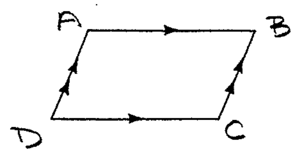
KEY

Geometry (H)
Section 5.1 Notes – Parallelograms

Directions: *Read the following notes about parallelograms and their properties. Then use the notes to complete the attached problems.*

Definition: A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

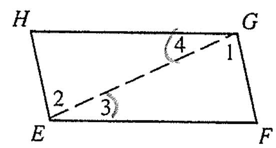


▱ABCD

Theorem 5.1: Opposite sides of a parallelogram are congruent.

Given: ▱ EFGH

Prove: $\overline{EF} \cong \overline{HG}$; $\overline{FG} \cong \overline{EH}$



Proof:

① Draw $\overline{EG}$

② ▱EFGH ③ → $\overline{HG} \parallel \overline{EF}$ ④ → $\angle 3 \cong \angle 4$

→ $\overline{HE} \parallel \overline{GF}$ → $\angle 1 \cong \angle 2$

⑤ $\overline{EG} \cong \overline{EG}$

⑥ → $\triangle HEG \cong \triangle FGE$ ⑦ → $\overline{EF} \cong \overline{HG}$, $\overline{FG} \cong \overline{HE}$

Reasons

① Two pts determine a line

② Given

③ Def ▱

④ $\parallel \Rightarrow$ alt int ∠s ≅

⑤ Reflexive

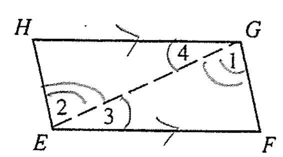
⑥ ASA

⑦ Def ≅ △s (CPCTC)

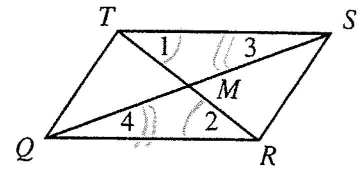
Theorem 5.2: Opposite angles of a parallelogram are congruent.

Given: ▱ EFGH

Prove: $\angle H \cong \angle F$ and $\angle E \cong \angle G$



Theorem 5.3: Diagonals of a parallelogram bisect each other.



Given: ▱QRST with diagonals $\overline{QS}$ and $\overline{TR}$

Prove: $\overline{QS}$ and $\overline{TR}$ bisect each other

Proof:

① ▱QRST → ② $\overline{ST} \parallel \overline{QR}$ → ③ $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

① → ④ $\overline{ST} \cong \overline{QR}$

→ ⑤ $\triangle TSM \cong \triangle RQM$ → ⑥ $\overline{TM} \cong \overline{MR}$, $\overline{SM} \cong \overline{MQ}$

→ ⑦ M midpt $\overline{TR}$ + $\overline{SQ}$ → ⑧ $\overline{QS}$ + $\overline{TR}$ bis each other

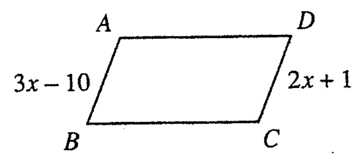
Reasons.

① Given
② Def ▱
③ ∥ → alt int ∠s ≅
④ ▱ → opp. sides ≅
⑤ ASA
⑥ Def ≅ △'s (CPCTC)
⑦ Def midpt
⑧ Def seg. bisector

Corollary: Consecutive angles in a parallelogram are supplementary.

Example 1

Find the lengths of $\overline{AB}$ and $\overline{DC}$ in ▱ABCD.



Solution

$3x - 10 = 2x + 1$ *Opposite sides of a ▱ have equal measure.*

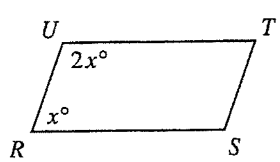
$x = 11$

$2(11) + 1 = 23$ *Replace x with 11 in 2x + 1.*

$AB = DC = 23$

Example 2

Find the measures of the four angles of ▱RSTU if the measure of ∠U is twice the measure of ∠R.



