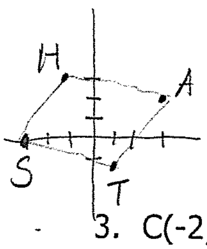
KEY

Geometry (H)
Section 5.4 – Identifying Quads in the Coordinate Plane

Directions: Graph each quadrilateral then determine the type of quadrilateral: rectangle, rhombus, square or parallelogram. Join the vertices in the given order. ~~Use~~ you must use slope or distance to verify the type of quadrilateral.

1. H(-1,3) A(3,2) T(1,-1) S(-3,0)



$m_{\overline{HS}} = \frac{3-0}{-1+3} = \frac{3}{2}$

$m_{\overline{ST}} = \frac{-1-0}{1+3} = \frac{-1}{4}$

not $\perp$.

$d_{\overline{HA}} = \sqrt{(-1-3)^2 + (3-2)^2} = \sqrt{16+1} = \sqrt{17}$

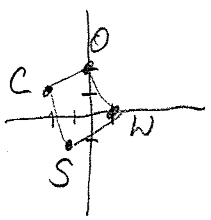
$d_{\overline{ST}} = \sqrt{(1+3)^2 + (-1-0)^2}$ $\sqrt{16+1} = \sqrt{17}$

$d_{\overline{HS}} = \sqrt{(-1+3)^2 + (3-0)^2}$ $\sqrt{4+9} = \sqrt{13}$

$d_{\overline{AT}} = \sqrt{(3-1)^2 + (2+1)^2}$ $\sqrt{4+9} = \sqrt{13}$

Since $\overline{HA} \cong \overline{ST}$ and $\overline{HS} \cong \overline{AT}$, HATS is a parallelogram

3. C(-2,1) O(0,2) W(1,0) S(-1,-1)



$m_{\overline{CW}} = \frac{1-0}{-2-1} = \frac{1}{-3}$

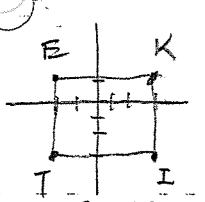
$m_{\overline{OS}} = \frac{2+1}{0+1} = \frac{3}{1}$

$d_{\overline{OS}} = \sqrt{(0+1)^2 + (2+1)^2} = \sqrt{1+9} = \sqrt{10}$

$d_{\overline{CW}} = \sqrt{(-2-1)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10}$

$\overline{CW} \perp \overline{OS}$ and $\overline{OS} \cong \overline{CW}$, therefore, COWS is a square.

4. K(3,1) I(3,-3) T(-2,-3) E(-2,1)



$d_{\overline{EK}} = \sqrt{(3+2)^2 + (1-1)^2} = \sqrt{25} = 5$

$d_{\overline{TI}} = \sqrt{(3+2)^2 + (-3+3)^2}$ $\sqrt{25} = 5$

$d_{\overline{ET}} = \sqrt{(-2+2)^2 + (-3-1)^2}$ $\sqrt{0+16} = 4$

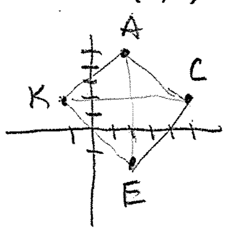
$d_{\overline{KI}} = \sqrt{(3-3)^2 + (1+3)^2}$ $\sqrt{0+16} = 4$

$m_{\overline{EK}} = \frac{1-1}{3+2} = 0$

$m_{\overline{ET}} = \frac{-3-1}{-2+2} = \frac{-4}{0}$

$\overline{EK} \cong \overline{TI}$, $\overline{ET} \cong \overline{KI}$, and $\overline{EK} \perp \overline{ET}$, KITE is a rectangle

6. C(5,2) A(2,5) K(-1,2) E(2,-1)



Square

$m_{\overline{AE}} = \frac{5+1}{2-2} = \frac{6}{0} = \text{undef}$

$m_{\overline{KC}} = \frac{2-2}{5+1} = \frac{0}{6} = 0$

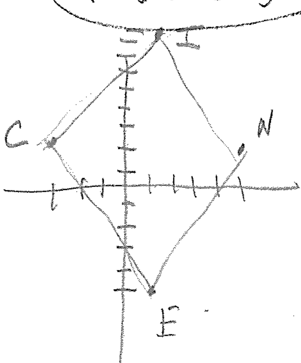
$d_{\overline{AE}} = \sqrt{(2-2)^2 + (5+1)^2}$ $\sqrt{36} = 6$

$d_{\overline{KC}} = \sqrt{(5+1)^2 + (2-2)^2} = \sqrt{36} = 6$

So, $\overline{AE} \perp \overline{KC}$ and $\overline{AE} \cong \overline{KC}$, → Square

