Name KEY Class Date

8.3 Medians and Altitudes of Triangles

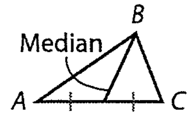
Essential Question: How can you find the balance point or *center of gravity* of a triangle?

Resource Locker

Explore Finding the Balance Point of a Triangle

If a triangle were cut out of a sheet of wood or paper, the triangle could be balanced around exactly one point inside the triangle.

A **median** of a triangle is a segment whose endpoints are a vertex of a triangle and the midpoint of the opposite side.



Every triangle has three distinct medians. You can use construction tools to show that the intersection of the three medians is the balance point of the triangle.

(A) Draw a large triangle on a sheet of construction paper. Label the vertices *A*, *B*, and *C*.

(B) Find the midpoint of the side opposite *A*, which is $\overline{BC}$. You may use a compass to find points equidistant to *B* and *C* and then draw the perpendicular bisector. Or you can use paper folding or a ruler. Write the label *X* for the midpoint.

(C) Draw a segment to connect *A* and *X*. The segment is one of the three medians of the triangle.

(D) Repeat Steps B and C, this time to draw the other two medians of the triangle. Write the label *Y* for the midpoint of the side opposite point *B*, and the label *Z* for the midpoint of the side opposite point *C*. Write the label *P* for the intersection of the three medians.

(E) Use a ruler to measure the lengths of each median and the subsegments defined by P in your triangle. Record your measurements in the table.

Median $\overline{AX}$:	$AX =$ ____	$AP =$ ____	$PX =$ ____
Median $\overline{BY}$:	$BY =$ ____	$BP =$ ____	$PY =$ ____
Median $\overline{CZ}$:	$CZ =$ ____	$CP =$ ____	$PZ =$ ____

(F) What pattern do you observe in the measurements?

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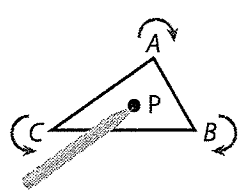
(G) Let AX be the length of any median of a triangle from a vertex A, and let P be the intersection of the three medians. Write an equation to describe the relationship between AP and PX.

(H) Let AX be the length of any median of a triangle from a vertex A, and let P be the intersection of the three medians. Write an equation to show the relationship between AX and AP.

(I) Cut out the triangle, and then punch a very small hole through P. Stick a pencil point through the hole, and then try to spin the triangle around the pencil point. How easily does it spin? Repeat this step with another point in the triangle, and compare the results.



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Reflect

1. Why is "balance point" a descriptive name for point P, the intersection of the three medians?

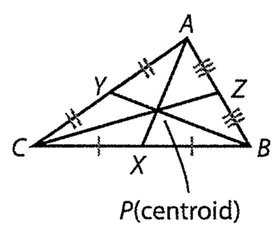
2. **Discussion** By definition, median $\overline{AX}$ intersects $\triangle ABC$ at points A and X. Could it intersect the triangle at a third point? Explain why or why not.

Explain 1 Using the Centroid Theorem

The intersection of the three medians of a triangle is the *centroid* of the triangle. The centroid is always inside the triangle and divides each median by the same ratio.

Centroid Theorem

The centroid theorem states that the **centroid** of a triangle is located $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.

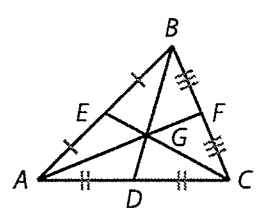


$AP = \frac{2}{3}AX$ $\quad$ $BP = \frac{2}{3}BY$ $\quad$ $CP = \frac{2}{3}CZ$

* Notice what part of the segment is $\frac{2}{3}$ of whole and what part is $\frac{1}{3}$ of whole.

Example 1 Use the Centroid Theorem to find the length.

$AF = 9$, and $CE = 7.2$



(A) AG

Centroid Theorem	$AG = \frac{2}{3}AF$
Substitute 9 for AF.	$AG = \frac{2}{3}(9)$
Simplify.	$AG = 6$

(B) GE

Centroid Theorem	$CG = \frac{2}{3}$ CE
Substitute for the given value.	$CG = \frac{2}{3}$ 7.2
Simplify.	$CG =$ 4.8
Segment Addition Postulate	$CG +$ GE $= CE$
Subtraction Property of Equality	$GE = CE -$ CG
Substitute for the value of CG.	$GE = 7.2 -$ 4.8
Simplify.	$GE =$ 2.4

Reflect

3. To find the centroid of a triangle, how many medians of the triangle must you construct?

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* **4.** Compare the lengths of $\overline{CG}$ and $\overline{GE}$ in Part B. What do you notice? (C to midpt $= \frac{1}{2}$ C to vertex)

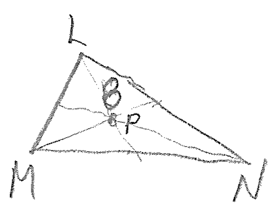
$CG = 4.8$, $GE = 2.4$ $GE = \frac{1}{2} CG$ or $CG = 2(GE)$

5. **Make a Conjecture** The three medians of $\triangle FGH$ divide the triangle into six smaller triangles. Is it possible for the six smaller triangles to be congruent to one another? If yes, under what conditions?