Solution

$x + 2x = 180$ *Consecutive angles of a ▱ are supplementary.*

$3x = 180$

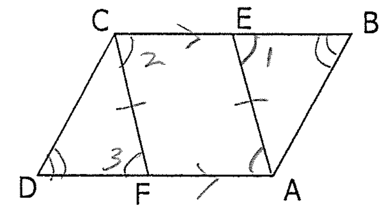
$x = 60$

$m\angle R = m\angle T = 60 \quad m\angle U = m\angle S = 120$

Geometry (H) Name: KEY

Section 5.1 – More practice

Write a flow proof for each of the following.



1. Given: ▱ABCD, ▱AECF

   Prove: $\triangle CDF \cong \triangle ABE$ AAS

① ▱ABCD → ② $\overline{CB} \parallel \overline{AD}$ → ③ $\angle 2 \cong \angle 3$, ⑤ $\angle 2 \cong \angle 1$ → ⑥ $\angle 1 \cong \angle 3$

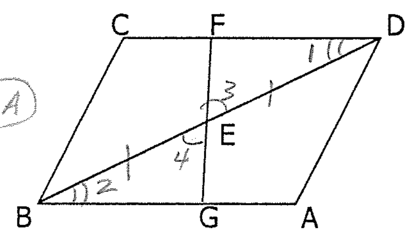
① ▱ABCD → ⑦ $\angle D \cong \angle B$

④ ▱AECF → ⑧ $\overline{CF} \cong \overline{EA}$

④ ▱AECF → ⑤ $\angle 2 \cong \angle 1$

⑥, ⑦, ⑧ → ⑨ $\triangle CDF \cong \triangle ABE$

① Given
② ▱ → opp sides ∥
③ 2 ∥ lines → corresp. ∠s ≅
④ 2 ∥ lines → alt. int. ∠s ≅
⑤ Transitive Property
⑥ In ▱ → opposite ∠s ≅.
⑦ Given
⑧ ▱ → opp sides ∥
⑨ AAS ≅ AAS



2. Given: ▱ABCD; $\overline{FG}$ bisects $\overline{DB}$

   Prove: $\overline{DB}$ bisects $\overline{FG}$ Show $\triangle DFE \cong \triangle BGE$ (ASA)

① ▱ABCD → ② $\overline{CD} \parallel \overline{AB}$ → ③ $\angle 1 \cong \angle 2$

④ $\overline{FG}$ bisects $\overline{DB}$ → ⑤ $\overline{BE} \cong \overline{ED}$

⑥ $\angle 3 \cong \angle 4$

③, ⑤, ⑥ → ⑦ $\triangle DFE \cong \triangle BGE$ → ⑧ $\overline{FE} \cong \overline{EG}$ → ⑨ $\overline{DB}$ bisects $\overline{FG}$

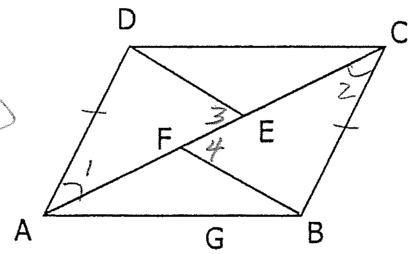
① Given
② ▱ → opp sides ∥.
③ In 2 ∥ lines → alt. int. ∠s ≅.
④ Given
⑤ Def of seg. bisector
⑥ Vertical ∠s ≅.
⑦ ASA ≅ ASA
⑧ CPCTC
⑨ Def of seg. bisector

5-1 KEY

3. Given: $\square ABCD$; $\overline{AF} \cong \overline{CE}$

Prove: $\overline{DE} \parallel \overline{BF}$

Show $\triangle ADE \cong \triangle CBF$ (SAS)



① $\boxed{\square}$ ABCD → ② $\overline{AD} \parallel \overline{CB}$ → ③ $\angle 1 \cong \angle 2$

