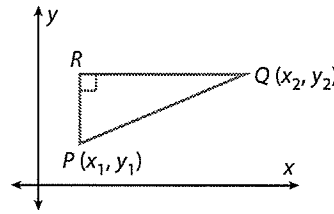
# Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

**1.** Explain how to derive the Distance Formula using $\triangle PQR$.

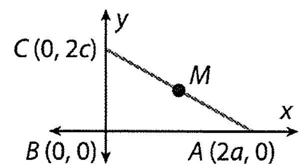



**Write each coordinate proof.**

**2.** **Given:** $\angle B$ is a right angle in $\triangle ABC$. $M$ is the midpoint of $\overline{AC}$.

**Prove:** $M$ is equidistant from all three vertices of $\triangle ABC$.

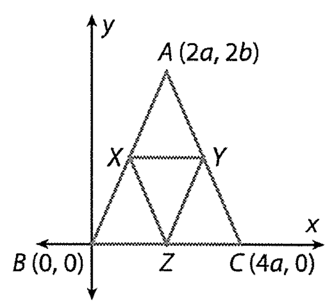
Use the coordinates that have been assigned in the figure.




**3.** **Given:** $\triangle ABC$ is isosceles. $X$ is the midpoint of $\overline{AB}$, $Y$ is the midpoint of $\overline{AC}$, $Z$ is the midpoint of $\overline{BC}$.

**Prove:** $\triangle XYZ$ is isosceles.

Use the coordinates that have been assigned in the figure.



4. **Given:** $\angle R$ is a right angle in $\triangle PQR$. $A$ is the midpoint of $\overline{PR}$. $B$ is the midpoint of $\overline{QR}$.

**Prove:** $\overline{AB}$ is parallel to $\overline{PQ}$.

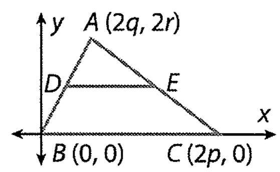
5. **Given:** $\triangle ABC$ is isosceles. $M$ is the midpoint of $\overline{AB}$. $N$ is the midpoint of $\overline{AC}$. $\overline{AB} \cong \overline{AC}$

**Prove:** $\overline{MC} \cong \overline{NB}$

6. Prove the Triangle Midsegment Theorem using the figure shown here.

**Given:** $\overline{DE}$ is a midsegment of $\triangle ABC$.

**Prove:** $\overline{DE} \parallel \overline{BC}$ and $DE = \frac{1}{2}BC$




7. **Critique Reasoning** A student proves the Concurrency of Medians Theorem by first assigning coordinates to the vertices of $\triangle PQR$ as $P(0, 0)$, $Q(2a, 0)$, and $R(2a, 2c)$. The student says that this choice of coordinates makes the algebra in the proof a bit easier. Do you agree with the student's choice of coordinates? Explain.

**Write each proof.**

**8.** **Given:** $J(-2, 2)$, $K(0, 1)$, $L(-3, -1)$, $P(4, -2)$, $Q(3, -4)$, $R(1, -1)$

**Prove:** $\angle JKL \cong \angle PQR$

**9.** **Given:** $D(-3, 2)$, $E(3, 3)$, $F(1, 1)$, $S(9, -2)$, $T(3, -1)$, $U(5, -3)$

**Prove:** $\angle FDE \cong \angle UST$

**10.** **Given:** $A(-2, 2)$, $B(4, 4)$, $M(-2, -1)$, $N(4, -3)$, $X$ is the midpoint of $\overline{AB}$, $Y$ is the midpoint of $\overline{MN}$.

**Prove:** $\angle ABY \cong \angle MNX$

**11.** **Given:** $J(-1, 4)$, $K(3, 0)$, $P(3, -6)$, $Q(-1, -2)$, $U$ is the midpoint of $\overline{JK}$, $V$ is the midpoint of $\overline{PQ}$.

