Geometry (H)
Section 5.5 – Trapezoids

Name: KEY

Use the notes from section 5.5 to help you solve the following problems.

1. Draw the trapezoid described. If such a trapezoid cannot be drawn, explain why not.

   a. with two right angles

   b. with both bases shorter than the legs

   yes

   c. with congruent bases

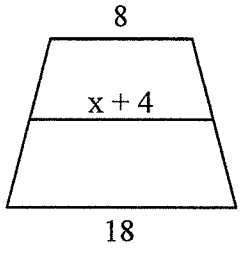
   Not possible; if bases are $\cong$, then legs will be $\cong$; then not trap.

   d. with three acute angles

   Not possible; can draw a quad w/ 3 acute $\angle$s but both pairs opp sides not parallel.

2. Each diagram shows a trapezoid and its median. Find the value of x.

   a.

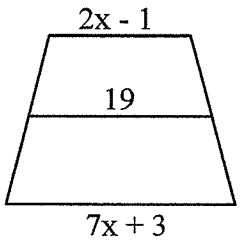
   

   $x+4 = \frac{1}{2}(8+18)$

   $x+4 = 13$

   $x = 9$

   b.

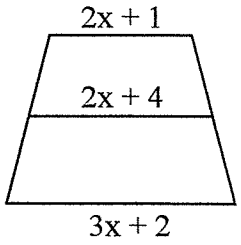
   $2(19) = 2x-1+7x+3$

   $38 = 9x+2$

   $36 = 9x$

   $x = 4$

   c.

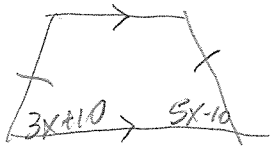
   

   $2x+4 = \frac{1}{2}(2x+1+3x+2)$

   $4x+8 = 5x+3$

   $5 = x$

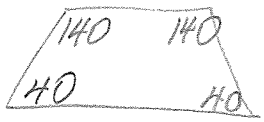
3. Two congruent angles of an isosceles trapezoid have measures 3x + 10 and 5x – 10. Find the value of x and then give the measures of all angles of the trapezoid.

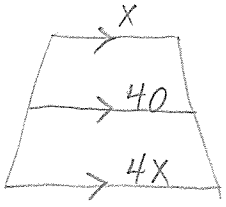
   $3x+10 = 5x-10$

   $20 = 2x$

   $10 = x$

4. The length of one base of a trapezoid is four times the length of the other base. If the length of the median is 40, find the length of each base.

   $40 = \frac{1}{2}(x+4x)$

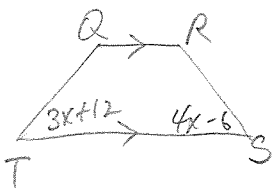
   $80 = 5x$

   $16 = x$

   base 1 = 16

   base 2 = 64

5. In isosceles trap QRST with $\overline{QR} \parallel \overline{TS}$, m∠T = 3x + 12, m∠S = 4x – 6. Find m∠Q.



$3x+12 = 4x-6$

$18 = x$

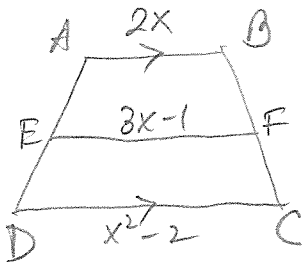
$3x+12 =$

$3(18)+12 = 66$

$180-66 = 114$

$m\angle Q = 114$

6. In trap ABCD with $\overline{AB} \parallel \overline{CD}$, and median $\overline{EF}$, AB = 2x, EF = 3x -1, DC = x^2 -2. Find the length of the median.



$3x-1 = \frac{1}{2}(x^2-2+2x)$

$6x-2 = x^2+2x-2$

$0 = x^2-4x$

$x(x-4)$

$x=0$ OMIT | $x-4=0$, $x=4$

$AB = 8$

$EF = 11$

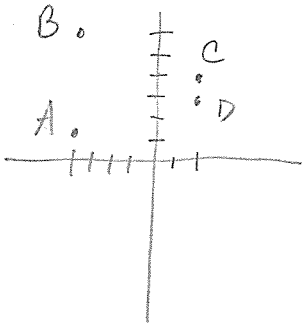
$DC = 14$

ck

$11 \overset{\checkmark}{=} \frac{1}{2}(8+14)$

7. Determine if the quadrilateral is a trapezoid. If it is, determine if it is an isosceles trapezoid.

A(-4,1). B(-4,6), C(2,4), D(2,3)