① → ④ $\overline{AD} \cong \overline{CB}$

⑤ $\overline{AF} \cong \overline{CE}$ → ⑥ $\overline{AE} \cong \overline{CF}$

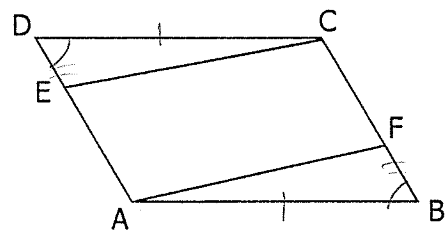
③, ④, ⑥ → ⑦ $\triangle ADE \cong \triangle CBF$ → ⑧ $\angle 3 \cong \angle 4$ → ⑨ $\overline{DE} \parallel \overline{BF}$

① Given
② $\square$ → opp sides $\parallel$
③ 2 $\parallel$ lines → alt. int. $\angle$s $\cong$.
④ $\square$ → opp sides $\cong$.
⑤ Given
⑥ common segment than
⑦ SAS $\cong$ SAS
⑧ CPCTC
⑨ 2 lines forms alt. int. $\angle$s $\cong$ → 2 $\parallel$ lines.

4. Given: $\square ABCD$; $\overline{AE} \cong \overline{CF}$

Prove: $\overline{AF} \cong \overline{CE}$

Show $\triangle ABF \cong \triangle CDE$ (SAS)



① $\square$ ABCD → ② $\overline{AD} \cong \overline{CB}$ → ③ $AD = CB$

④ $AD = DE + EA$, $CB = CF + FB$

③, ④ → ⑤ $DE + EA = CF + FB$

⑥ $\overline{AE} \cong \overline{CF}$ → ⑦ $AE = CF$

⑤, ⑦ → ⑧ $DE = FB$ → ⑨ $\overline{DE} \cong \overline{FB}$

① → ⑩ $\angle D \cong \angle B$

① → ⑪ $\overline{DC} \cong \overline{AB}$

⑨, ⑩, ⑪ → ⑫ $\triangle ABF \cong \triangle CDE$ → ⑬ $\overline{AF} \cong \overline{CE}$

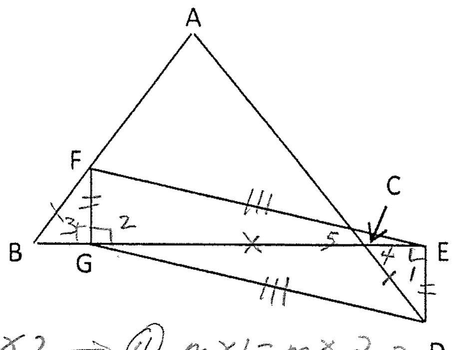
① Given
② $\square$ → opp sides $\cong$
③ Def of $\cong$ seg.
④ Partition Postulate
A whole qty is = to sum of its parts
⑤ Substitution
⑥ Given
⑦ Def of $\cong$ seg.
⑧ Subtraction Prop.
⑨ Def $\cong$ seg
⑩ $\square$ → opp $\angle$s $\cong$
⑪ $\square$ → opp sides $\cong$.
⑫ SAS $\cong$ SAS
⑬ CPCTC

Re-do!

5. Given: ▱EFGD; $\overline{ED} \perp \overline{BE}$; $\overline{BF} \cong \overline{CD}$

Prove: $\triangle ABC$ isosceles

Show $\triangle BFG \cong \triangle CDE$ (hyleg)



Careful!

* easy to confuse $\measuredangle FGD$ with $\measuredangle 2$ (right $\measuredangle$)

(1) ▱EFGD → (2) $\overline{FG} \parallel \overline{ED}$ → (3) $\measuredangle 1 \cong \measuredangle 2$ → (4) $m\measuredangle 1 = m\measuredangle 2$

(5) $\overline{ED} \perp \overline{BE}$ → (6) $\measuredangle 1$ is Rt $\measuredangle$ → (7) $m\measuredangle 1 = 90$

(4), (7) → (8) $m\measuredangle 2 = 90$

(9) $\measuredangle 2$ is Rt → (10) $\overline{FG} \perp \overline{BE}$ → (11) $\measuredangle 3$ is Rt $\measuredangle$ → (12) $\triangle BFG$ Rt.

