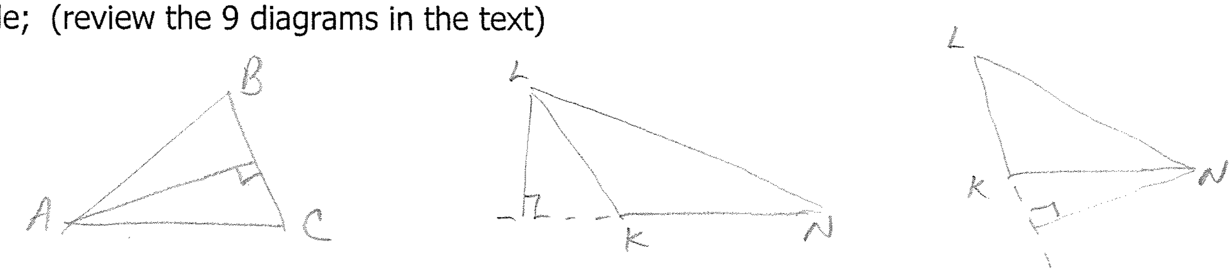
KEY for Notes

Geometry (H)

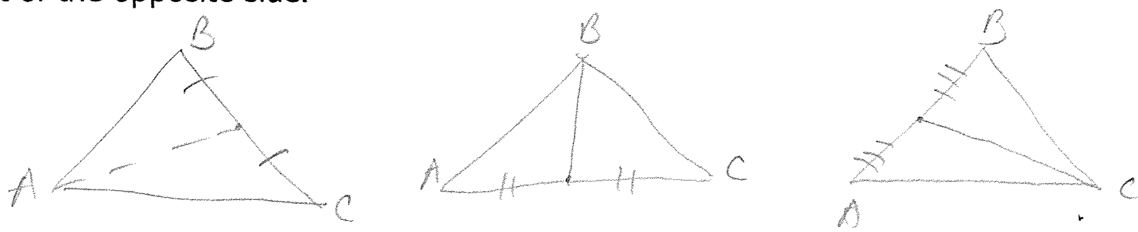
Section 4.7 – Altitudes, Medians, and the Equidistance Theorems

Definitions:

Altitude – in a triangle, it is the segment from a vertex perpendicular to the opposite side; (review the 9 diagrams in the text)

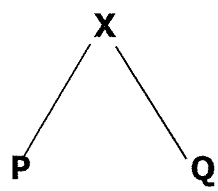


Median – a segment associated with triangles; a segment that connects a vertex to the midpoint of the opposite side.

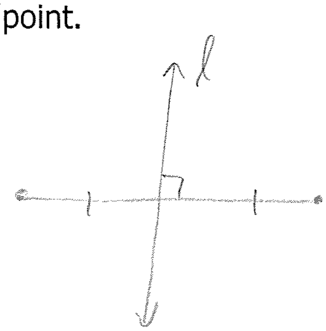


The **distance** between two objects is the length of the shortest path joining them. (Postulate: The shortest path between two points is the segment joining them.)

If two points P and Q are the same distance from a third point X, then X is said to be **equidistant** from P and Q.



Perpendicular Bisector of a segment – a line, ray, or segment that is perpendicular to the segment at its midpoint.

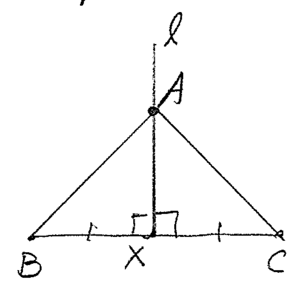


Theorems:

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of that segment.

Given: Line ℓ is the $\perp$ bisector of $\overline{BC}$
A is on ℓ.

Diagram:



Prove: $AB = AC$

① Line ℓ is the $\perp$ bisector of $\overline{BC}$. → ② $\angle AXC$ & $\angle AXB$ are right $\angle$s. → ③ $\angle AXC \cong \angle AXB$

④ $\overline{BX} \cong \overline{XC}$

⑤ $\overline{AX} \cong \overline{AX}$

→ ⑥ $\triangle ABX \cong \triangle ACX$

⑦ $\overline{AB} \cong \overline{AC}$ → ⑧ $AB = AC$

① Given
② $\perp$ lines form Rt $\angle$s
③ All Rt. $\angle$s $\cong$.
④ Def. of seg. bisector
⑤ Reflexive prop
⑥ SAS $\cong$ SAS
⑦ CPCTC
⑧ If segmts $\cong$, their measures are =.

