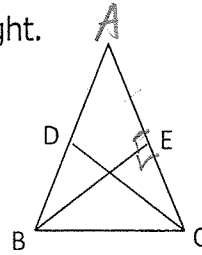


Geometry (H)

Altitude, Median & The Equidistant Theorems – Problems

For example 1 and 2 using the diagram at the right.



1. Supposed that \overline{BE} and \overline{CD} are medians. If $AD = 2x + 3$, $BD = 3x - 2$, $AE = 4x - 7$, explain why $AB = AC$.

$AD = BD$ $AD = 13$ $AE = 13$ $AB = 26$
 $2x + 3 = 3x - 2$ $BD = 13$ Since $AE = 13$, EC also is 13.
 $5 = x$ $AE + EC = 26$. Since $AB = 26$
 and $AC = 26$, $AB = AC$.

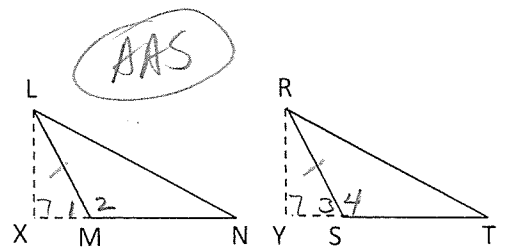
2. Suppose that \overline{BE} is an altitude. $m\angle AEB = 4x + 10$, $AD = 2x + 5$, $BD = 3x - 15$, is \overline{CD} a median?

$m\angle AEB = 90$ $AD = 2(20) + 5 = 45$
 $4x + 10 = 90$ $BD = 3(20) - 15 = 45$
 $4x = 80$ Since $AD = BD$, \overline{CD} is a median.
 $x = 20$

3. Write a flow proof.

Given: $\triangle LMN \cong \triangle RST$; \overline{LX} and \overline{RY} are altitudes

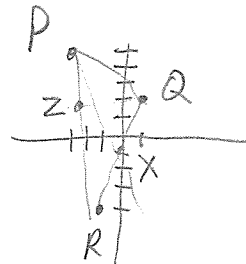
Prove: $\overline{LX} \cong \overline{RY}$



- ① $\angle 1$ & $\angle 2$ linear pair \rightarrow ② $\angle 1$ supp $\angle 2$
- $\angle 3$ & $\angle 4$ linear pair \rightarrow $\angle 3$ supp $\angle 4$
- ③ $\triangle LMN \cong \triangle RST \rightarrow$ ④ $\angle 2 \cong \angle 4$
- ⑦ \overline{LX} & \overline{RY} altitudes \rightarrow ⑦a $\overline{LX} \perp \overline{MN}$ \rightarrow ⑧ $\angle 3$ & $\angle 4$ are Rt \angle s. \rightarrow ⑨ $\angle 3 \cong \angle 4$
- \rightarrow ⑤ $\angle 1 \cong \angle 3$
- \rightarrow ⑩ $\triangle LXN \cong \triangle RYS$
- \rightarrow ⑪ $\overline{LX} \cong \overline{RY}$

- ① Def. linear pair
- ② linear pair postulate
- ③ Given
- ④ CPCTC
- ⑤ CPCTC
- ⑦ Given
- ⑦a Def. of altitude
- ⑧ \perp lines form Rt \angle s.
- ⑨ All Rt. \angle s \cong .
- ⑩ AAS \cong AAS
- ⑪ CPCTC

4. In $\triangle PQR$, $P(-3,5)$, $Q(1,2)$ and $R(-1,-4)$



a. Find the equation of the altitude \overline{QZ} .

Slope of \overline{PR}

$$m_{\overline{PR}} = \frac{5+4}{-3+1} = \frac{9}{-2}$$

$Q(1,2)$

$$y\text{-int}$$

$$y = mx + b$$

$$2 = \frac{2}{9}(1) + b$$

$$\frac{16}{9} = b$$

altitude \overline{QZ}

$$y = \frac{2}{9}x + \frac{16}{9}$$

* $m_{\perp} = \frac{2}{9}$

b. Find the equation of the median \overline{PX} .

midpoint of \overline{QR}

$$\left(\frac{1-1}{2}, \frac{2-4}{2}\right)$$

$X(0, -1)$

slope of \overline{PX}

$$m_{\overline{PX}} = \frac{5+1}{-3-0} = \frac{6}{-3} = -2$$

y-int

$$-1 = (-2)(0) + b$$

$$-1 = b$$

median \overline{PX}

$$y = -2x - 1$$

c. Find the equation of the perpendicular bisector of \overline{PQ} . (does not necessarily go through the vertex)