When the initial $\triangle$ is an equilateral, it is possible to divide into 6 smaller $\cong$ $\triangle$s.

Your Turn

6. Vertex L is 8 units from the centroid of $\triangle LMN$. Find the length of the median that has one endpoint at L. Let x = length of median



$LP = 8$

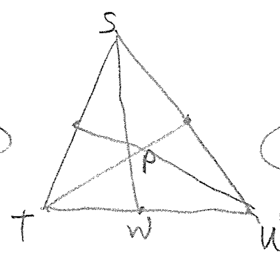
$LP = \frac{2}{3}x$

$8 = \frac{2}{3}x$

$12 = x$

median = 12

7. Let P be the centroid of $\triangle STU$, and let $\overline{SW}$ be a median of $\triangle STU$. If $SW = 18$, find SP and PW.



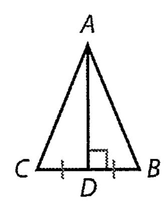
$SP = \frac{2}{3}SW$

$= \frac{2}{3}(18)$

$SP = 12$

$PW = 6$

8. In $\triangle ABC$, the median $\overline{AD}$ is perpendicular to $\overline{BC}$. If $AD = 21$ feet, describe the position of the centroid of the triangle.



Centroid $= \frac{2}{3}(21)$

$= 14$

The centroid is 14 feet above D or 7 feet from A on $\overline{AD}$.

Your Turn

Find the orthocenter for the triangles described by each set of vertices.

13. $Q(4, -3)$, $R(8, 5)$, $S(8, -8)$

14. $K(2, -2)$, $L(4, 6)$, $M(8, -2)$

Elaborate

15. Could the centroid of a triangle be coincident with the orthocenter? If so, give an example.

16. Describe or sketch an example in which the orthocenter P of $\triangle ABC$ is far away from the triangle. That is, PA, PB, and PC are each greater than the length of any side of the triangle.

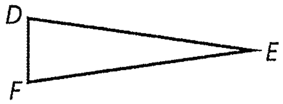
17. A sculptor is assembling triangle-shaped pieces into a mobile. Describe circumstances when the sculptor would need to identify the centroid and orthocenter of each triangle.

18. **Essential Question Check-In** How can you find the centroid, or balance point, of a triangle?

Evaluate: Homework and Practice

Use a compass and a straightedge to draw the medians and identify the centroid of the triangle. Label the centroid *P*.

1.



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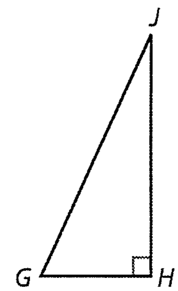
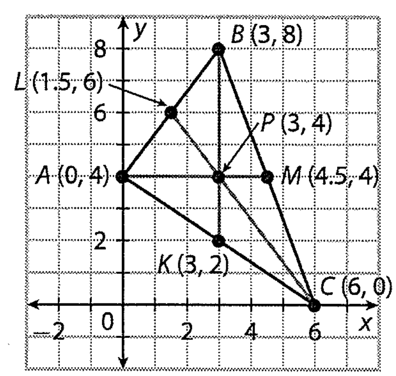


3. **Critique Reasoning** Paul draws $\triangle ABC$ and the medians from vertices A and B. He finds that the medians intersect at a point, and he labels this point X. Paul claims that point X lies outside $\triangle ABC$. Do you think this is possible? Explain.

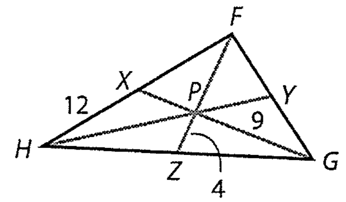
No, not possible. All medians will be inside the triangle. Therefore, where they intersect will also be inside the Δ.

4. For $\triangle ABC$ and its medians, match the segment on the left with its length.

A. *AM*	________	1.5
B. *AP*	________	2
C. *PM*	________	2.5
D. *BK*	________	3
E. *BP*	________	4
F. *PK*	________	4.5
G. *CL*	________	5
H. *CP*	________	6
I. *PL*	________	7.5



The diagram shows $\triangle FGH$, its medians, centroid P, and the lengths of some of the subsegments. Apply the Centroid Theorem to find other lengths.

