

Sequences/Compositions of Transformations

Combining Rigid Transformations

Ex: A GLIDE REFLECTION!

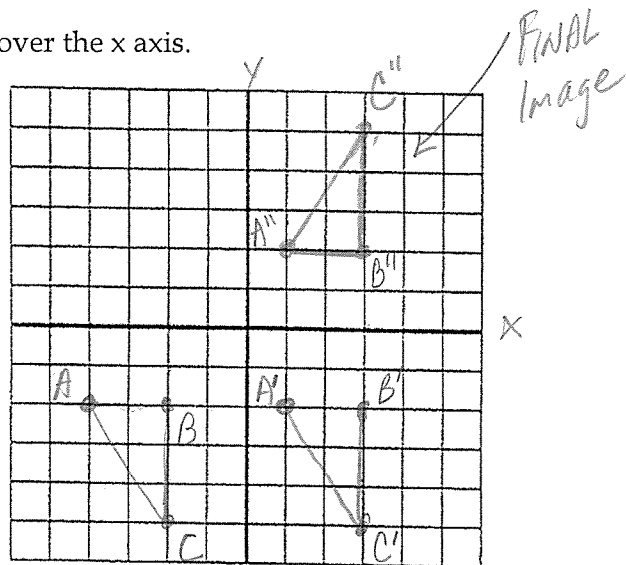
A glide reflection is a transformation in which every point P is mapped onto a point P'' by the following steps:

Is a rigid motion/isometry.

- a) *perform a translation*
 - b) *carry out a reflection*
- } order can be reversed.*

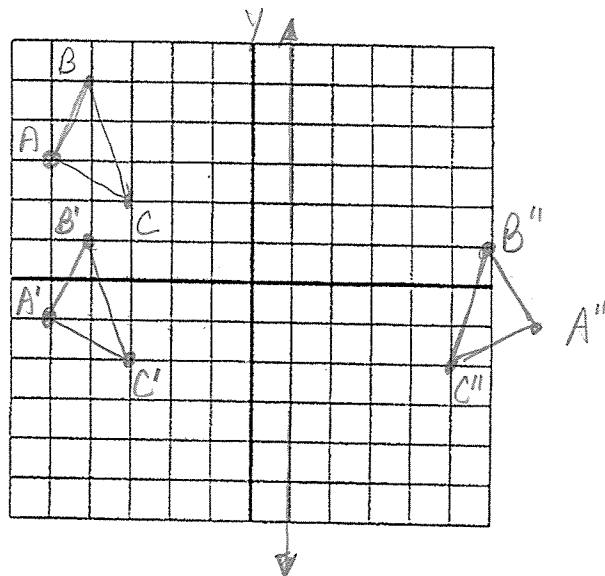
1. Translate $\triangle ABC$ by the vector $\langle 5, 0 \rangle$ and then reflect over the x axis.

$$\begin{aligned} A(-4, -2) &\rightarrow A'(1, -2) \\ B(-2, -2) &\rightarrow B'(3, -2) \\ C(-2, -5) &\rightarrow C'(3, -5) \end{aligned}$$



2. Translate $\triangle ABC$ by the vector $\langle 0, -4 \rangle$ and then reflect over $x=1$.

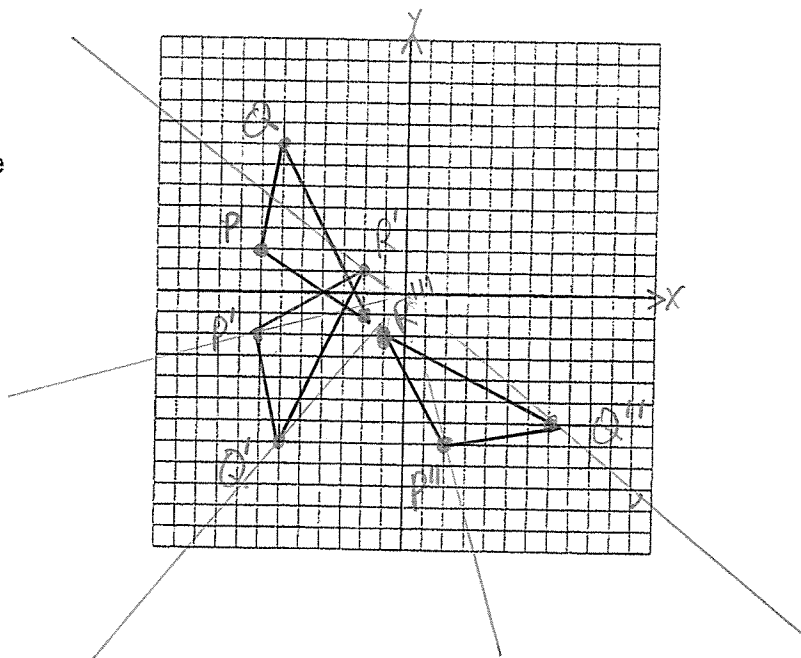
$$\begin{aligned} A(-5, 3) &\rightarrow A'(-5, -1) \\ B(-4, 5) &\rightarrow B'(-4, 1) \\ C(-3, 2) &\rightarrow C'(-3, -2) \end{aligned}$$



Now, let's take a look at drawing other compositions.