slope of \overline{PQ}

$$m_{\overline{PQ}} = \frac{5-2}{-3-1} = \frac{3}{-4}$$

* $m_{\perp} = \frac{4}{3}$

midpoint of \overline{PQ}

$$\left(\frac{-3+1}{2}, \frac{5+2}{2}\right)$$

$$\left(-1, \frac{7}{2}\right)$$

y-int

$$m = \frac{4}{3}, \left(-1, \frac{7}{2}\right)$$

$$\frac{7}{2} = \frac{4}{3}(-1) + b$$

$$\frac{21}{6} + \frac{8}{6} = b$$

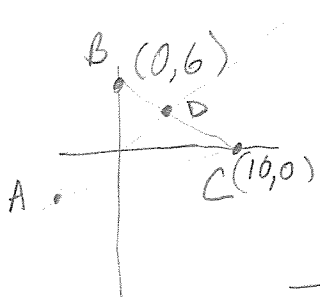
$$\frac{29}{6} = b$$

\perp bisector

$$y = \frac{4}{3}x + \frac{29}{6}$$

lies on \perp bisector

5. Find the coordinates of the point that is equidistant from the points (0,6) and (10,0) and lies on the line $y=x$.



$$m_{BC} = \frac{6-0}{0-10} = -\frac{3}{5}$$

$$* m_{\perp} = \frac{5}{3}$$

midpt of \overline{BC}
 $(\frac{0+10}{2}, \frac{6+0}{2})$

$$D(5,3)$$

intersects with $y=x$

y -int. of \perp bisector

$$b = -\frac{16}{3}$$

EQ of \perp bisector

$$y = \frac{5}{3}x - \frac{16}{3}$$

Solve system:

$$\begin{cases} y=x \\ y = \frac{5}{3}x - \frac{16}{3} \end{cases}$$

$$x = \frac{5}{3}x - \frac{16}{3}$$

$$8 = x$$

$$y = 8$$

point is (8,8)

6. Write a flow proof for each of the following.

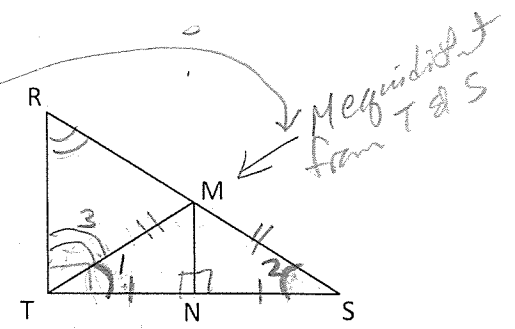
Given: $\overline{RT} \perp \overline{TS}$

\overline{MN} is the perpendicular bisector of \overline{TS}

Prove: \overline{TM} is a median

(Hint: use parallel lines)

Plan: ① Prove $\triangle TMN \cong \triangle SMN$ (hyleg, SSS, SAS, ASA)
 ② show $\sphericalangle 2$ comp. $\sphericalangle 3$
 $\sphericalangle 2$ comp. $\sphericalangle R$ } $\rightarrow \sphericalangle 3 \cong \sphericalangle R$
 ③ $\overline{TM} \cong \overline{RM} \cong \overline{MS} \rightarrow \overline{TM}$ median.



$\sphericalangle 3$ comp. $\sphericalangle 2$
 $\sphericalangle R$ comp. $\sphericalangle 2$

① $\overline{MN} \perp$ bisector of \overline{TS} . \rightarrow ② $\overline{TN} \cong \overline{NS}$
 ③ $\overline{MN} \perp \overline{TS} \rightarrow$ ④ $\sphericalangle TNM \cong \sphericalangle SNM$
 ⑤ $\sphericalangle TNM \cong \sphericalangle SNM$
 me Rt \sphericalangle s. ⑥ $\overline{MN} \cong \overline{MN}$ } \rightarrow ⑦ $\triangle TMN \cong \triangle SMN$
 ⑧ $\overline{RT} \perp \overline{TS}$
 \rightarrow ⑩ $\sphericalangle RTN$ is Rt. \rightarrow ⑪ $\triangle RTS$ is Rt. \rightarrow ⑫ $\sphericalangle R$ comp. $\sphericalangle 2$
 ⑬ $\sphericalangle 1 \cong \sphericalangle 2$ } \rightarrow ⑭ $\sphericalangle 3 \cong \sphericalangle R$
 ⑮ $\overline{TM} \cong \overline{RM}$ } \rightarrow ⑰ $\overline{RM} \cong \overline{MS} \rightarrow$ ⑱ M is the midpt of \overline{RS}
 ⑯ $\overline{TM} \cong \overline{MS}$ }
 ⑲ \overline{TM} is median.

Problem 6 Reasons.

- ① Given
- ② Def of \perp bisector
- ③ Def of \perp bisector
- ④ \perp lines form Rt \angle s.
- ⑤ All Rt \angle s \cong .
- ⑥ Reflexive Prop.
- ⑦ SAS \cong SAS
- ⑧ Given
- ⑨ If exterior sides form acute adj. \angle s are $\perp \rightarrow$ complement. \angle s.
- ⑩ \perp lines form Rt \angle s.
- ⑪ Def of Rt Δ
- ⑫ In a Rt Δ , acute \angle s complementary.
- ⑬ CPCTC
- ⑭ \cong Complements Thm
- ⑮ If \angle s \cong , then sides opp. \cong .
- ⑯ CPCTC
- ⑰ Transitive Prop.
- ⑱ Def of midpoint
- ⑲ A median is a seg. that connects a midpt of a segment & the opp. vertex.