$m_{\overline{AB}} = \frac{6-1}{-4+4} = \frac{5}{0}$ = undefined

$m_{\overline{CD}} = \frac{4-3}{2-2} = \frac{1}{0}$ = undefined

} $\overline{AB} \parallel \overline{CD}$

$m_{\overline{BC}} = \frac{6-4}{-4-2} = \frac{2}{-6} = -\frac{1}{3}$

$m_{\overline{AD}} = \frac{1-3}{-4-2} = \frac{-2}{-6} = \frac{1}{3}$

} $\overline{AD} \not\parallel \overline{BC}$

$d_{\overline{BC}} = \sqrt{(-4-2)^2+(6-4)^2}$

$= \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

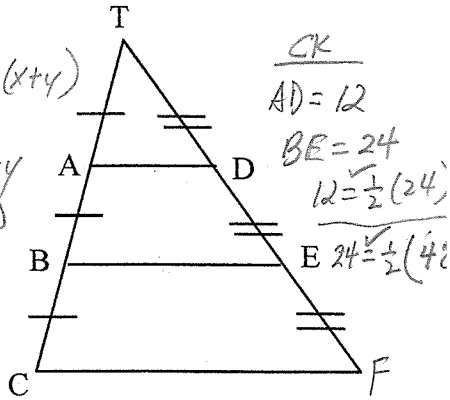
$d_{\overline{AD}} = \sqrt{(-4-2)^2+(1-3)^2}$

$= \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

$\overline{AD} \cong \overline{BC}$

ABCD is an isosceles trapezoid.

8. Use the diagram at the right to solve the following problems.



a. If AD = x + 3, BE = x + y, and CF = 36, find x and y.

$AD = \frac{1}{2}BE$

$x+3 = \frac{1}{2}(x+y)$

$2x+6 = x+y$

$x-y = -6$

$BE = \frac{1}{2}(AD+CF)$

$x+y = \frac{1}{2}(x+3+36)$

$2x+2y = x+39$

$x+2y = 39$

$x-y = -6$

$2x-2y = -12$

$x+2y = 39$

$3x = 27$

$x = 9$

$x+3 = \frac{1}{2}(x+y)$

$9+3 =$

$24 = 9+y$

$15 = y$

ck

$AD = 12$

$BE = 24$

$12 \overset{\checkmark}{=} \frac{1}{2}(24)$

$24 \overset{\checkmark}{=} \frac{1}{2}(48)$

b. If AD = x + y, BE = 20 and CF = 4x – y, then find CF.

$AD = \frac{1}{2}BE$

$x+y = \frac{1}{2}(20)$

$x+y = 10$

$y = 2$

$BE = \frac{1}{2}(AD+CF)$

$20 = \frac{1}{2}(x+y+4x-y)$

$40 = 5x$

$8 = x$

$CF = 32-2$

$CF = 30$

$AD = 10$

ck

$10 = \frac{1}{2}(20)$

$10 = 10$

$20 = \frac{1}{2}(10+30)$

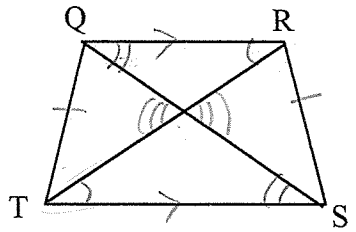
$20 = 20$

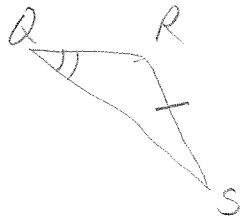
9. Write a flow proof for the following theorem.

The diagonals of an isosceles trapezoid are congruent.

Given: Isos trap TQRS with $\overline{QR} \parallel \overline{TS}$

Prove: $\overline{TR} \cong \overline{QS}$ (Prove $\triangle TQR \cong \triangle SRQ$ or $\triangle QTS \cong \triangle RST$)





① Isos trap TQRS w/ $\overline{QR} \parallel \overline{TS}$ → ② $\overline{QT} \cong \overline{RS}$

→ ③ $\measuredangle TQR \cong \measuredangle SRQ$

④ $\overline{QR} \cong \overline{QR}$

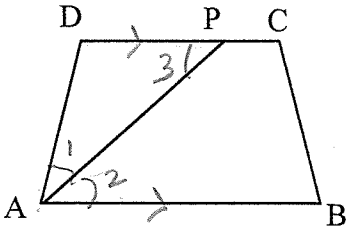
→ ⑤ $\triangle TQR \cong \triangle SRQ$ → ⑥ $\overline{TR} \cong \overline{QS}$