3. $P(-7, 2)$, $Q(-6, 7)$, $R(-2, -1)$

P' Reflect: x-axis
 P'' Rotate: 90° counterclockwise
about the origin



Non-Rigid Transformations

Horizontal stretch/shrink: When the x value get multiplied by a factor. $(x, y) \rightarrow (ax, y)$

Stretch if $a > 1$

Shrink if $0 < a < 1$ (fraction)

Changes shape and size!

Vertical stretch/shrink : When the y value get multiplied by a factor. $(x, y) \rightarrow (x, ay)$

Stretch if $a > 1$

Shrink if $0 < a < 1$ (fraction)

Changes shape and size!

Dilation: When both x and y values are multiplied by the same factor. $(x, y) \rightarrow (ax, ay)$

Enlargement if $a > 1$

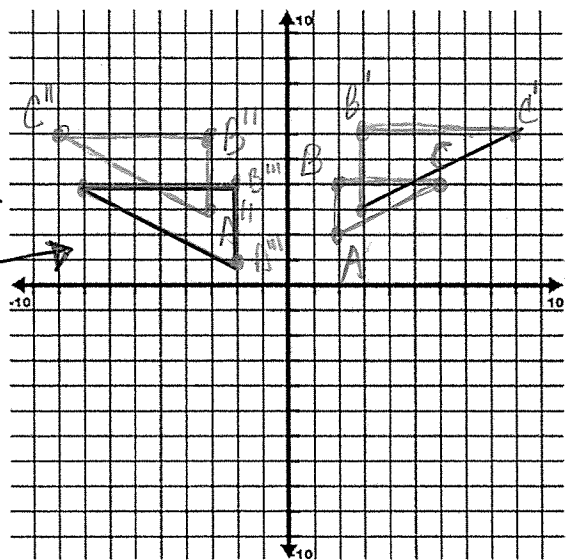
Reduction if $0 < a < 1$ (fraction)

Changes size but not shape!

4. Draw the image of the figure in the plane after the given combination of transformations and describe the sequence.

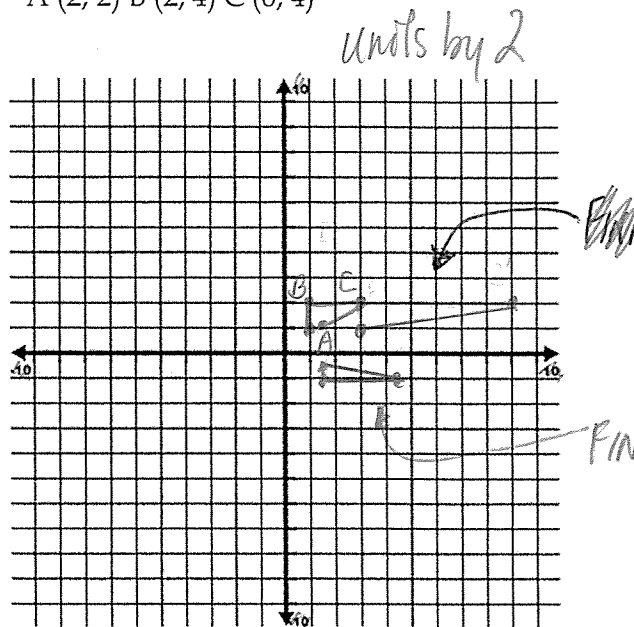
a) $(x, y) \xrightarrow{A'} \left(\frac{3}{2}x, \frac{3}{2}y\right) \xrightarrow{A''} (-x, y) \xrightarrow{A'''} (x + 1, y - 2)$

A (2, 2) B (2, 4) C (6, 4)



b) $(x, y) \xrightarrow{A'} (3x, y) \xrightarrow{A''} \left(\frac{1}{2}x, -\frac{1}{2}y\right)$

A (2, 2) B (2, 4) C (6, 4)



$A'(3, 3) \rightarrow A''(-3, 3) \rightarrow A'''(-2, 1)$
 $B'(3, 6) \rightarrow B''(-3, 6) \rightarrow B'''(-2, 4)$
 $C'(9, 6) \rightarrow C''(-9, 6) \rightarrow C'''(-8, 4)$

$A'(6, 2) \rightarrow A''(3, 7)$
 $B'(6, 4) \rightarrow B''(3, 2)$
 $C'(18, 4) \rightarrow C''(9, 2)$