(13) $\triangle CDE$ Rt

(14) $\overline{FG} \cong \overline{ED}$

(15) $\overline{BF} \cong \overline{CD}$

(12)–(15) → (16) $\triangle BFG \cong \triangle CDE$

(17) $\measuredangle B \cong \measuredangle 4$

(18) $\measuredangle 4 \cong \measuredangle 5$

(17), (18) → (19) $\measuredangle B \cong \measuredangle 5$ → (20) $\overline{AB} \cong \overline{AC}$ → (21) $\triangle ABC$ isosceles.

(1) Given

(2) ▱ → opp sides ∥.

(3) 2 ∥ lines → alt. int. $\measuredangle$s ≅

(4) Def of ≅ $\measuredangle$s

(5) Given

(6) ⊥ lines form Rt $\measuredangle$s

(7) Def Rt. $\measuredangle$

(8) Substitution

(9) Def of Rt. $\measuredangle$

(10) Def ⊥ lines

(11) ⊥ lines form Rt. $\measuredangle$.

(12) Def of Rt. △

(13) Def of Rt. △

(14) ▱ → opp sides ≅.

(15) Given

(16) hyleg ≅ hyleg

(17) CPCTC

(18) Vertical $\measuredangle$s ≅.

(19) Transitive Prop

(20) If 2 $\measuredangle$s ≅, sides opp ≅.

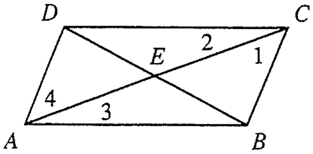
(21) Def of isosceles △.

Exercises

KEY

A

***ABCD* is a parallelogram. State the theorem that justifies each conclusion.**

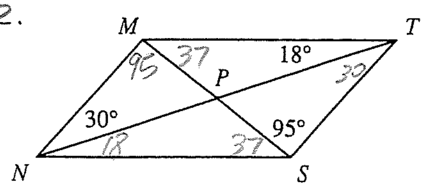


1. $\angle DAB \cong \angle DCB$ 2. $\overline{BE} \cong \overline{ED}$
3. $\overline{AD} \cong \overline{BC}$ 4. $\overline{DC} \cong \overline{AB}$

Complete each statement.

5. If $AD = 20$, $BC =$ <u>20</u>. 6. If $m\angle ADC = 115$, $m\angle ABC$ <u>115</u>.
7. If $DB = 22$, $DE =$ <u>11</u>. 8. If $AE = 18$, $AC =$ <u>36</u>.
9. If $m\angle DAB = 75$, $m\angle ADC =$ <u>105</u>.
10. If $m\angle 2 = 30$, $m\angle 3 =$ <u>30</u>.
11. If $BD = 10$ and $AE = 8$, $AC =$ <u>16</u>.
12. If $m\angle ABC = 2(m\angle BCD)$, $m\angle ADC =$ <u>120</u>
13. If $m\angle ADC = 130$, $m\angle 1 = 35$, $m\angle 2 =$ <u>15</u>.

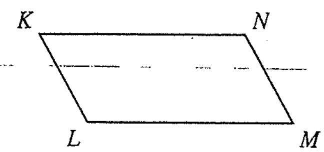
Find the measure of each angle in ▱ *MNST*.



14. $m\angle TMN$ 15. $m\angle TSN$
16. $m\angle MSN$ 17. $m\angle SPN$

(14) $m\angle TMN = 132$
(15) $m\angle TSN = 132$
(16) $m\angle MSN = 37$
(17) $m\angle SPN = 125$

Complete each statement about ▱ *KLMN*.