AC = AO

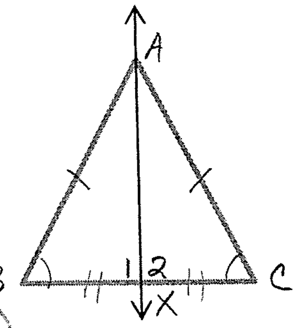
If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment.

bisector of vertex ∡ of isos △ is ⊥ to base at midpt

Given: AB = AC

Prove: A is on the perpendicular bisector of $\overline{BC}$.

Diagram:



(Draw $\overline{AX}$ such that it goes thru midpt of BC)
(Postulate: A segment has one & only 1 midpt)

Part I

① Draw $\overline{AX}$ so that it goes thru midpt of $\overline{BC}$ → ② $\overline{BX} \cong \overline{XC}$

③ $AB = AC$ → ④ $\overline{AB} \cong \overline{AC}$ → ⑤ $\angle B \cong \angle C$

→ ⑥ $\overline{AB} \cong \overline{AC}$

→ ⑦ $\triangle ABX \cong \triangle ACX$

⑧ $\angle 1 \cong \angle 2$ → ⑨ $\overline{AX} \perp \overline{BC}$ → ⑩ A is on the ⊥ bisector of $\overline{BC}$

Reasons

① A segmt has one & only 1 midpt.
② Def midpoint
③ Given
④ Def ≅ segments.
⑤ If 2 sides ≅, ∡s opp ≅
⑥ Def ≅ segmts
⑦ SAS ≅ SAS
⑧ CPCTC
⑨ If 2 lines form ≅ adj ∡s → ⊥ lines.
⑩ Def. of ⊥ bisector.

Part II: ① Draw $\overline{AX} \perp \overline{BC}$

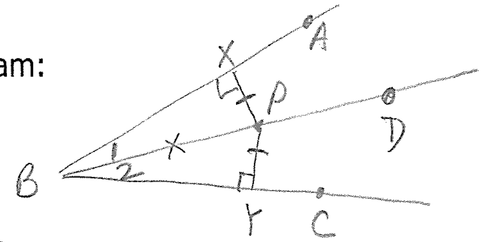
Show that $\overline{BX} \cong \overline{XC}$

If a point is equidistant from the sides of an angle, then it is on the angle bisector.

Given: $\overline{PX} \perp \overrightarrow{BA}$; $\overline{PY} \perp \overrightarrow{BC}$

$PX = PY$

Diagram:



hyleg

Prove: $\overrightarrow{BD}$ bisects $\angle ABC$

① $\overline{PX} \perp \overrightarrow{BA}$ → ② $\angle BXP$ is Rt → ③ $\triangle BXP$ Rt. $\triangle$

$\overline{PY} \perp \overrightarrow{BC}$ → $\angle BYP$ is Rt. → $\triangle BYP$ Rt. $\triangle$

④ $PX = PY$ → ⑤ $\overline{PX} \cong \overline{PY}$

⑥ $\overline{BP} \cong \overline{BP}$

→ ⑦ $\triangle BXP \cong \triangle BYP$ → ⑧ $\angle 1 \cong \angle 2$ → ⑨ $\overrightarrow{BP}$ bisects $\angle ABC$

① Given
② $\perp$ lines form Rt $\angle$s
③ $\triangle$ Rt. Definition
④ Given
⑤ Def $\cong$ segments
⑥ Reflexive Prop.
⑦ hyleg $\cong$ hyleg
⑧ CPCTC
⑨ Def of $\angle$ bisector

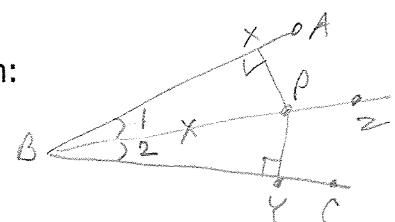
If a point is on an angle bisector, then it is equidistant from the sides of the angle.