**Prove:** $\angle KVJ \cong \angle QUP$

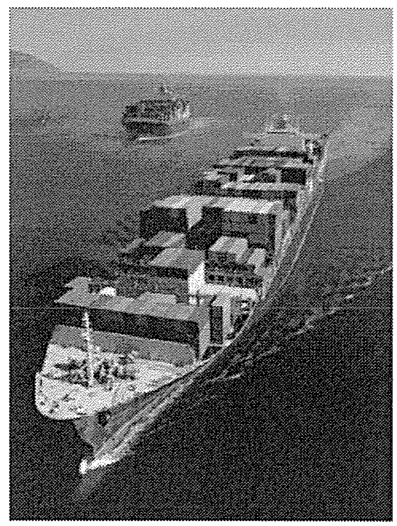
**Prove or disprove each statement.**

**12.** The triangle with vertices $R(-2, -2)$, $S(1, 4)$, and $T(4, -5)$ is an equilateral triangle.

**13.** The triangle with vertices $J(-2, 2)$, $K(2, 3)$, and $L(-1, -2)$ is an isosceles triangle.

**14.** The triangle with vertices $A(-1, 3)$, $B(2, 1)$, and $C(0, -2)$ is a scalene triangle.

**15.** Two container ships depart from a port at $P(20, 10)$. The first ship travels to a location at $A(-30, 50)$, and the second ship travels to a location at $B(70, -30)$. Each unit represents one nautical mile. Find the distance between the ships to the nearest nautical mile. Verify that the port is the midpoint between the two ships.

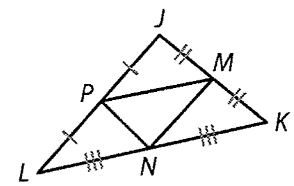


**16.** The support structure for a hammock includes a triangle whose vertices have coordinates $G(-1, 3)$, $H(-3, -2)$, and $J(1, -2)$.

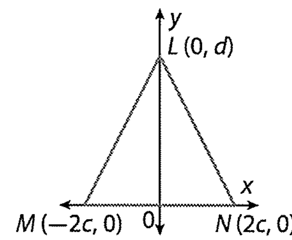
**a.** Classify the triangle and justify your answer.

**b.** Algebra Each unit of the coordinate plane represents one foot. To the nearest tenth of a foot, how much metal is needed to make one of the triangular parts for the support structure?

**17.** **Communicate Mathematical Ideas** Explain how the perimeter of $\triangle JKL$ compares to the perimeter of $\triangle MNP$.




**18.** The coordinates of the vertices of $\triangle LMN$ are shown in the figure. Determine whether each statement is true or false. Select the correct answer for each lettered part.




**a.** $\triangle LMN$ is isosceles. ○ True ○ False

**b.** One side of $\triangle LMN$ has a length of $2c$ units. ○ True ○ False

**c.** If $P$ is the midpoint of $\overline{LN}$, then $\overline{OP}$ is parallel to $\overline{LM}$. ○ True ○ False

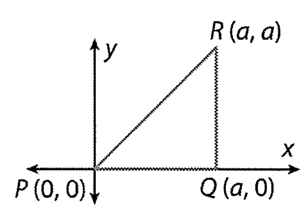
**d.** The area of $\triangle LMN$ is $4cd$ square units. ○ True ○ False

**e.** The midpoint of $\overline{MN}$ is the origin. ○ True ○ False



**H.O.T. Focus on Higher Order Thinking**

**19.** **Explain the Error** A student assigns coordinates to a right triangle as shown in the figure. Then he uses the Distance Formula to show that $PQ = a$ and $RQ = a$. Since $PQ = RQ$, the student says he has proved that every right triangle is isosceles. Explain the error in the student's proof.