18. If $KN = 3x - 5$, $LM = x + 9$, $KN =$ <u>16</u>.
19. If $KL = \frac{x}{2}$, $MN = 2x - 9$, $KL =$ <u>3</u>.
20. If $KL = 8$, $MN = \frac{x^2}{2}$, $x =$ <u>+4 or −4</u>
21. If $m\angle K = 4x + 11$, $m\angle L = 6x - 1$, $m\angle K =$ <u>79</u>.
22. If $m\angle K = 31$, $m\angle M = 2x^2 - 1$, $x =$ <u>±4</u>.
23. If $m\angle L = x - 40$, $m\angle N = \frac{3x}{4}$, $m\angle L =$ <u>120</u>.

(18) $3x - 5 = x + 9$
$2x = 14$
$x = 7$

(20) $8 = \frac{x^2}{2}$
$16 = x^2$
$\pm 4 = x$

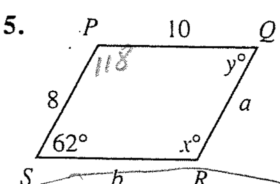
(21) $4x + 11 + 6x - 1 = 180$
$10x = 170$
$x = 17$

(23) $x - 40 = \frac{3}{4}x$
$\frac{1}{4}x = 40$
$x = 160$

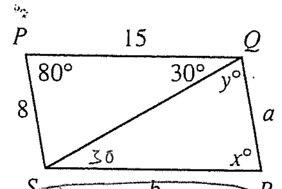
(19) $\frac{x}{2} = 2x - 9$
$x = 4x - 18$
$18 = 3x$
$x = 6$

(22) $31 = 2x^2 - 1$
$32 = 2x^2$
$16 = x^2$
$\pm 4 = x$

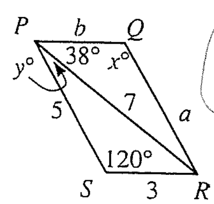
In Exercises 5–10 quad. *PQRS* is a parallelogram. Find the values of *a*, *b*, *x*, and *y*.

5.


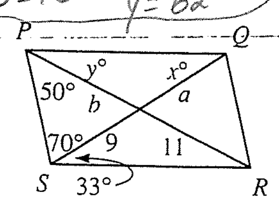
$a = 8$ $x = 118°$
$b = 10$ $y = 62°$

6.


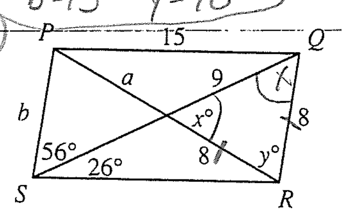
$a = 8$ $x = 80$
$b = 15$ $y = 70$

7.


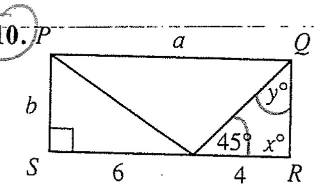
$a = 5$ $x = 120$
$b = 3$ $y = 22$

8.


$a = 9$ $x = 33$
$b = 11$ $y = 27$

9.


$a = 8$ $x = 56$
$b = 8$ $y = 68$

10.


$a = 10$ $x = 90$
$b = 4$ $y = 45$

#17, 18

The coordinates of three vertices of $\square ABCD$ are given. Plot the points and find the coordinates of the fourth vertex.

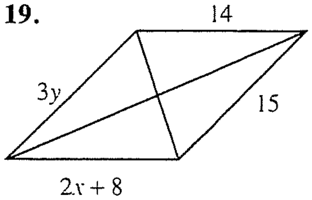
17. $A(1, 0)$, $B(5, 0)$, $C(7, 2)$, $D(\underline{\ ?\ }, \underline{\ ?\ })$ D(3, 2)

18. $A(3, 2)$, $B(8, 2)$, $C(\underline{\ ?\ }, \underline{\ ?\ })$, $D(0, 5)$ C(5, 5)

19 - 24

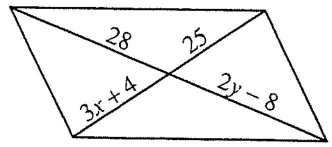
Each figure in Exercises 19–24 is a parallelogram with its diagonals drawn. Find the values of x and y. See below.