7. N(5,2) I(1,9) C(-3,2) E(1,-5)

rhombus



$m_{\overline{CN}} = \frac{2-2}{5+3} = 0$

$m_{\overline{IE}} = \frac{9+5}{1-1} = \frac{14}{0} = \text{undef}$

$\overline{CN} \perp \overline{IE}$

$\overline{CI} \cong \overline{CE} \cong \overline{IN} \cong \overline{NE}$

$d_{\overline{CI}} = \sqrt{(1+3)^2 + (9-2)^2} = \sqrt{16+49} = \sqrt{65}$

$d_{\overline{IN}} = \sqrt{(5-1)^2 + (2-9)^2}$ $\sqrt{16+49} = \sqrt{65}$

$d_{\overline{CE}} = \sqrt{(-3-1)^2 + (2+5)^2}$ $\sqrt{16+49} = \sqrt{65}$

$d_{\overline{NE}} = \sqrt{(5-1)^2 + (2+5)^2}$ $\sqrt{16+49} = \sqrt{65}$

(5-4)

a rhombus

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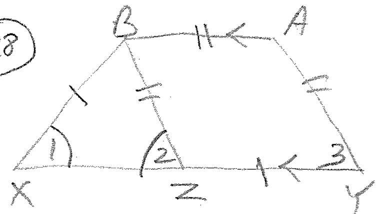

① ▱ABZY → ② $\overline{ZY} \cong \overline{BA}$, ③ $\overline{ZY} \cong \overline{BX}$ → ④ $\overline{BA} \cong \overline{BX}$

⑤ $\angle 1 \cong \angle 2$ → ⑥ $\overline{BX} \cong \overline{BZ}$

④, ⑥ → ⑦ $\overline{BA} \cong \overline{BZ}$

⑦, ⑧ ▱ABZY → ⑨ ABZY is a rhombus

① Given
② ▱ → opp sides ≅.
③ Given
④ Transitive Prop
⑤ Given
⑥ If 2 ∠s of △ ≅ → sides opp ≅.
⑦ Transitive Prop
⑧ Given
⑨ ▱ → if 2 consecutive sides ≅ → a rhombus.

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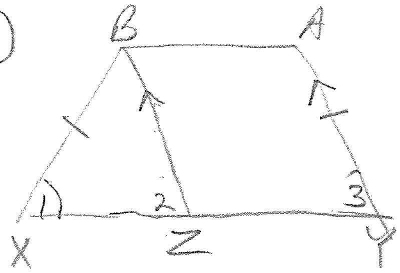

① ▱ABZY → ② $\overline{AY} \cong \overline{BZ}$, ③ $\overline{AY} \cong \overline{BX}$ → ④ $\overline{BZ} \cong \overline{BX}$ → ⑤ $\angle 1 \cong \angle 2$

① → ⑥ $\overline{AY} \parallel \overline{BZ}$ → ⑦ $\angle 2 \cong \angle 3$

⑤, ⑦ → ⑧ $\angle 1 \cong \angle 3$

① Given
② ▱ → opp sides ≅.
③ Given
④ Transitive prop.
⑤ If 2 ∠s of a △ ≅, then sides opp ≅.
⑥ ▱ → opp sides ∥
⑦ 2 ∥ lines → corresp. ∠s ≅.
⑧ Transitive prop.

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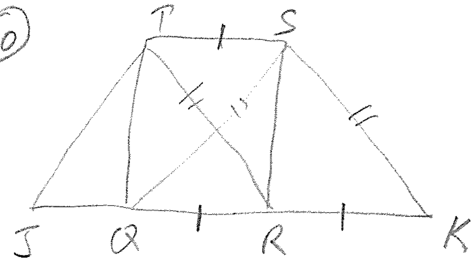

① Rectangle QRST → ② $\overline{QS} \cong \overline{TR}$ } → ⑤ $\overline{QS} \cong \overline{SK}$ → ⑥ $\triangle QSK$ is isosceles.

③ ▱ RKST → ④ $\overline{SK} \cong \overline{TR}$ }

① Given

② In a rectangle → diagonals are ≅.

③ Given

④ ▱ → opp sides ≅.

⑤ Transitive Prop.

⑥ Def of isosceles △.

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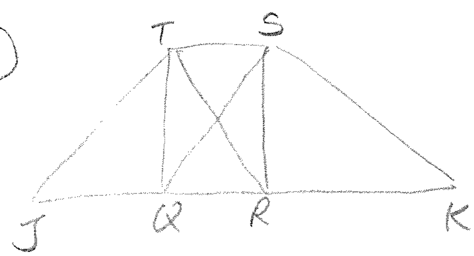


① Rect. QRST → ② $\overline{QS} \cong \overline{TR}$ } → ⑤ $\overline{QS} \cong \overline{SK}$ } → ⑧ $\overline{JT} \cong \overline{KS}$

③ ▱ RKST → ④ $\overline{TR} \cong \overline{SK}$ }

⑥ ▱ JQST → ⑦ $\overline{JT} \cong \overline{QS}$ }

① Given

② In a rect. → diagonals ≅.