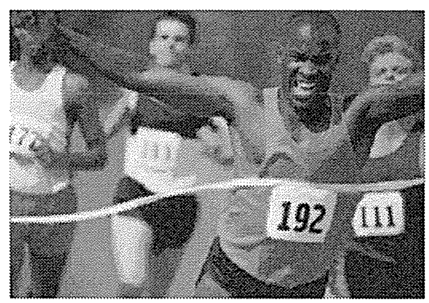


**20.** A carpenter wants to make a triangular bracket to hold up a bookshelf. The plan for the bracket shows that the vertices of the triangle are $R(-2, 2)$, $S(1, 4)$, and $T(1, -2)$. Can the carpenter conclude that the bracket is a right triangle? Explain.

**21.** **Analyze Relationships** The vertices chosen to represent an isosceles right triangle for a coordinate proof are at $(-2s, 2s)$, $(0, 2s)$, and $(0, 0)$. What other coordinates could be used so that the coordinate proof would be easier to complete? Explain.

## Lesson Performance Task

A triathlon course was mapped on a coordinate grid marked in 1-kilometer units. The starting point was (0, 0). The triathlon was broken into three stages:




- Stage 1: Contestants swim from (0, 0) to (0.6, 0.8).
- Stage 2: Contestants bicycle from the previous stopping point to (30.6, 16.8).
- Stage 3: Contestants run from the previous stopping point to (25.6, 28.8).

The winner averaged 4 kilometers per hour for Stage 1, 50 kilometers per hour for Stage 2, and 13 kilometers per hour for Stage 3. What was the winner's time for the entire race? (Assume that no time elapsed between stages.) Explain how you found the answer.

**Reflect**

7. **Discussion** Why is it more convenient to assign vertex $P$ the coordinates $(2a, 2b)$ and vertex $Q$ the coordinates $(2c, 2d)$ rather than using the coordinates $(a, b)$ and $(c, d)$?

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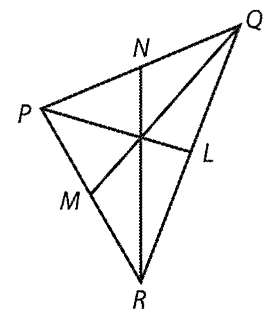
## Explain 3 Proving the Concurrency of Medians Theorem

You used the Concurrency of Medians Theorem in Module 8 and proved it in Module 9. Now you will prove the theorem again, this time using coordinate methods.

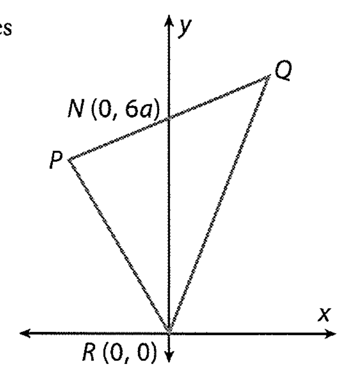
**Example 3** **Prove the Concurrency of Medians Theorem.**

**Given:** $\triangle PQR$ with medians $\overline{PL}$, $\overline{QM}$, and $\overline{RN}$

**Prove:** $\overline{PL}$, $\overline{QM}$, and $\overline{RN}$ are concurrent.




Place $\triangle PQR$ so that vertex $R$ is at the origin. Also, place the triangle so that point $N$ lies on the $y$-axis. For convenience, assign point $N$ the vertices $(0, 6a)$. (The factor of 6 will result in easier calculations later.)



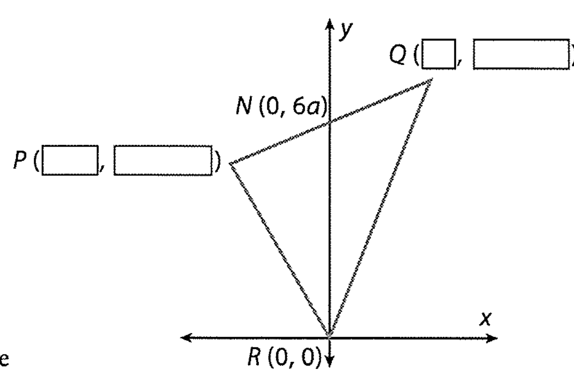

Since $N$ is the midpoint of $\overline{PQ}$, assign coordinates to $P$ and $Q$ as follows.

