Review: Properties of Proportions

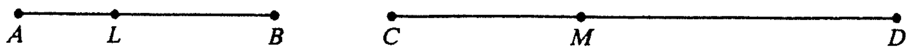
① $\frac{a}{b} = \frac{c}{d} \rightarrow$ a) $ad = bc \rightarrow$ b) $\frac{a}{c} = \frac{b}{d} \rightarrow$ c) $\frac{b}{a} = \frac{d}{c} \rightarrow$ d) $\frac{a+b}{b} = \frac{c+d}{d}$

Geometry (H)

Section 7.6 – Proportional Lengths

Name: NOTES

Point L and M lie on $\overline{AB}$ and $\overline{CD}$, respectively. If $\frac{AL}{LB} = \frac{CM}{MD}$, we say that $\overline{AB}$ and $\overline{CD}$ are divided proportionally.

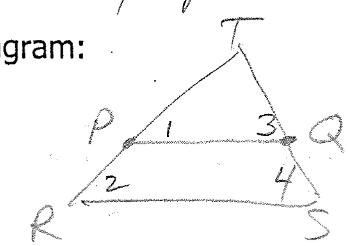


Triangle Proportionality Theorem:

If a line ∥ to one side of a Δ intersects the other 2 sides, then it divides those sides proportionally.

Given: $\triangle RST$; $\overleftrightarrow{PQ} \parallel \overline{RS}$

Diagram:



Prove: $\frac{RP}{PT} = \frac{SQ}{QT}$

① $\overline{PQ} \parallel \overline{RS} \rightarrow$ ② $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$ $\rightarrow$ ③ $\triangle TPQ \sim \triangle TRS$

$\rightarrow$ ④ $\frac{RT}{PT} = \frac{TS}{TQ}$

⑤ $RT = RP + PT$, $ST = SQ + QT$

$\rightarrow$ ⑥ $\frac{RP+PT}{PT} = \frac{SQ+QT}{QT}$

⑦ $\frac{RP}{PT} = \frac{SQ}{QT}$

① Given

② 2 ∥ lines → corresp. ∠s ≅.

③ AA ~ AA

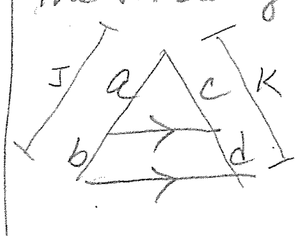
④ ~Δs → corresp. parts proportional.

⑤ Segment addition postulate

⑥ Substitution

⑦ Property (1d) of proportions.

Use the T.P.T. to justify the following:



$\frac{a}{J} = \frac{c}{K}$ $\quad$ $\frac{a}{c} = \frac{J}{K}$

$\frac{b}{J} = \frac{d}{K}$ $\quad$ $\frac{a}{b} = \frac{c}{d}$

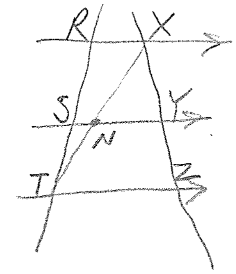
$\frac{a}{c} = \frac{b}{d}$ $\quad$ $\frac{b}{d} = \frac{J}{K}$

Ch5 Thm: If 3 ∥ lines cut off ≅ seg. on one trans., then they cut off ≅ seg. on every trans.

Corollary: If three parallel lines intersect two transversals, then they divide the transversals proportionally.

Given: $\overleftrightarrow{RX} \parallel \overleftrightarrow{SY} \parallel \overleftrightarrow{TZ}$

Diagram:



Prove: $\frac{RS}{ST} = \frac{XY}{YZ}$

① Draw $\overline{XT}$
② $\overline{SY} \parallel \overline{RX}$
→ ③ $\frac{RS}{ST} = \frac{XN}{NT}$

④ $\overline{XT}$
⑤ $\overline{SY} \parallel \overline{TZ}$
→ ⑥ $\frac{XY}{YZ} = \frac{XN}{NT}$

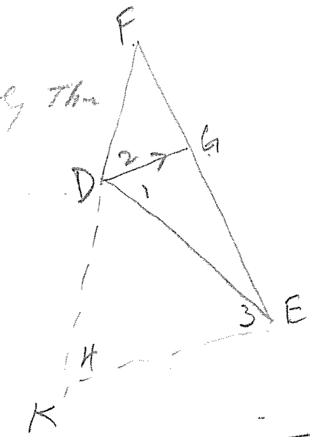
→ ⑦ $\frac{RS}{ST} = \frac{XY}{YZ}$

① 2 pts deter. a line
② Given
③ △ Proportionality Thm
④ 2 pts deter a line.
⑤ Given
⑥ △ Prop. Thm.
⑦ Transitive Prop of Equality

<u>Triangle Angle Bisector Theorem</u>:
If a ray bisects an angle of a △, then it divides the opp side into segments proportional to the other 2 sides.

Given: $\triangle DEF$; $\overrightarrow{DG}$ bisects $\angle FDE$

Diagram:



Prove: $\frac{FG}{GE} = \frac{FD}{DE}$

Triangle Proportionality Thm

Hint: ① Draw $\overline{EK} \parallel \overline{DG}$.
② Extend FD to meet at K.
→ ③ $\frac{FG}{GE} = \frac{FD}{DK}$

Prove $\triangle DEK$ is isos.

④ $\angle 2 \cong \angle 4$
⑤ $\overrightarrow{DG}$ bis. $\angle FDE$ → ⑥ $\angle 2 \cong \angle 1$
→ ⑦ $\angle 4 \cong \angle 1$
⑧ $\angle 1 \cong \angle 3$
→ ⑨ $\angle 4 \cong \angle 3$ → ⑩ $\overline{DK} \cong \overline{DE}$ → ⑪ $DK = DE$

⑫ $\frac{FG}{GE} = \frac{FD}{DE}$

① Thru a pt outside a line, there is exactly one line ∥ to given line.
② 2 pts determine a line.
③ △ Proportionality Thm
④ 2 ∥ lines → Corresp ∠s ≅.
⑤ Given
⑥ Def of ∠ bisector
⑦ Transitive Prop.
⑧ 2 ∥ lines → alt. int. ∠s ≅.
⑨ Transitive
⑩ In a △, if 2 ∠s ≅, sides opp ≅.
⑪ def of ≅ seg.
⑫ Substitution