③ Given

④ ▱ → opp sides ≅.

⑤ Transitive Prop

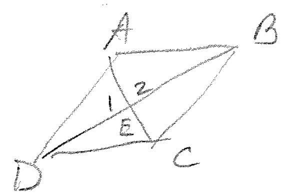
⑥ Given

⑦ ▱ → opp sides ≅

⑧ Transitive Prop.

(34) Given: ▱ABCD
$\overline{AC} \perp \overline{BD}$

Prove: ABCD is a rhombus



use: If a ▱ has consecutive ≅ sides → then a rhombus.

① $\overline{AC} \perp \overline{BD}$ → ② ∡1 is Rt, ∡2 is Rt → ③ ∡1 ≅ ∡2

④ ▱ABCD → ⑤ $\overline{AC}$ bisects $\overline{BD}$. → ⑥ $\overline{DE} \cong \overline{EB}$

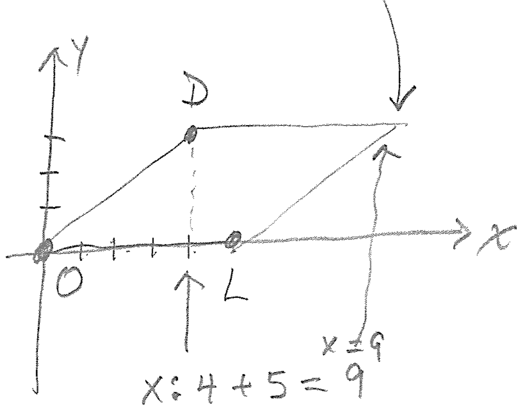
⑦ $\overline{AE} \cong \overline{AE}$

③, ⑥, ⑦ → ⑧ △ADE ≅ △ABE → ⑨ $\overline{AB} \cong \overline{AD}$

⑨, ⑩ ▱ABCD → ⑪ ABCD is a rhombus.

① Given
② Def of ⊥ lines
③ All rt ∡s ≅.
④ Given
⑤ ▱ → diag. bisect each other.
⑥ Def of seg. bisector
⑦ Reflexive prop.
⑧ SAS ≅ SAS
⑨ CPCTC
⑩ Given
⑪ A rhombus is a ▱ with 2 consecutive sides ≅.

(38) Answer: (9, 3)



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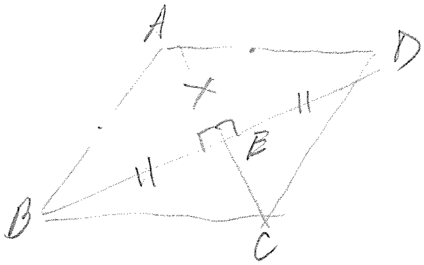


Given: ▱ABCD

$\overline{AC} \perp \overline{BD}$

(using definition)

(Shorter to use thm: ▱ w/ 2 consecutive sides ≅)

① ▱ABCD → ② $\overline{BE} \cong \overline{ED}$ →

③ $\overline{AC} \perp \overline{BD}$ → ④ ∡AED & ∡AEB are right → ⑤ ∡AED ≅ AEB

⑥ $\overline{AE} \cong \overline{AE}$

→ ⑦ △AED ≅ △AEB

⑧ $\overline{AD} \cong \overline{AB}$

⑨ $\overline{BE} \cong \overline{ED}$

⑩ ∡BEC & ∡DEC are Rt ∡s. → ⑪ ∡BEC ≅ ∡DEC

⑫ $\overline{EC} \cong \overline{EC}$

→ ⑬ △BEC ≅ △DEC → ⑭ $\overline{BC} \cong \overline{CD}$

⑮ $\overline{AE} \cong \overline{EC}$

⑯ ∡AED ≅ ∡DEC

⑰ $\overline{DE} \cong \overline{DE}$

→ ⑱ △AED ≅ △CED → ⑲ $\overline{AD} \cong \overline{CD}$

→ ⑳ $\overline{AD} \cong \overline{AB} \cong \overline{BC} \cong \overline{CD}$

→ ㉑ ▱ABCD is a rhombus

Geometry (H)

Section 5.4 – Special Parallelogram continued ...