① Given
② In isos trap, legs are $\cong$.
③ In isos trap, base $\measuredangle$s $\cong$.
④ Reflexive property
⑤ SAS $\cong$ SAS
⑥ CPCTC

10. Write a flow proof for each the following.

a. Given: Trap ABCD with bases $\overline{AB}$ and $\overline{CD}$
$\overline{AP}$ bisects $\angle A$

Prove: $\triangle APD$ is isosceles



① Trap ABCD w/ bases $\overline{AB}$ & $\overline{CD}$ → ② $\overline{AB} \parallel \overline{CD}$ → ③ $\measuredangle 3 \cong \measuredangle 2$

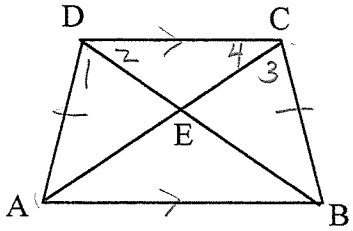
④ $\overline{AP}$ bisects $\measuredangle A$ → ⑤ $\measuredangle 1 \cong \measuredangle 2$

→ ⑥ $\measuredangle 1 \cong \measuredangle 3$ → ⑦ $\overline{AD} \cong \overline{DP}$ → ⑧ $\triangle APD$ is isosceles.

① Given
② In a trap, bases $\cong$.
③ 2 $\parallel$ lines → alt. int. $\measuredangle$s $\cong$
④ Given
⑤ Def of $\measuredangle$ bisector
⑥ Transitive Prop.
⑦ In a $\triangle$, if 2 $\measuredangle$s $\cong$, then sides opp $\cong$.
⑧ Def of isos. $\triangle$.

b. Given: Isos. Trap ABCD with bases $\overline{AB}$ and $\overline{CD}$

Prove: ΔCDE is isosceles



① Isos. trap. ABCD w/ bases $\overline{AB}$ & $\overline{CD}$. → ② $\overline{AD} \cong \overline{BC}$; ③ $\overline{BD} \cong \overline{AC}$; ④ $\overline{AB} \cong \overline{AB}$ → ⑤ $\triangle ADB \cong \triangle BCA$ → ⑥ $\angle 1 \cong \angle 3$

① Isos. trap. ABCD → ⑦ $\angle ADC \cong \angle BCD$; ⑧ $\angle ADC \cong \angle 1 + \angle 2$, $\angle BCD \cong \angle 3 + \angle 4$ → ⑨ $\angle 1 + \angle 2 \cong \angle 3 + \angle 4$

⑥ and ⑨ → ⑩ $\angle 2 \cong \angle 4$

⑪ $\overline{DE} \cong \overline{CE}$

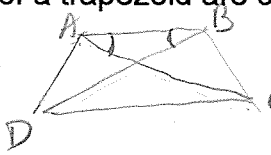
⑫ ΔCDE is isosceles.

① Given
② In an isos. trap, legs ≅.
③ In an isos. trap, diagonals ≅.
④ Reflexive Prop.
⑤ SSS ≅ SSS
⑥ CPCTC
⑦ In isos trap, base ∠s ≅.
⑧ Angle Addition Prop.
⑨ Substitution
⑩ Subtraction Postulate
⑪ If 2 ∠s ≅, sides opp ≅
⑫ Def. of isos. Δ.

11. Determine if the following statements are sometimes, always or never true. Justify your answer.

a. If the diagonals of a trapezoid are congruent, then the trapezoid is isosceles.

Always True



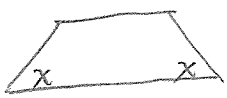
If can be proven that $\triangle ABC \cong \triangle BAD$ by SAS. Therefore, legs ≅ by CPCTC.

b. If a quadrilateral has more than two right angles, then it is not a trapezoid.

Always true. If a quad has 3 right ∠s, the 4th ∠ will be 90. This would make it a rectangle.

c. The opposite angles of an isosceles trapezoid are supplementary.

Always true



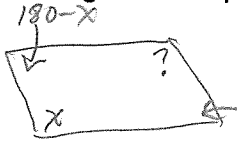
If base ∠ = x, then the 2 base ∠s = 2x. The top 2 ∠s = 360 − 2x. If (360−2x) ÷ 2, each opp. ∠ = 180 − x.

d. The diagonals of an isosceles trapezoid bisect each other.

Never

e. If the consecutive angles of a quadrilateral are supplementary, it is a trapezoid.

Never. Will be a ▱.



$360 - [2(180 - x) + x] = ?$

$360 - (360 - x) = ?$

$360 - 360 + x = ?$ $\quad ? = x$