Given: $\overrightarrow{BZ}$ bisects $\angle ABC$

P lies on $\overrightarrow{BZ}$

$\overline{PX} \perp \overrightarrow{BA}$; $\overline{PY} \perp \overrightarrow{BC}$

Diagram:



AAS

Prove: $PX = PY$

① $\overline{PX} \perp \overrightarrow{BA}$ → ② $\angle BXP$ Rt $\angle$

$\overline{PY} \perp \overrightarrow{BC}$ → $\angle BYP$ Rt $\angle$

→ ③ $\angle BXP \cong \angle BYP$

④ $\overrightarrow{BZ}$ bisects $\angle ABC$ → ⑤ $\angle 1 \cong \angle 2$

⑥ $\overline{BP} \cong \overline{BP}$

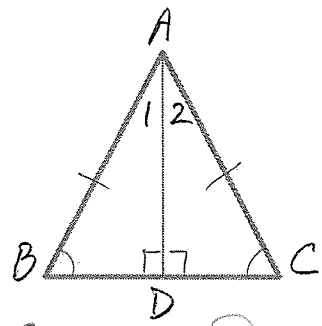
→ ⑦ $\triangle BXP \cong \triangle BYP$ → ⑧ $\overline{PX} \cong \overline{PY}$ → ⑨ $PX = PY$

① Given
② $\perp$ lines for Rt $\angle$s
③ All Rt $\angle$s $\cong$
④ Given
⑤ Def. $\angle$ bisector
⑥ Reflexive Prop.
⑦ AAS $\cong$ AAS
⑧ CPCTC
⑨ Def $\cong$ segments

In an isosceles triangle, the altitude to the base is the median to the base and bisects the vertex angle.

Given: Isosceles $\triangle ABC$ with vertex A and $\overline{AD}$ altitude from A.

Diagram:



Prove: $\overline{BD} \cong \overline{CD}$, $\angle 1 \cong \angle 2$

(1) Isosceles $\triangle ABC$ $\rightarrow$ (2) $\overline{AB} \cong \overline{AC}$ $\rightarrow$ (3) $\angle B \cong \angle C$

(2) $\overline{AB} \cong \overline{AC}$

(4) $\overline{AD}$ altitude from A $\rightarrow$ (5) $\angle ADB$ is Rt, $\angle ADC$ is Rt $\rightarrow$ (6) $\angle ADB \cong \angle ADC$

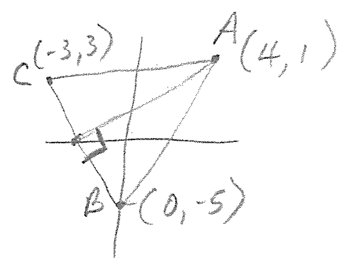
$\rightarrow$ (7) $\triangle ABD \cong \triangle ACD$ (AAS)

(8) $\overline{BD} \cong \overline{CD}$

(9) $\angle 1 \cong \angle 2$

(1) Given
(2) If $\triangle$ isos., 2 sides $\cong$.
(3) If 2 sides $\cong$, $\angle$s opp $\cong$.
(4) Given
(5) def. of altitude
(6) All Rt. $\angle$s $\cong$.
(7) AAS $\cong$ AAS
(8) CPCTC
(9) CPCTC

Ex: In $\triangle ABC$, $A(4,1)$, $B(0,-5)$, $C(-3,3)$



a. Find the equation of the altitude from A.

Need slope $\overline{CB}$

$m_{\overline{CB}} = \frac{3+5}{-3} = \frac{8}{-3}$

Slope alt.

$m_{\perp} = \frac{3}{8}$

$A(4,1)$

$y = mx + b$

$1 = \frac{3}{8}(4) + b$

$1 = \frac{3}{2} + b$

$1 - 1\frac{1}{2} = b$

$-\frac{1}{2} = b$

Altitude

$\boxed{y = \frac{3}{8}x - \frac{1}{2}}$

b. Find the equation of the median from B.

Need midpoint of $\overline{AC}$

$\left(\frac{4-3}{2}, \frac{1+3}{2}\right)$

midpoint $\left(\frac{1}{2}, 2\right)$

Need slope of $B \rightarrow$ midpoint

$\left(\frac{1}{2}, 2\right)$ $(0,-5)$

$m = \frac{2+5}{\frac{1}{2}} = 7 \times 2 = 14$

$y = mx + b$

$-5 = 14(0) + b$

$-5 = b$

$\boxed{y = 14x - 5}$

* Careful! Perpendicular bisector often does not pass thru vertex.