Name: KEY

1. Put an X in the box if the shape has the given properties.

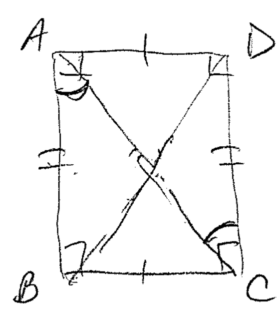
Property	Parallelogram	Rectangle	Rhombus	Square
Both pairs of opposite sides //	X	X	X	X
Diagonals are ⊥			X	X
Diagonals are ≅		X		X
Diagonals bisect each other	X	X	X	X
All angles are right angles		X		X
One pair of opposite sides ≅				
All sides are ≅			X	X
Both pairs of opposite angles ≅	X	X	X	X
Diagonals bisect the angles they are drawn from			X	X
All angles are ≅		X		X

2. Let's prove the properties we discovered yesterday!

Thm: The diagonals of a rectangle are congruent.

Given: rectangle ABCD

Diagram:



Prove: $\overline{AC} \cong \overline{BD}$

Prove: $\triangle ABC \cong \triangle DCB$ (overlap) (SAS)

① Rectangle ABCD → ② $\overline{AB} \cong \overline{DC}$

③ $\angle ABC$ & $\angle DCB$ are right $\angle$s. → ④ $\angle ABC \cong \angle DCB$

⑤ $\overline{BC} \cong \overline{BC}$

→ ⑥ $\triangle ABC \cong \triangle DCB$ → ⑦ $\overline{AC} \cong \overline{BD}$

① Given

② Rect. → opp sides ≅.

③ Def. of rectangle

④ All right $\angle$s ≅

⑤ Reflexive Prop

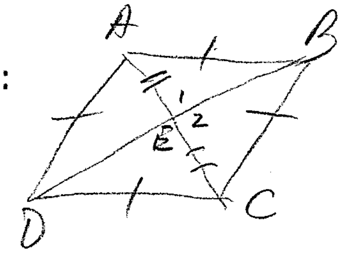
⑥ SAS ≅ SAS

⑦ CPCTC

Thm: The diagonals of a rhombus are perpendicular.

Given: rhombus ABCD

Diagram:



Prove: $\overline{AC} \perp \overline{BD}$

Prove: $\triangle ABE \cong \triangle CBE$ (SSS)

Use "if 2 lines form $\cong$ adj. $\angle$s, then 2 lines $\perp$.

① rhombus ABCD → ② $\overline{AB} \cong \overline{BC}$ →

↳ ③ ▱ABCD → ④ $\overline{AC}$ & $\overline{BD}$ bisect other other → ⑤ $\overline{AE} \cong \overline{EC}$

⑥ $\overline{BE} \cong \overline{BE}$ } ⑦ $\triangle ABE \cong \triangle CBE$

→ ⑧ $\angle 1 \cong \angle 2$ → ⑨ $\overline{AC} \perp \overline{BD}$

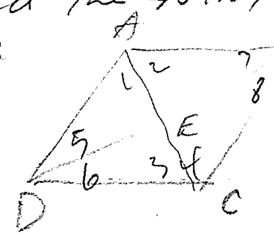
① Given
② rhombus → all sides $\cong$
③ Def of a rhombus: A ▱ w/ 4 $\cong$ sides
④ ▱ → diag. bisect each other.
⑤ Def. of bisector
⑥ Reflexive Prop.
⑦ SSS $\cong$ SSS
⑧ CPCTC
⑨ If 2 lines form $\cong$ adj. $\angle$s, then lines $\perp$.

Thm: Each diagonal of a rhombus bisects the angles they are drawn from.

If a quad. is a rhombus, then each diag. bisect the $\angle$s they are drawn from.

Given: rhombus ABCD

Diagram:



Prove: $\overline{AC}$ bisects $\angle DAB$ & $\angle BCD$
$\overline{BD}$ bisects $\angle ADC$ & $\angle ABC$

Ⓘ ① Rhom ABCD → ② $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$
③ $\overline{BD} \cong \overline{BD}$ } → ④ $\triangle ABD \cong \triangle CBD$ → ⑤ $\angle 5 \cong \angle 6$, $\angle 7 \cong \angle 8$ → ⑥ $\overline{BD}$ bis. $\angle ADC$ & $\angle ABC$

→ ⑦ $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$
⑧ $\overline{AC} \cong \overline{AC}$ } → ⑨ $\triangle ADC \cong \triangle ABC$ → ⑩ $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$ → ⑪ $\overline{AC}$ bis. $\angle DAB$ & $\angle BCD$

① Given
② In a rhombus, all sides $\cong$.
③ Reflexive property
④ SSS $\cong$ SSS
⑤ CPCTC
⑥ Def. $\angle$ bisector
⑦ same as #2
⑧ Reflexive prop.
⑨ SSS $\cong$ SSS
⑩ CPCTC
⑪ Def $\angle$ bisector