Name Notes Class Date

10.5 Perimeter and Area on the Coordinate Plane

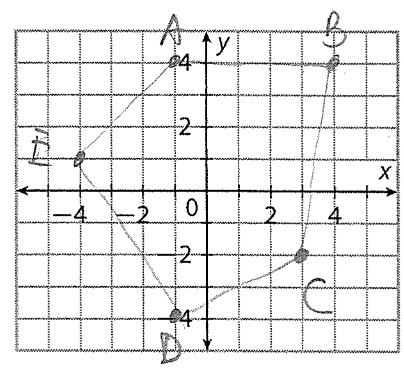
Resource Locker

Essential Question: How do you find the perimeter and area of polygons in the coordinate plane?

Explore Finding Perimeters of Figures on the Coordinate Plane

Recall that the perimeter of a polygon is the sum of the lengths of the polygon's sides. You can use the Distance Formula to find perimeters of polygons in a coordinate plane.

Follow these steps to find the perimeter of a pentagon with vertices $A(-1, 4)$, $B(4, 4)$, $C(3, -2)$, $D(-1, -4)$, and $E(-4, 1)$. Round to the nearest tenth.



(A) Plot the points. Then use a straightedge to draw the pentagon that is determined by the points.

(B) Are there any sides for which you do not need to use the Distance Formula? Explain, and give their length(s). For $\overline{AB}$

length = 5

(C) Use the Distance Formula to find the remaining side lengths. Round your answers to the nearest tenth.

(D) Find the sum of the side lengths.

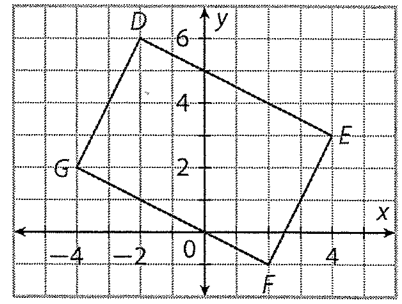
P= 25.6 units

Reflect

1. Explain how you can find the perimeter of a rectangle to check that your answer is reasonable.

(B) **Step 1** Find the coordinates of the vertices of *DEFG*.

$D(-2, 6)$, $E(4, 3)$, $F(2, -1)$, $G(-4, 2)$



Step 2 *DEFG* appears to be a rectangle. Use slopes to check that adjacent sides are perpendicular.

slope of $\overline{DE}$: $\dfrac{\boxed{3} - \boxed{6}}{4 - (-2)} = \dfrac{\boxed{-3}}{6} = \boxed{-\frac{1}{2}}$; slope of $\overline{EF}$: $\dfrac{\boxed{-1} - 3}{2 - \boxed{4}} = \dfrac{\boxed{-4}}{\boxed{-2}} = \boxed{2}$

slope of $\overline{FG}$: $\dfrac{2 - \boxed{-1}}{-4 - \boxed{2}} = \dfrac{\boxed{3}}{\boxed{-6}} = \boxed{-\frac{1}{2}}$; slope of $\overline{DG}$: $\dfrac{2 - \boxed{6}}{\boxed{-4} - \boxed{-2}} = \dfrac{\boxed{-4}}{\boxed{-2}} = \boxed{2}$

so *DEFG* is a rectangle (A quad. with 4 right ∡s.)

Step 3 Find the area of *DEFG*. 36 + 9

$b = FG = \sqrt{\left(2 - \boxed{-4}\right)^2 + \left(\boxed{-1} - 2\right)^2} = \sqrt{\boxed{45}} = \boxed{3}\sqrt{\boxed{5}}$

4 16

$h = GD = \sqrt{\left(\boxed{-2} - (-4)\right)^2 + \left(6 - \boxed{2}\right)^2} = \sqrt{\boxed{20}} = \boxed{2}\sqrt{\boxed{5}}$

Area of *DEFG*: $A = bh = \left(\boxed{3}\sqrt{\boxed{5}}\right)\left(\boxed{2}\sqrt{\boxed{5}}\right) = \boxed{30}$ square units

Reflect

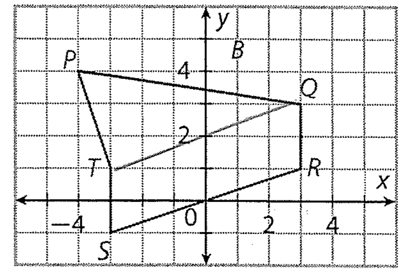
2. In Part A, is it possible to use another side of $\triangle ABC$ as the base? If so, what length represents the height of the triangle?

Yes.

3. **Discussion** In Part B, why was it necessary to find the slopes of the sides?

To confirm that the 2 adjacent sides are perpendicular – Since we are using them for base and height.

B $PQRST$ can be divided into a parallelogram and a triangle.



