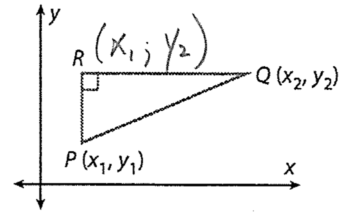
Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. Explain how to derive the Distance Formula using $\triangle PQR$.



Get coordinates of R. Write PR and RQ.
Use Pyth. Thm and Plug in PR and RQ.

$PQ^2 = PR^2 + RQ^2$

$PQ^2 = (y_2 - y_1)^2 + (x_2 - x_1)^2$

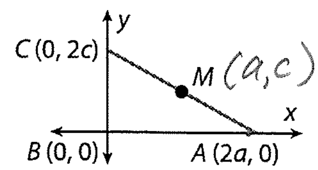
$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

take the square root on both sides.

Write each coordinate proof.

2. **Given:** $\angle B$ is a right angle in $\triangle ABC$. M is the midpoint of $\overline{AC}$.

Prove: M is equidistant from all three vertices of $\triangle ABC$.



Use the coordinates that have been assigned in the figure.

Coord. of $M = \left(\frac{0+2a}{2}, \frac{2c+0}{2}\right) = (a, c)$

Find distances of $\overline{MC}$, $\overline{MB}$, and $\overline{MA}$.

$MC = \sqrt{(a-0)^2 + (c-2c)^2} = \sqrt{a^2 + c^2}$

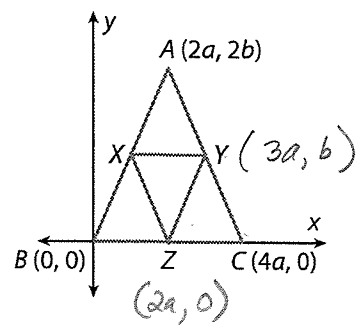
$MB = \sqrt{(a-0)^2 + (c-0)^2} = \sqrt{a^2 + c^2}$

$MA = \sqrt{(a-2a)^2 + (c-0)^2} = \sqrt{a^2 + c^2}$

Since MC, MB, and MA each equals $\sqrt{a^2+c^2}$, $MC = MB = MA$ by the transitive property of equality. Therefore, M is equidistant from A, B, and C.

3. **Given:** $\triangle ABC$ is isosceles. X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$, Z is the midpoint of $\overline{BC}$.

Prove: $\triangle XYZ$ is isosceles.



Use the coordinates that have been assigned in the figure.

midpoints:

$X = \left(\frac{2a+0}{2}, \frac{2b+0}{2}\right) = (a, b)$

$Y = \left(\frac{2a+4a}{2}, \frac{2b+0}{2}\right) = (3a, b)$

$Z = \left(\frac{0+4a}{2}, \frac{0+0}{2}\right) = (2a, 0)$

distances:

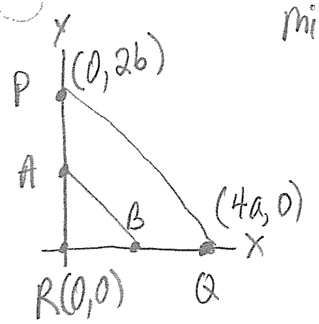
$XZ = \sqrt{(a-2a)^2 + (b-0)^2} = \sqrt{a^2+b^2}$

$YZ = \sqrt{(3a-2a)^2 + (b-0)^2} = \sqrt{a^2+b^2}$

Since $XZ = YZ$, $\overline{XZ} \cong \overline{YZ}$. Therefore, $\triangle XYZ$ is isosceles by the definition of an isos. $\triangle$ which is if a $\triangle$ has at least 2 $\cong$ sides, then the $\triangle$ is isosceles.

4. **Given:** $\angle R$ is a right angle in $\triangle PQR$. A is the midpoint of $\overline{PR}$. B is the midpoint of $\overline{QR}$.

Prove: $\overline{AB}$ is parallel to $\overline{PQ}$.



midpt $A = \left(\frac{0+0}{2}, \frac{2b+0}{2}\right) = (0, b)$

$B = \left(\frac{0+4a}{2}, \frac{0+0}{2}\right) = (2a, 0)$

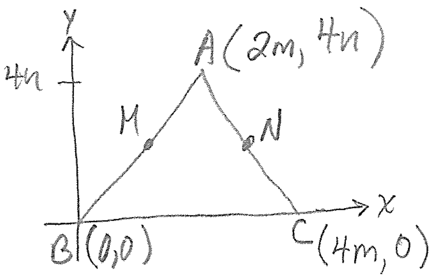
$\text{slope}_{\overline{AB}} = \frac{b-0}{0-2a} = \frac{b}{-2a}$

$\text{slope}_{\overline{PQ}} = \frac{0-2b}{4a-0} = \frac{-2b}{4a} = -\frac{b}{2a}$

Since slope of $\overline{AB}$ = slope of $\overline{PQ}$, $\overline{AB} \parallel \overline{PQ}$.

5. **Given:** $\triangle ABC$ is isosceles. M is the midpoint of $\overline{AB}$. N is the midpoint of $\overline{AC}$. $\overline{AB} \cong \overline{AC}$

Prove: $\overline{MC} \cong \overline{NB}$



midpt $M = \left(\frac{0+2m}{2}, \frac{0+4n}{2}\right) = (m, 2n)$

midpt $N = \left(\frac{2m+4m}{2}, \frac{4n+0}{2}\right) = (3m, 2n)$

$MC = \sqrt{(m-4m)^2 + (2n-0)^2} = \sqrt{9m^2 + 4n^2}$

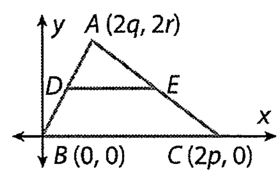
$NB = \sqrt{(3m-0)^2 + (2n-0)^2} = \sqrt{9m^2 + 4n^2}$

Since $MC = NB$, $\overline{MC} \cong \overline{NB}$

6. Prove the Triangle Midsegment Theorem using the figure shown here.

Given: $\overline{DE}$ is a midsegment of $\triangle ABC$.

Prove: $\overline{DE} \parallel \overline{BC}$ and $DE = \frac{1}{2}BC$



7. **Critique Reasoning** A student proves the Concurrency of Medians Theorem by first assigning coordinates to the vertices of $\triangle PQR$ as $P(0, 0)$, $Q(2a, 0)$, and $R(2a, 2c)$. The student says that this choice of coordinates makes the algebra in the proof a bit easier. Do you agree with the student's choice of coordinates? Explain.