$\hookrightarrow$ c. Find the equation of the perpendicular bisector of $\overline{AB}$. (Not from vertex C)

slope of $\overline{AB}$

$m_{\overline{AB}} = \frac{1+5}{4-0} = \frac{6}{4} = \frac{3}{2}$

slope of $\perp$

* $m = -\frac{2}{3}$

① midpoint of $\overline{AB}$

$\left(\frac{4+0}{2}, \frac{1-5}{2}\right)$

* $(2, -2)$

y-int

$y = mx + b$

$-2 = -\frac{2}{3}(2) + b$

$-2 + \frac{4}{3} = b$

$-\frac{2}{3} = b$

$\perp$ bisector

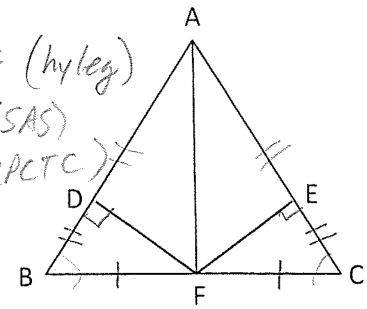
$\boxed{y = -\frac{2}{3}x - \frac{2}{3}}$

Write a flow proof for the following.

Plan
$\triangle DBF \cong \triangle ECF$ (hy leg)
$\triangle ABF \cong \triangle ACF$ (SAS)
$\angle AFB \cong \angle AFC$ (CPCTC)
$\overline{AF} \perp \overline{BC}$

(2) Given: F is the midpoint of $\overline{BC}$
$\overline{DB} \cong \overline{EC}$; $\overline{DB} \perp \overline{DF}$; $\overline{EC} \perp \overline{EF}$

Prove: $\overline{AF} \perp \overline{BC}$



(1) $\overline{DB} \perp \overline{DF}$ → (2) $\angle BDF$ Rt. → (3) $\triangle BDF$ is Rt.
$\overline{EC} \perp \overline{EF}$ → $\angle CEF$ Rt. → $\triangle CEF$ is Rt.
(4) F midpt. $\overline{BC}$ → (5) $\overline{BF} \cong \overline{FC}$
(6) $\overline{DB} \cong \overline{EC}$
→ (7) $\triangle BDF \cong \triangle CEF$

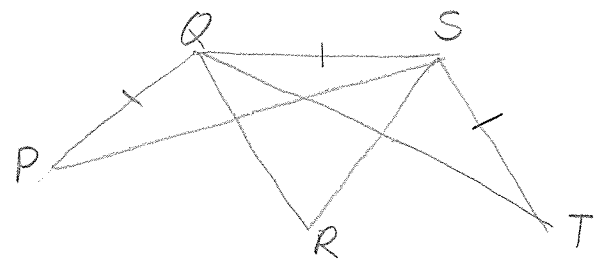
(8) $\angle B \cong \angle C$ → (9) $\overline{AB} \cong \overline{AC}$
(8) $\angle B \cong \angle C$
(5) $\overline{BF} \cong \overline{FC}$
→ (10) $\triangle ABF \cong \triangle ACF$ → (11) $\angle AFB \cong \angle AFC$

(12) $\overline{AF} \perp \overline{BC}$

#2 Reasons

1. Given
2. $\perp$ lines form rt $\angle$s.
3. Rt $\triangle$ has a rt. $\angle$.
4. Given
5. Midpt divides a segmt into 2 $\cong$ parts.
6. Given
7. hyleg thm
8. CPCTC
9. In a $\triangle$, if 2 $\angle$s $\cong$, sides opp $\cong$.
10. SAS post.
11. CPCTC
12. 2 lines form $\cong$ adjacent $\angle$s $\rightarrow$ $\perp$ lines.

3



(1) $\overline{SR} \perp$ bisector $\overline{QT}$ $\rightarrow$ (2) $QS = ST$

(3) $\overline{QR} \perp$ bisector $\overline{SP}$ $\rightarrow$ (4) $QS = QP$

$\rightarrow$ (5) $ST = QP$ $\rightarrow$ (6) $\overline{ST} \cong \overline{QP}$

1. Given
2. Point on $\perp$ bisector of seg $\rightarrow$ it is equidistant from endpts.
3. Given
4. Point on $\perp$ bisector of seg $\rightarrow$ it is equidistant from endpts.
5. Transitive Prop.
6. $\cong$ seg. have = measures.