P(−4, 4) R(3, 1)
T(−3, 1) S(−3, −1)
Q(3, 3)

$\triangle PQT$ appears to be a right triangle. Check that $\overline{PT}$ and $\boxed{\overline{TQ}}$ are perpendicular:

slope of $\overline{PT}$: $\dfrac{1-\boxed{4}}{-3-\boxed{-4}} = \dfrac{\boxed{-3}}{\boxed{1}} = \boxed{-3}$

slope of $\boxed{\overline{TQ}}$: $\dfrac{\boxed{1}-3}{-3-\boxed{3}} = \dfrac{\boxed{-2}}{\boxed{-6}} = \boxed{\tfrac{1}{3}}$

$\triangle PQT$ is a right triangle with base $\overline{PT}$ and height $\boxed{\overline{TQ}}$.

$$PT = \sqrt{\left(-3-\boxed{-4}\right)^2 + \left(1-\boxed{4}\right)^2} = \sqrt{\boxed{10}}$$

$$\boxed{TQ} = \sqrt{\left(-3-\boxed{3}\right)^2 + \left(\boxed{1}-3\right)^2} = \sqrt{\boxed{40}} = \boxed{2}\sqrt{\boxed{10}}$$

area of $\triangle PQT$: $A = \frac{1}{2}bh = \frac{1}{2}\left(\sqrt{\boxed{10}}\right)\left(\boxed{2}\sqrt{\boxed{10}}\right) = \boxed{10}$

$\overline{QR} \parallel \overline{TS}$ since both sides are vertical.

slope of $\boxed{\overline{RS}} = \dfrac{\boxed{}-1}{-3-\boxed{}} = \dfrac{\boxed{}}{\boxed{}} = \boxed{\tfrac{1}{3}}$, so $\overline{QT} \parallel \boxed{\overline{RS}}$. Therefore, $QRST$ is a parallelogram.

$\overline{RT}$ is a~~n~~ diagonal of ~~△~~ parallelogram $QRST$ and is horizontal. Because $\overline{RT} \perp \overline{RQ}$, $\triangle QRT$ is a right triangle with base $\boxed{\overline{RT}}$ and height $\boxed{\overline{QR}}$. Therefore, the area of $\triangle QRST = 2 \cdot (\text{area of } \triangle QRT)$.

$RT = \boxed{6}$, $QR = \boxed{2}$, so the area of $\triangle QRT = \frac{1}{2}\left(\boxed{6}\right)\left(\boxed{2}\right) = 6$.

$\triangle QRST = 2 \cdot (\text{area of } QRT) = 2 \cdot \boxed{6} = 12$

area of $PQRST$: $A = \boxed{10} + \boxed{12} = \boxed{22}$ square units

Reflect

5. Discussion How could you use subtraction to find the area of a figure on the coordinate plane?

For some figures, you can draw a rectangle around the figure and create triangles around the figure. Then find area of the rectangle and subtract areas of △s from it.

Analyze Information

Students do this to see another way.

Identify the important information.

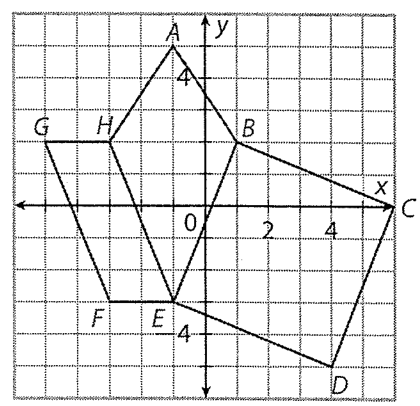
- The vertices are $A(\boxed{-1}, 5)$, $B(1, \boxed{2})$, $C(6, \boxed{0})$, $D(4, \boxed{-5})$, $E(-1, -3)$, $F(\boxed{-3}, -3)$, $G(-5, \boxed{2})$, and $H(\boxed{-3}, 2)$.
- The cost of turf is $\boxed{3.25}$ per square yard.
- The cost of the ornamental stones is $\boxed{7.95}$ per yard.

Formulate a Plan

- Divide the garden up into ________________.
- Add up the _______ of the smaller figures.
- Find the cost of turf by ______________ the total area by the cost per square yard.
- Find the perimeter of the garden by adding the _________ of the sides.
- Find the cost of the border by ______________ the perimeter by the cost per yard.
- Find total cost by adding the __________________ and ____________________.

Solve

Divide the garden into smaller figures.
The garden can be divided into square *BCDE*, kite *ABEH*, and parallelogram *EFGH*.