The horizontal distance from $N$ to $P$ must be the same as the horizontal distance from $N$ to $Q$. Let this distance be $2b$.

Then the $x$-coordinate of point $P$ is $-2b$ and the $x$-coordinate of point $Q$ is ______.

The vertical distance from $N$ to $P$ must be the same as the vertical distance from $N$ to $Q$. Let this distance be $2c$.

Then the $y$-coordinate of point $P$ is $6a - 2c$ and the $y$-coordinate of point $Q$ is ______.




Complete the figure by writing the coordinates of points $P$ and $Q$.

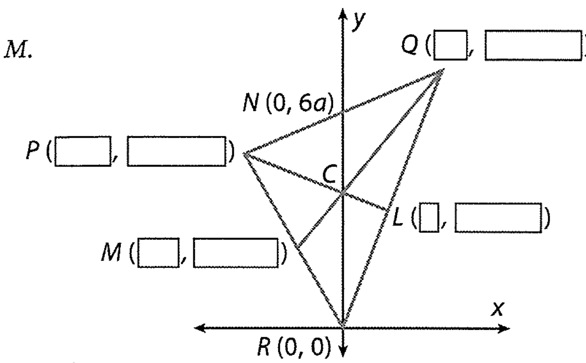
Now use the Midpoint Formula to find the coordinates of $L$ and $M$.

The midpoint of $\overline{RQ}$ is $L\left(\frac{\square + \square}{2}, \frac{\square + \square}{2}\right) = L\left(\square, \square\right)$.

The midpoint of $\overline{RP}$ is $M\left(\frac{\square + \square}{2}, \frac{\square + \square}{2}\right) = M\left(\square, \square\right)$.

Complete the figure by writing the coordinates of points $L$ and $M$.

To complete the proof, write the equation of $\overleftrightarrow{QM}$ and use the equation to find the coordinates of point $C$, which is the intersection of the medians $\overline{QM}$ and $\overline{RN}$. Then show that point $C$ lies on $\overleftrightarrow{PL}$ .




Write the equation of $\overleftrightarrow{QM}$ using point-slope form.

The slope of $\overleftrightarrow{QM}$ is $\frac{(6a + 2c) - (3a - c)}{2b - (-b)} = \frac{3\square + 3\square}{3\square} = \frac{\square + \square}{\square}$.

Use the coordinates of point $Q$ for the point on $\overleftrightarrow{QM}$ .

Therefore, the equation of $\overleftrightarrow{QM}$ is $y - \square = \frac{\square + \square}{\square} \cdot \left(x - \square\right)$.

Since point $C$ lies on the $y$-axis, the $x$-coordinate of point $C$ is 0. To find the $y$-coordinate of $C$, substitute $x = 0$ in the equation of $\overleftrightarrow{QM}$ and solve for $y$.

Substitute $x = 0$. $\quad y - \square = \frac{\square + \square}{\square} \cdot \left(0 - \square\right)$

Simplify the right side of the equation. $\quad y - \square = -2\square$

Distributive property $\quad y - \square = -2\square - 2\square$

Add $6a + 2c$ to each side and simplify. $\quad y = \square$

So, the coordinates of point $C$ are $C\left(\square, \square\right)$.

Now write the equation of $\overleftrightarrow{PL}$ using point-slope form.

The slope of $\overleftrightarrow{PL}$ is $\frac{(6a - 2c) - (3a + c)}{-2b - b} = \frac{3\square - 3\square}{-3\square} = \frac{\square - \square}{-\square}$.

Use the coordinates of point $P$ for the point on $\overleftrightarrow{PL}$ .

Therefore, the equation of $\overleftrightarrow{PL}$ is $y - \square = \frac{\square - \square}{-\square} \cdot \left(x + \square\right)$.