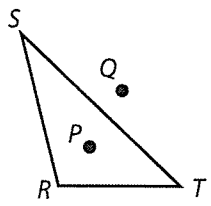
## Elaborate

**10.** $P$ and $Q$ are the circumcenter and incenter of $\triangle RST$, but not necessarily in that order. Which point is the circumcenter? Which point is the incenter? Explain how you can tell without constructing any bisectors.




_____

_____

**11.** Complete the table by filling in the blanks to make each statement true.

| | Circumcenter | Incenter |
|---|---|---|
| Definition | The point of concurrency of the _____ | The point of concurrency of the _____ |
| Distance | Equidistant from the _____ | Equidistant from the _____ |
| Location (Inside, Outside, On) | Can be _____ the triangle | Always _____ the triangle |

**12.** **Essential Question Check-In** How do you know that the intersection of the bisectors of the angles of a triangle is equidistant from the sides of the triangle?

_____

_____

## Evaluate: Homework and Practice


- Online Homework
- Hints and Help
- Extra Practice

**1.** Use a compass and straightedge to investigate points on the bisector of an angle. On a separate piece of paper, draw a large angle $A$.

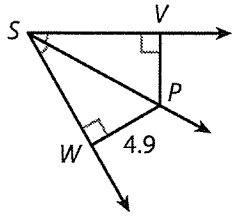
**a.** Construct the bisector of $\angle A$.

**b.** Choose a point on the angle bisector you constructed. Label it $P$. Construct a perpendicular through $P$ to each side of $\angle A$.

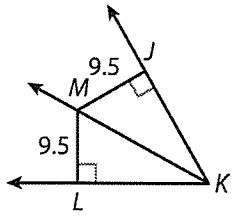
**c.** Explain how to use a compass to show that $P$ is equidistant from the sides of $\angle A$.

**Find each measure.**

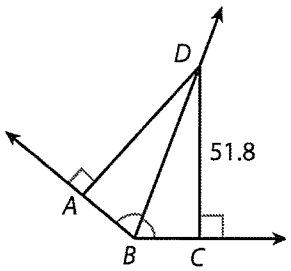
**2.** $VP$



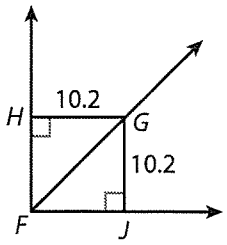

**3.** $m\angle LKM$, given that $m\angle JKL = 63°$



**4.** $AD$

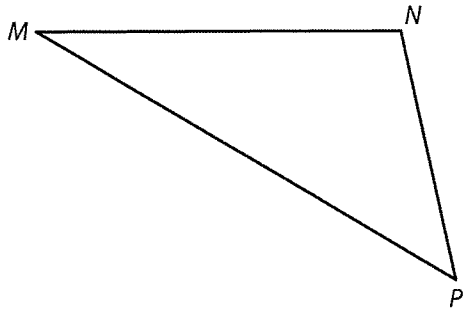



**5.** $m\angle HFJ$, given that $m\angle GFJ = 45°$

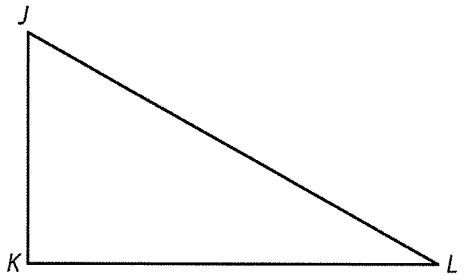



**Construct an inscribed circle for each triangle.**

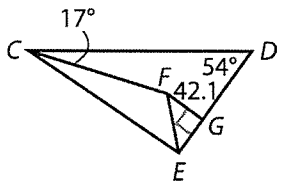
**6.**




**7.**




**$\overline{CF}$ and $\overline{EF}$ are angle bisectors of $\triangle CDE$. Find each measure.**



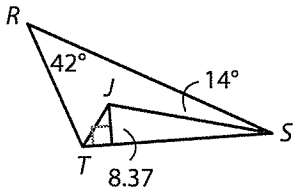

**8.** the distance from $F$ to $\overline{CD}$

**9.** $m\angle FED$

**$\overline{TJ}$ and $\overline{SJ}$ are angle bisectors of $\triangle RST$. Find each measure.**

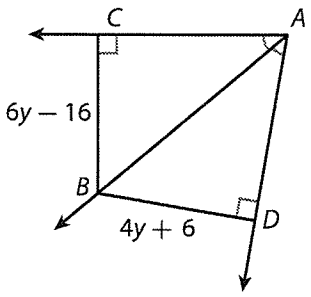
**10.** the distance from $J$ to $\overline{RS}$