Find the area of each smaller figure.

area of *BCDE*:

slope of $\overline{BC}$: $\dfrac{\boxed{} - 2}{6 - \boxed{}} = \boxed{-\frac{2}{3}}$

slope of $\boxed{}$: $\dfrac{\boxed{} - 0}{4 - \boxed{}} = \boxed{\frac{5}{2}}$

Also, $BC = \sqrt{\left(\boxed{} - 1\right)^2 + \left(0 - \boxed{}\right)^2} = \sqrt{\boxed{29}}$ and

$CD = \sqrt{\left(4 - \boxed{}\right)^2 + \left(\boxed{} - \boxed{}\right)^2} = \sqrt{\boxed{29}}$.

So *BCDE* is a square, with area $A = s^2 = \left(\sqrt{\boxed{29}}\text{ yd}\right)^2 = \boxed{29}\text{ yd}^2$.

Justify and Evaluate

The area can be checked by subtraction:

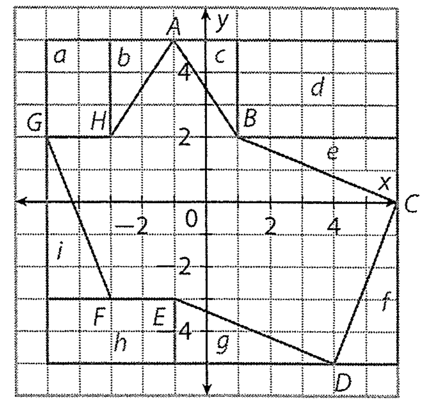
area of large rectangle = (11)(10) = 110 square units

$$\text{area} = (11)(\square) - (\square)(3) - \tfrac{1}{2}(2)(\square) - \tfrac{1}{2}(\square)(\square)$$

$$-(5)(\square) - \tfrac{1}{2}(\square)(2) - \tfrac{1}{2}(\square)(5)$$

$$-\tfrac{1}{2}(\square)(\square) - (\square)(\square) - \tfrac{1}{2}(\square)(\square)$$

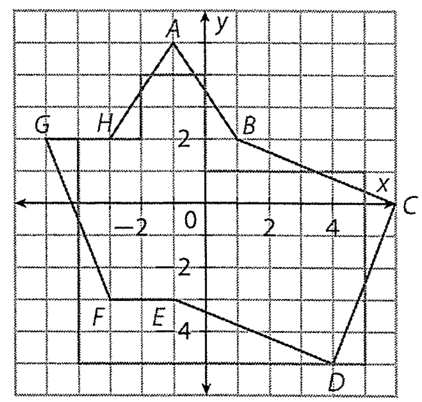
$$= \square - \square - \square - \square - \square - \square - \square - \square - \square - \square = \boxed{55}$$



The perimeter is approximately the perimeter of the polygon shown:

The perimeter of the polygon shown is $\boxed{36}$,

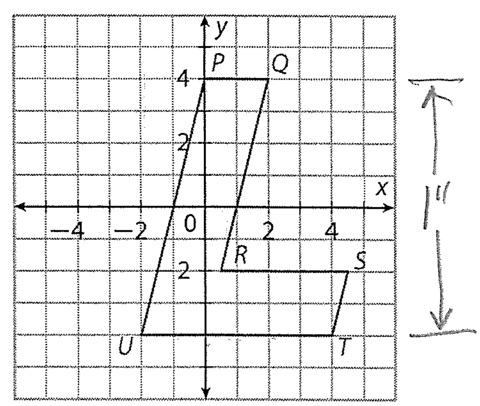
so the answer is reasonable.



Your Turn

8. A designer is making a medallion in the shape of the letter "L." Each unit on the coordinate grid represents an eighth of an inch, and the medallion is to be cut from a 1-in. square of metal. How much metal is wasted to make each medallion? Write your answer as a decimal.

use S(4.5, −2) and R(.5, −2)



Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Find the perimeter of the figure with the given vertices. Round to the nearest tenth.

1. $D(0, 1)$, $E(5, 4)$, and $F(2, 6)$

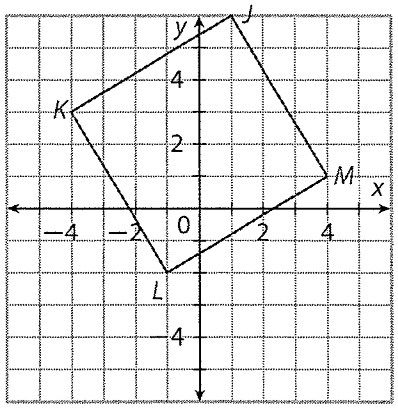
2. $P(2, 5)$, $Q(-3, 0)$, $R(2, -5)$, and $S(6, 0)$

3. $M(-3, 4)$, $N(1, 4)$, $P(4, 2)$, $Q(4, -1)$, and $R(2, 2)$

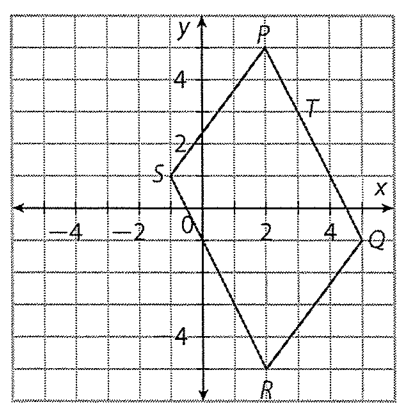
4. $A(-5, 1)$, $B(0, 3)$, $C(5, 1)$, $D(4, -2)$, $E(0, -4)$, and $F(-2, -4)$

Find the area of each figure.