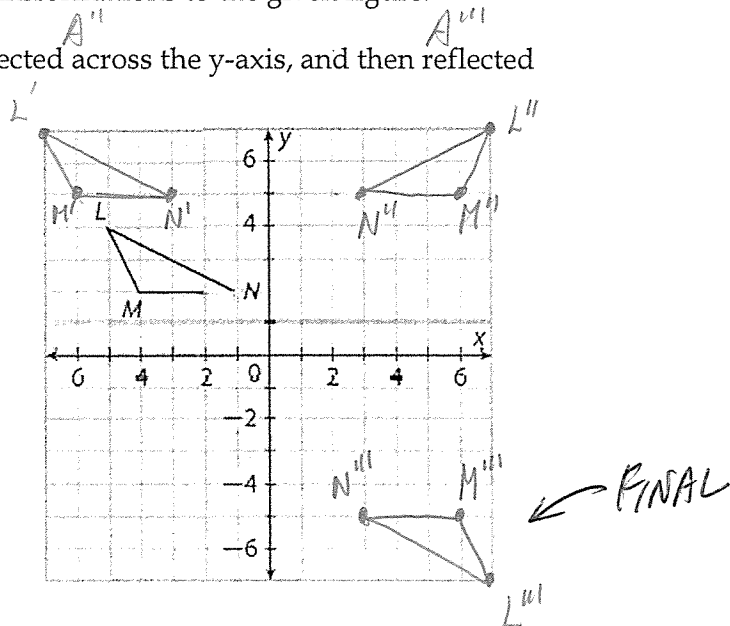
Dilation, reflection across y-axis, translation. horizontal stretch,

5. Predict the result of applying the sequence of transformations to the given figure.

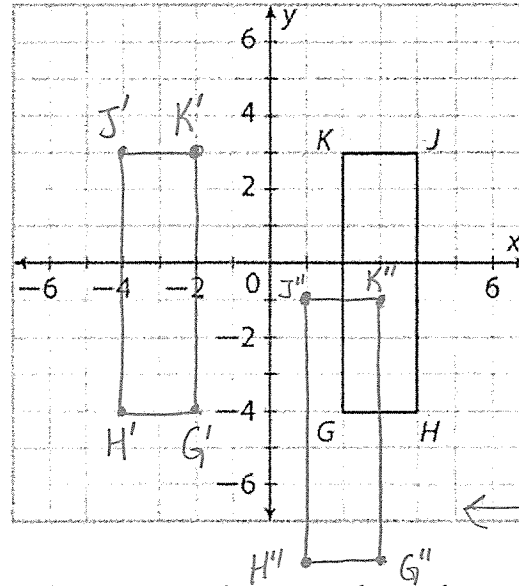
a) $\triangle LMN$ is translated along the vector $\langle -2, 3 \rangle$, reflected across the y-axis, and then reflected across the x-axis.

$L(-5, 4) \rightarrow L'(-7, 7) \rightarrow L''(7, 7)$
 $M(-4, 2) \rightarrow M'(-6, 5) \rightarrow M''(6, 5)$
 $N(-1, 2) \rightarrow N'(-3, 5) \rightarrow N''(3, 5)$

$\rightarrow L'''(7, -7)$
 $\rightarrow M'''(6, -5)$
 $\rightarrow N'''(3, -5)$



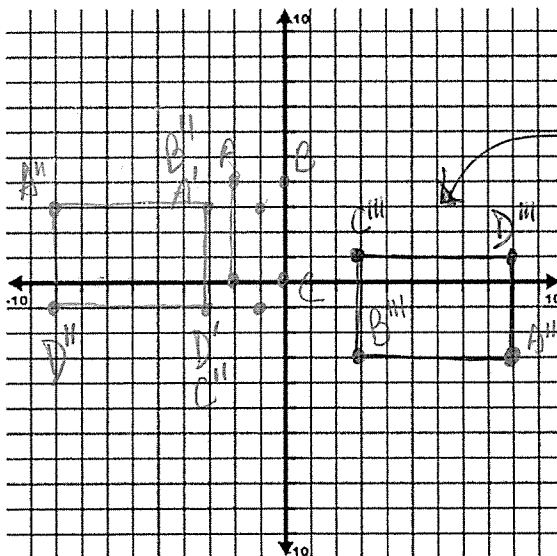
b) Rectangle GHJK is reflected across the y-axis, and translated along Vector $\langle 5, -4 \rangle$.



What figure

Try: Draw the image of the figure in the plane after the given combination of transformations and describe the sequence.

$$(x, y) \xrightarrow{A'} (x - 1, y - 1) \xrightarrow{A''} (3x, y) \xrightarrow{A'''} (-x, -y)$$



$$\begin{aligned} A(-2, 4) &\rightarrow A'(-3, 3) \rightarrow A''(-9, 3) \rightarrow A'''(9, -3) \\ B(0, 4) &\rightarrow B'(-1, 3) \rightarrow B''(-3, 3) \rightarrow B'''(3, -3) \\ C(0, 0) &\rightarrow C'(-1, -1) \rightarrow C''(-3, -1) \rightarrow C'''(3, 1) \\ D(-2, 0) &\rightarrow D'(-3, -1) \rightarrow D''(-9, -1) \rightarrow D'''(9, 1) \end{aligned}$$