC}$ → $\overline{AN} \cong \overline{NC}$ → ③ $\overline{MN} \parallel \overline{BC}$

① Given
② Def of midpt
③ A line that contains midpt of 1 side of △ and is ∥ to another side passes thru midpt of 3rd side.

Part #2



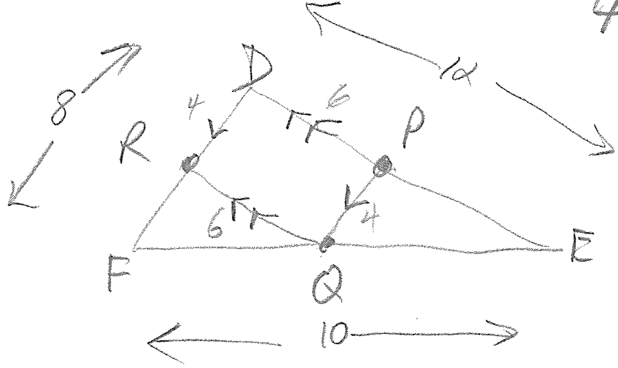
① L is midpt of $\overline{BC}$. Draw $\overline{NL}$ → ② $\overline{NL} \parallel \overline{AB}$, ③ $\overline{MN} \parallel \overline{BC}$ → ④ ▱BMNL → ⑤ $\overline{MN} \cong \overline{BL}$

→ ⑥ $BL = \frac{1}{2} BC$ → ⑦ $MN = \frac{1}{2} BC$

① A segmt has one & only 1 midpt.
② Segmt that joins midpts of 2 sides of △ is ∥ to 3rd side.
③ Given (proven in part 1)
④ Def of ▱
⑤ ▱ → opp sides ≅.
⑥ Midpt theorem
⑦ Substitution.

Explore: P, Q, and R are midpoints of the sides of ΔDEF, respectively.

a. What kind of figure is DPQR? ▱ DPQR

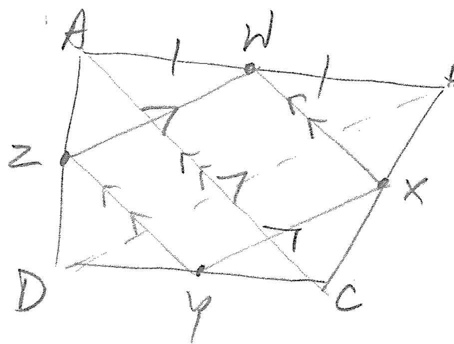


b) What is perimeter of DPQR?

20 units.

Explore: W, X, Y, and Z are the midpoints of quad ABCD.

What kind of figure is the smaller quadrilateral formed by joining WXYZ?



a parallelogram

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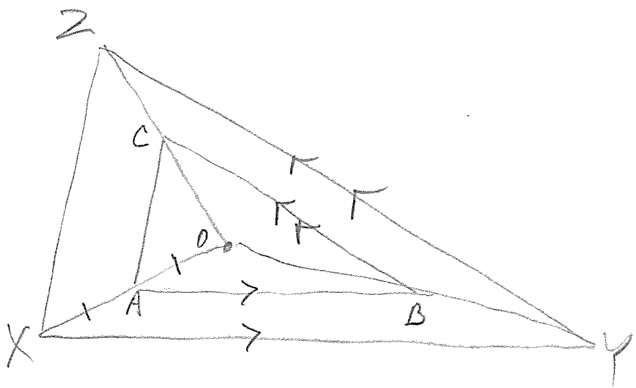
Given: A is the midpoint of $\overline{OX}$;
$\overline{AB} \parallel \overline{XY}$; $\overline{BC} \parallel \overline{YZ}$

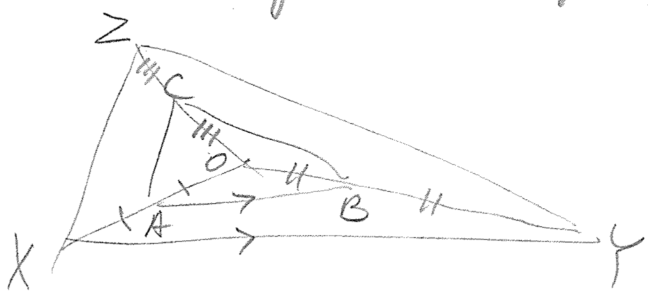
Prove: $\overline{AC} \parallel \overline{XZ}$



Notes:

B midpt of $\overline{OY}$ (line that has midpt of 1 side of △ & ∥ to 2nd side goes thru midpt of 3rd side)

C midpt of $\overline{OZ}$



$\Rightarrow \overline{AC} \parallel \overline{XZ}$ (segmt that joins 2 midpts of 2 sides of △ is ∥ to 3rd side)

① A midpt of $\overline{OX}$, $\overline{AB} \parallel \overline{XY}$ → ② B midpt of $\overline{OY}$, ③ $\overline{BC} \parallel \overline{YZ}$ → ④ C midpt of $\overline{OZ}$, ⑤ A midpt of $\overline{OX}$ → ⑥ $\overline{AC} \parallel \overline{XZ}$