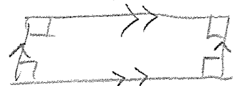
Geometry (H)
Section 5.4 – Special Parallelograms

Name: KEY

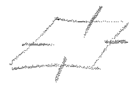
Today we will be studying the properties of special parallelogram.

Define each of the following and draw a diagram, mark any parts that are congruent.

Rectangle – quadrilateral with 4 right ∡s.



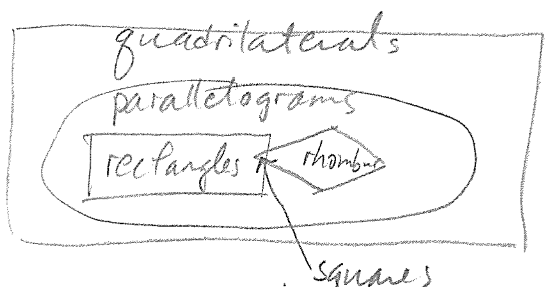
Rhombus – quad. with 4 ≅ sides.



Square – quad w/ 4 ≅ sides & ≅ ∡s

How is it a ▱?
- opp sides ≅
- opp ∡s ≅
- *- opp sides ∥ (definition)
- 1 pr. sides ≅ & ∥
- diagonals bisect each other

A Venn diagram can be drawn to show the relationship between all the quadrilaterals we've studied so far.



- special quads can go into room of parallelograms
- only special ▱ can go into room of rect.

etc.

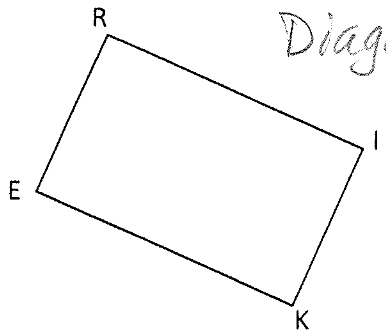
Use the Venn diagram to determine if the statement is sometimes, always or never true.

1. A square is a parallelogram. A
2. A rectangle is a square. S
3. A square is a rectangle. A
4. A parallelogram is a rhombus. S
5. A square is a rhombus. A
6. A rhombus has opposite angles congruent. A
7. A rectangle is a rhombus. S

Because a rhombus, rectangle and square are parallelograms they take on the properties of a parallelogram. However, the rectangle and rhombus have some additional properties relating to their diagonals.

Look at the rectangle below. Draw in the diagonals and measure them. What seems to be true out them?

RK = 4.3 cm
EI = 4.3 cm



Diagonals have equal measures → $\cong$.

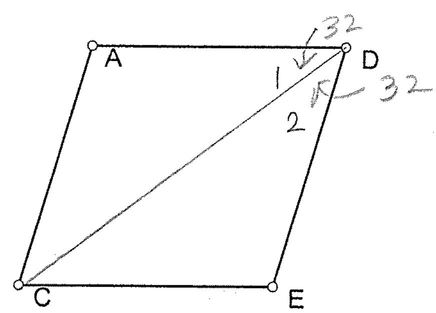
Look at the rhombus below. Draw in the diagonals. Are they congruent? NO
Measure one angle at the intersection of the diagonals. What seems to be true?

measure = 90°

Now measure the angles formed at D and C. What do you observe with their measurements?

$m\angle D = m\angle C$
$m\angle 1 = m\angle 2$

CD = 5.4 CM
AE = 4 CM



Note: A square is a rectangle and rhombus so it has all the properties of both.

Let summarize our findings:

Properties of a Rectangle:

Thm: diagonals are $\cong$.

Properties of a Rhombus:

Thm: diagonals are $\perp$
Thm: diagonals bisect their $\angle$s

Properties of a Square:

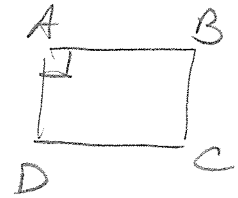
has properties of
- rectangle
- rhombus
- square

All of the above properties can be proven. Let's look at 2 additional theorems.

Thm: If an angle of a parallelogram is a right angle, then the parallelogram is a rectangle.

Given: ▱ ABCD, ∡A is right

Prove: ABCD is right. (Prove the definition: 4 right ∡s)

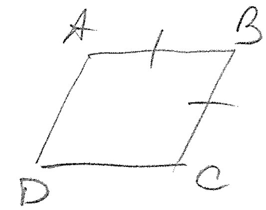
Diagram: 

Proof on next page ⟶

Thm: If two consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus.

Given: ▱ ABCD, $\overline{AB} \cong \overline{BC}$

Prove: ABCD is a rhombus (show 4 sides ≅)

Diagram: 

① ▱ ABCD → ② $\overline{AB} \cong \overline{DC}$, ③ $\overline{AB} \cong \overline{BC}$ → ④ $\overline{DC} \cong \overline{BC}$

① ▱ ABCD → ⑤ $\overline{AD} \cong \overline{BC}$, ⑥ $\overline{AB} \cong \overline{BC}$ → ⑦ $\overline{AD} \cong \overline{AB}$

⑧ $\overline{AB} \cong \overline{BC}$

④, ⑦, ⑧ → ⑨ $\overline{AB} \cong \overline{BC} \cong \overline{DC} \cong \overline{AD}$ → ⑩ ABCD is a rhombus.

① Given
② ▱ → opp sides ≅
③ Given
④ Transitive prop.
⑤ ▱ → opp sides ≅
⑥ Given
⑦ Transitive prop
⑧ Given
⑨ Transitive property
⑩ Definition of a rhombus

Thm #1   This proof uses a theorem we're rarely used. We can always benefit from seeing different ways

(1) $\boxed{p}ABCD \rightarrow$ (2) $\angle A \cong \angle C \rightarrow$ (3) $m\angle A = m\angle C$

(4) $\angle A$ is right. $\rightarrow$ (5) $m\angle A = 90$

$\} \rightarrow$ (6) $m\angle C = 90 \rightarrow$ (7) $\angle C$ is Rt.

(8) $\angle A$ is Rt. $\rightarrow$ (9) $\overline{DA} \perp \overline{BA}$

(10) $\overline{AB} \parallel \overline{DC}$

$\} \rightarrow$ * (11) $\overline{AD} \perp \overline{DC} \rightarrow$ (12) $\angle D$ is Rt $\angle$ $\rightarrow$ (13) $m\angle D = 90$

(14) $\angle D \cong \angle B \rightarrow$ (15) $m\angle D = m\angle B$

$\} \rightarrow$ (16) $m\angle B = 90 \rightarrow$ (17) $\angle B$ Rt.

(18) $\angle A$ Rt.

$\} \rightarrow$ (19) ABCD is rectangle

Reasons

(1) Given

(2) $\boxed{p} \rightarrow$ opp $\angle$s $\cong$.

(3) Def of $\cong$ $\angle$s

(4) Given

(5) Def of Rt. $\angle$

(6) Substitution

(7) Def of Rt $\angle$

(8) Given

(9) Def of $\perp$ lines

(10) $\boxed{p} \rightarrow$ opp sides $\parallel$

*(11) If a transversal is $\perp$ to one of 2 $\parallel$ lines, then it is $\perp$ to other also. (Thm 3-4)

(12) $\perp$ lines form Rt $\angle$s.

(13) Def of Rt $\angle$.

(14) $\boxed{p} \rightarrow$ opp $\angle$s $\cong$

(15) Def of $\cong$ $\angle$s

(16) Substitution

(17) Def of Rt $\angle$

(18) Given

(19) Def. of rectangle