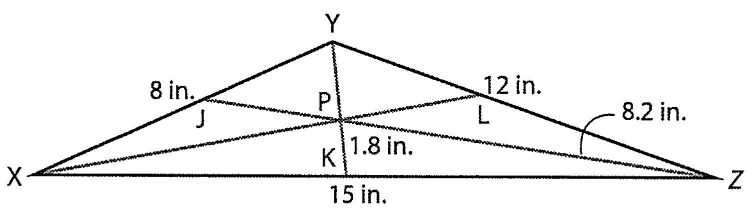


5. FH = 24
2(XH) = FH

6. PF = 8
PF = 2(PZ)
= 2(4)

7. GX = GP + ½(GP)
GX = 13.5

The diagram shows $\triangle XYZ$, which has side lengths of 8 inches, 12 inches, and 15 inches. The diagram also shows the medians, centroid P, and the lengths of some of the subsegments. Apply the Centroid Theorem to find other lengths.



8. LY = ½ ZY
LY = 6 in.

9. KY = 5.4 in.

10. ZJ = 12.3 in.

11. The diagram shows $\triangle ABC$, its medians, centroid P, and the lengths of some of the subsegments as expressions of variables x and y. Apply the Centroid Theorem to solve for the variables and to find other lengths.

a. x = 11

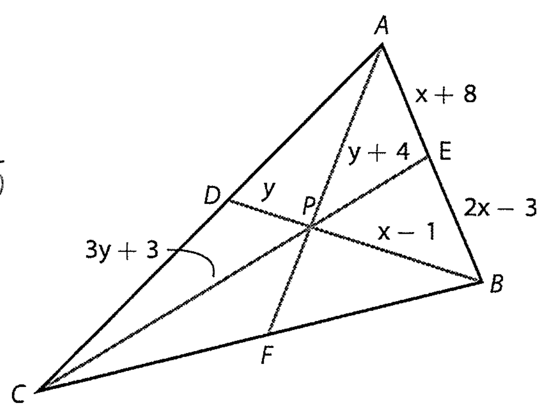
b. y = 5

c. BP = 10

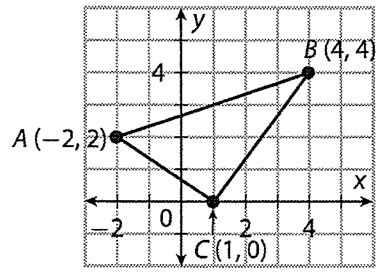
d. BD = 15

e. CP = 18

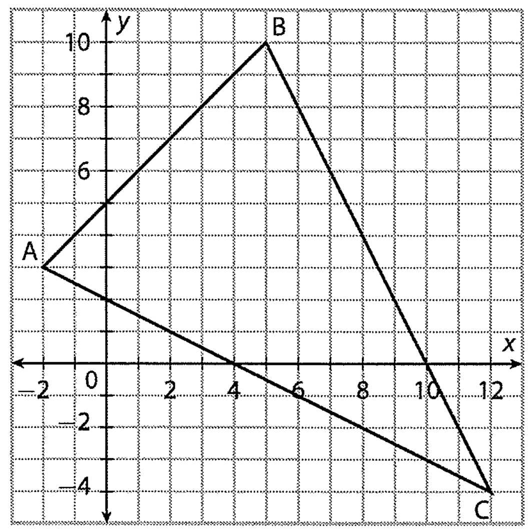
f. PE = 9



12. Draw the medians from A to $\overline{BC}$ and from C to $\overline{AB}$.



The vertices of a triangle are $A(-2, 3)$, $B(5, 10)$, and $C(12, -4)$. Find the coordinates or equations for each feature of the triangle.



13. the coordinates of the midpoint of $\overline{AC}$

14. the coordinates of the midpoint of $\overline{BC}$

15. the equation of the line that contains the median through point B

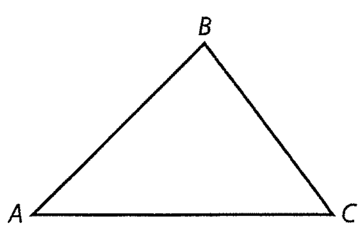
16. the equation of the line that contains the median through point A

17. the coordinates of the intersection of the two medians

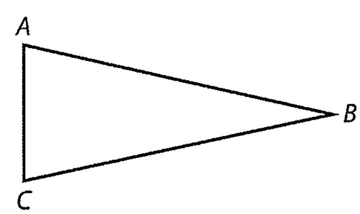
18. the coordinates of the center of balance of the triangle

For each triangle, draw the three altitudes and find the orthocenter. Label it P.

19.



20.



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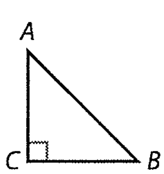

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