**11.** $m\angle RTJ$

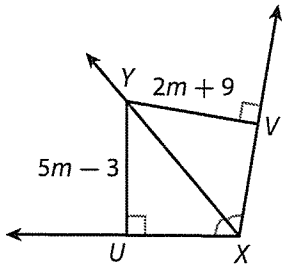


**Find each measure.**

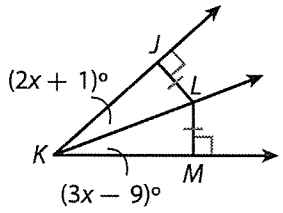
**12.** $BC$



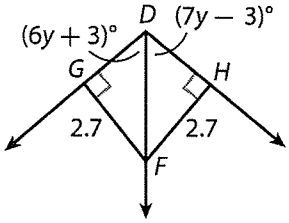

**13.** $VY$



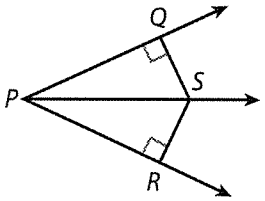

**14.** $m\angle JKL$




**15.** $m\angle GDF$




**16.** Complete the following proof of the Angle Bisector Theorem.




Given: $\overrightarrow{PS}$ bisects $\angle QPR$.

$\overline{SQ} \perp \overrightarrow{PQ}$, $\overline{SR} \perp \overrightarrow{PR}$

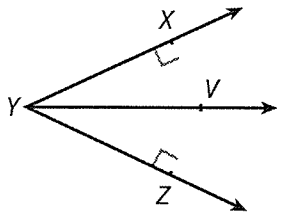
Prove: $SQ = SR$

| Statements | Reasons |
|---|---|
| 1. $\overrightarrow{PS}$ bisects $\angle QPR$. $\overline{SQ} \perp \overrightarrow{PQ}$, $\overline{SR} \perp \overrightarrow{PR}$ | 1. |
| 2. $\angle QPS \cong \angle RPS$ | 2. |
| 3. $\angle SQP$ and $\angle SRP$ are right angles. | 3. Definition of perpendicular |
| 4. $\angle SQP \cong \angle SRP$ | 4. All right angles are congruent. |
| 5. | 5. Reflexive Property of Congruence |
| 6. | 6. AAS Triangle Congruence Theorem |
| 7. $\overline{SQ} \cong \overline{SR}$ | 7. |
| 8. $SQ = SR$ | 8. Congruent segments have the same length. |

**17.** Complete the following proof of the Converse of the Angle Bisector Theorem.

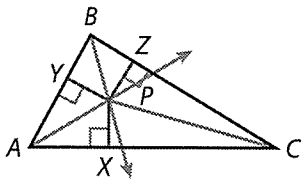
Given: $\overline{VX} \perp \overrightarrow{YX}$, $\overline{VZ} \perp \overrightarrow{YZ}$, $VX = VZ$.

Prove: $\overrightarrow{YV}$ bisects $\angle XYZ$.




| Statements | Reasons |
|---|---|
| 1. $\overline{VX} \perp \overrightarrow{YX}, \overline{VZ} \perp \overrightarrow{YZ}, VX = VZ$ | 1. |
| 2. $\angle VXY$ and $\angle VZY$ are right angles. | 2. |
| 3. $\overline{YV} \cong \overline{YV}$ | 3. |
| 4. $\triangle YXV \cong \triangle YZV$ | 4. |
| 5. $\angle XYV \cong \angle ZYV$ | 5. |
| 6. | 6. |

**18.** Complete the following proof of the Incenter Theorem.

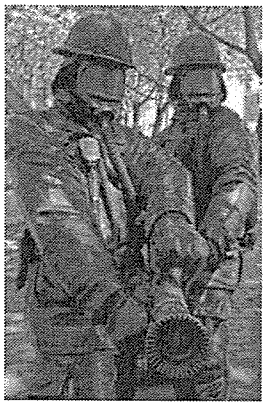


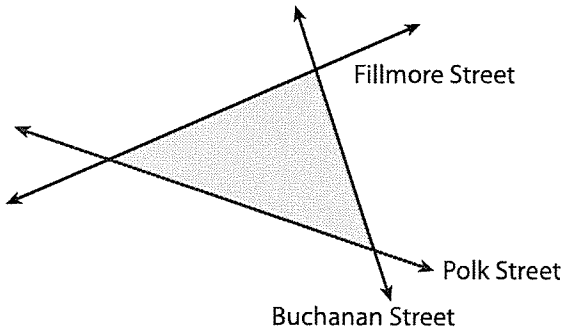

Given: $\overrightarrow{AP}$, $\overrightarrow{BP}$, and $\overrightarrow{CP}$ bisect $\angle A$, $\angle B$ and $\angle C$, respectively.
$\overline{PX} \perp \overline{AC}$, $\overline{PY} \perp \overline{AB}$, $\overline{PZ} \perp \overline{BC}$