5.

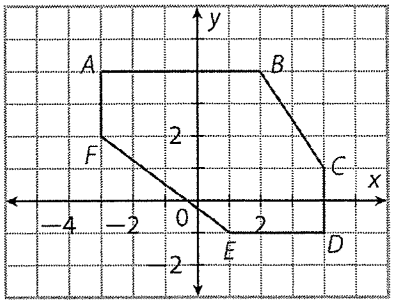


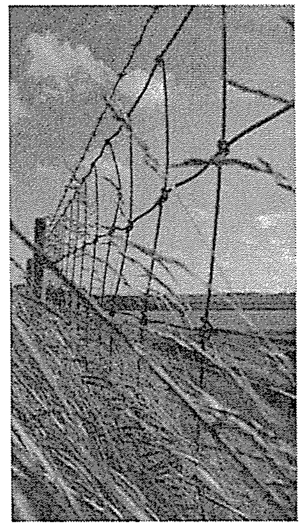
6.



11. Fencing costs \$1.45 per yard, and each unit on the grid represents 50 yd. How much will it cost to fence the plot of land represented by the polygon $ABCDEF$?

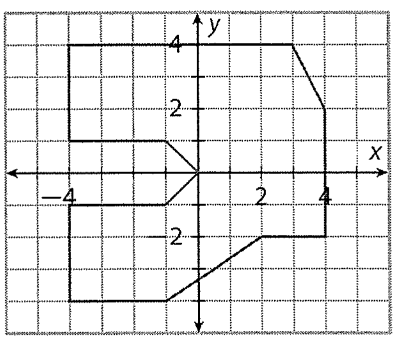
perimeter: 20.61
length of fencing: 1030.5 yds.
cost of fencing: \$1,494.23





12. A machine component has a geometric shaped plate, represented on the coordinate grid. Each unit on the grid represents 1 cm. Each plate is punched from an 8-cm square of alloy. The cost of the alloy is \$0.43/cm^2, but \$0.28/cm^2 can be recovered on wasted scraps of alloy. What is the net cost of alloy for each component?

Net cost: \$23.32



13. $\triangle ABC$ with vertices $A(1, 1)$ and $B(3, 5)$ has an area of 10 units^2. What is the location of the third vertex? Select all that apply.

B, C, D

A. $C(-5, 5)$

B. $C(3, -5)$

C. $C(-2, 5)$

D. $C(6, 1)$

E. $C(3, -3)$

H.O.T. Focus on Higher Order Thinking

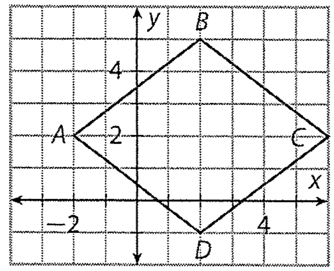
18. Explain the Error Wendell is trying to prove that $ABCD$ is a rhombus and to find its area. Identify and correct his error. (*Hint:* A rhombus is a quadrilateral with four congruent sides.)

$$AB = \sqrt{\left(2-(-2)\right)^2 + (5-2)^2} = \sqrt{25} = 5,$$

$$BC = \sqrt{(6-2)^2 + (2-5)^2} = \sqrt{25} = 5$$

$$CD = \sqrt{(2-6)^2 + (-1-2)^2} = \sqrt{25} = 5,$$

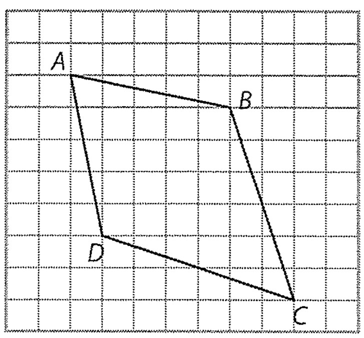
$$AD = \sqrt{\left(2-(-2)\right)^2 + \left(-1-(2)\right)^2} = \sqrt{25} = 5$$



So $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$, and therefore $ABCD$ is a rhombus.

area of $ABCD$: $b = AB = 5$ and $h = BC = 5$, so $A = bh = (5)(5) = 25$

19. Communicate Mathematical Ideas Using the figure, prove that the area of a kite is half the product of its diagonals. (Do not make numerical calculations.)



20. Justify Reasoning Use the Trapezoid Midsegment Theorem to show that the area of a trapezoid is the product of its midsegment and its height.