19.

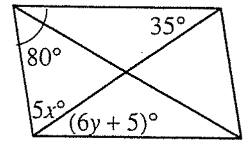


$3y = 15$
$y = 5$
$2x + 8 = 14$
$2x = 6$
$x = 3$

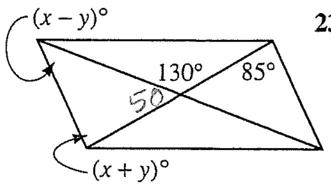
20.



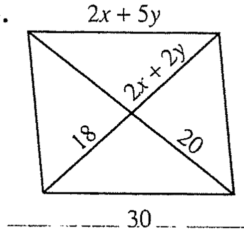
21.



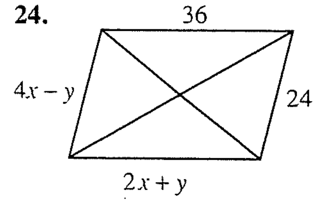
22.



23.

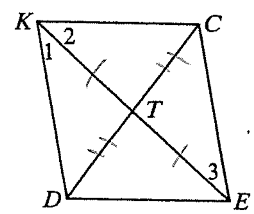


24.



#25-28

Quad. *DECK* is a parallelogram. Complete.



25. If $KT = 2x + y$, $DT = x + 2y$, $TE = 12$, and $TC = 9$, then $x = \underline{\ ?\ }$ and $y = \underline{\ ?\ }$.

26. If $DE = x + y$, $EC = 12$, $CK = 2x - y$, and $KD = 3x - 2y$, then $x = \underline{\ ?\ }$, $y = \underline{\ ?\ }$, and the perimeter of $\square DECK = \underline{\ ?\ }$.

27. If $m\angle 1 = 3x$, $m\angle 2 = 4x$, and $m\angle 3 = x^2 - 70$, then $x = \underline{\ ?\ }$ and $m\angle CED = \underline{\ ?\ }$ (numerical answers).

28. If $m\angle 1 = 42$, $m\angle 2 = x^2$, and $m\angle CED = 13x$, then $m\angle 2 = \underline{\ ?\ }$ or $m\angle 2 = \underline{\ ?\ }$ (numerical answers).

(25) $\boxed{\begin{array}{l} 2x + y = 12 \\ x + 2y = 9 \end{array}}$
$y = 2$
$x = 5$

(26) $\boxed{\begin{array}{l} x + y = 2x - y \\ 3x - 2y = 12 \end{array}}$
$x = 6$
$y = 3$
perimeter = 42

(27) $\boxed{3x = x^2 - 70}$
$x^2 - 3x - 70 = 0$
$x = 10$ $x = -7$ OMIT
$m\angle CED = 70$

(28) $\boxed{x^2 + 42 = 13x}$
$x^2 - 13x + 42 = 0$
$x = 6$ $x = 7$
$m\angle 2 = 36$
$m\angle 2 = 49$

(20) $2y - 8 = 28$
$2y = 36$
$y = 18$
$3x + 4 = 25$
$3x = 21$
$x = 7$

(21) $6y + 5 = 35$
$6y = 30$
$y = 5$
$5x + 35 + 80 = 180$
$5x = 65$
$x = 13$

(22) $\boxed{\begin{array}{l} x - y + x + y + 50 = 180 \\ x + y = 85 \end{array}}$
$x = 65$
$y = 20$

(23) $\boxed{\begin{array}{l} 2x + 2y = 18 \\ 2x + 5y = 30 \end{array}}$
$x = 5$
$y = 4$

(24) $\boxed{\begin{array}{l} 4x - y = 24 \\ 2x + y = 36 \end{array}}$
$x = 10$
$y = 16$