Prove: $PX = PY = PZ$

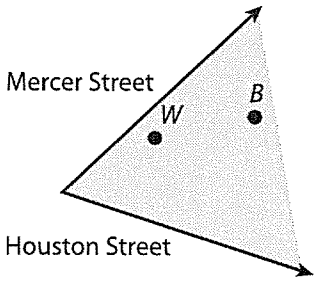
Let $P$ be the incenter of $\triangle ABC$. Since $P$ lies on the bisector of $\angle A$, $PX = PY$ by the _____ Theorem. Similarly, $P$ also _____, so $PY = PZ$. Therefore, $PX = PY = PZ$, by the _____.

**19.** A city plans to build a firefighter's monument in a triangular park between three streets. Draw a sketch on the figure to show where the city should place the monument so that it is the same distance from all three streets. Justify your sketch.

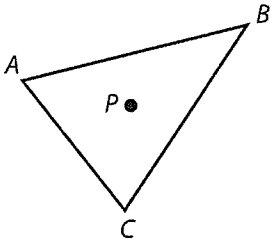




**20.** A school plans to place a flagpole on the lawn so that it is equidistant from Mercer Street and Houston Street. They also want the flagpole to be equidistant from a water fountain at $W$ and a bench at $B$. Find the point $F$ where the school should place the flagpole. Mark the point on the figure and explain your answer.




**21.** $P$ is the incenter of $\triangle ABC$. Determine whether each statement is true or false. Select the correct answer for each lettered part.




**a.** Point $P$ must lie on the perpendicular bisector of $\overline{BC}$. ○ True ○ False

**b.** Point $P$ must lie on the angle bisector of $\angle C$. ○ True ○ False

**c.** If $AP$ is 23 mm long, then $CP$ must be 23 mm long. ○ True ○ False

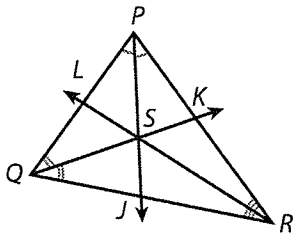
**d.** If the distance from point $P$ to $\overline{AB}$ is $x$, then the distance from point $P$ to $\overline{BC}$ must be $x$. ○ True ○ False

**e.** The perpendicular segment from point $P$ to $\overline{AC}$ is longer than the perpendicular segment from point $P$ to $\overline{BC}$. ○ True ○ False

## H.O.T. Focus on Higher Order Thinking

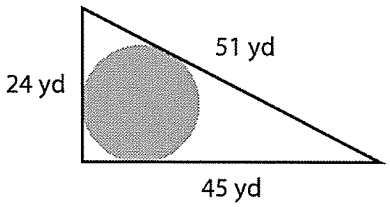
**22.** **What If?** In the Explore, you constructed the angle bisector of acute $\angle ABC$ and found that if a point is on the bisector, then it is equidistant from the sides of the angle. Would you get the same results if $\angle ABC$ were a straight angle? Explain.

**23.** **Explain the Error** A student was asked to draw the incircle for $\triangle PQR$. He constructed angle bisectors as shown. Then he drew a circle through points $J$, $K$, and $L$. Describe the student's error.




# Lesson Performance Task

Teresa has just purchased a farm with a field shaped like a right triangle. The triangle has the measurements shown in the diagram. Teresa plans to install central pivot irrigation in the field. In this type of irrigation, a circular region of land is irrigated by a long arm of sprinklers—the radius of the circle—that rotates around a central pivot point like the hands of a clock, dispensing water as it moves.




**a.** Describe how she can find where to locate the pivot.

**b.** Find the area of the irrigation circle. To find the radius, $r$, of a circle inscribed in a triangle with sides of length $a$, $b$, and $c$, you can use the formula $r = \frac{\sqrt{k(k-a)(k-b)(k-c)}}{k}$, where $k = \frac{1}{2}(a + b + c)$.

**c.** About how much of the field will *not* be irrigated?