Name KEY/NOTES Class ________ Date ________

6.1 Justifying Constructions

Essential Question: How can you be sure that the result of a construction is valid?

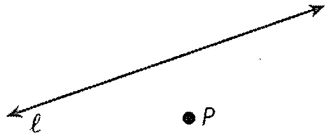
Resource Locker

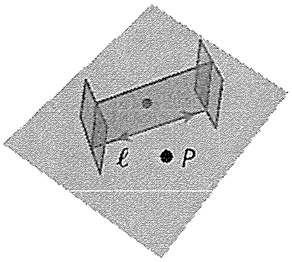
Explore 1 Using a Reflective Device to Construct a Perpendicular Line

You have constructed a line perpendicular to a given line through a point not on the line using a compass and straightedge. You can also use a reflective device to construct perpendicular lines.

Ⓐ **Step 1** Place the reflective device along line ℓ. Look through the device to locate the image of point P on the opposite side of line ℓ. Draw the image of point P and label it P'.

Step 2 Use a straightedge to draw $\overleftrightarrow{PP'}$.





Explain why $\overleftrightarrow{PP'}$ is perpendicular to line ℓ.

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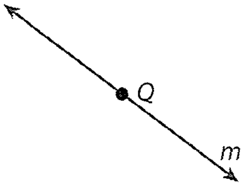
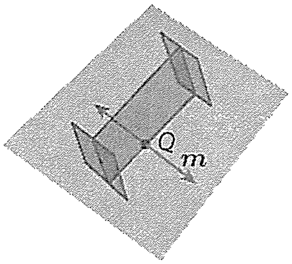
Ⓑ Place the reflective device so that it passes through point Q and is approximately perpendicular to line m. Adjust the angle of the device until the image of line m coincides with line m. Draw a line along the reflective device and label it line n. Explain why line n is perpendicular to line m.





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Reflect

1. How can you check that the lines you drew are perpendicular to lines ℓ and m?

2. Use the reflective device to draw two points on line ℓ that are reflections of each other. Label the points X and X'. What is true about PX and PX'? Why? Use a ruler to check your prediction.

3. Describe how to construct a perpendicular bisector of a line segment using paper folding. Use a rigid motion to explain why the result is a perpendicular bisector.

Explore 2 Justifying the Copy of an Angle Construction

You have seen how to construct a copy of an angle, but how do you know that the copy must be congruent to the original? Recall that to construct a copy of an angle A, you use these steps.

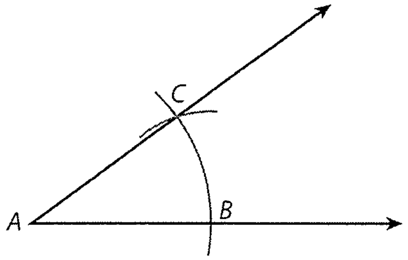
Step 1 Draw a ray with endpoint D.

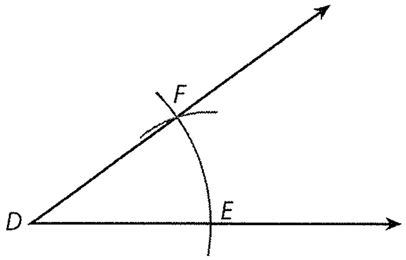
Step 2 Draw an arc that intersects both rays of $\angle A$. Label the intersections B and C.

Step 3 Draw the same arc on the ray. Label the point of intersection E.

Step 4 Set the compass to the length BC.

Step 5 Place the compass at E and draw a new arc. Label the intersection of the new arc F. Draw $\overrightarrow{DF}$. $\angle D$ is congruent to $\angle A$.

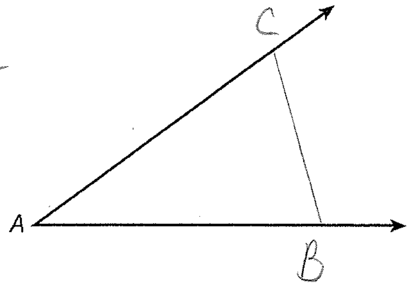


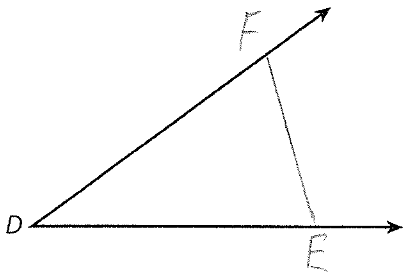


Ⓐ Sketch and name the two triangles that are created when you construct a copy of an angle.

ΔABC

ΔDEF





Ⓑ What segments do you know are congruent? Explain how you know.

$\overline{AB} \cong \overline{DE}$ (1) Created w/ same compass setting (2) arcs that created those segs have same radius.

$\overline{AC} \cong \overline{DF}$ " "

$\overline{BC} \cong \overline{EF}$ " "

Ⓒ Are the triangles congruent? How do you know?

Yes. ≅ by SSS Triangle Congruence Thm.

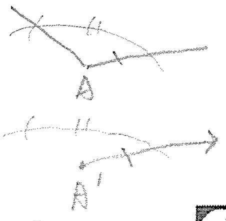
Reflect

4. **Discussion** Suppose you used a larger compass setting to create $\overline{AB}$ than another student when copying the same angle. Will your copied angles be congruent?

Yes, ∠D of S1 ≅ ∠A original and ∠D of S2 ≅ ∠A. ∴ ∠D of S1 ≅ ∠D of S2 by transitive Property.

5. Does the justification above for constructing a copy of an angle work for obtuse angles?

Yes.

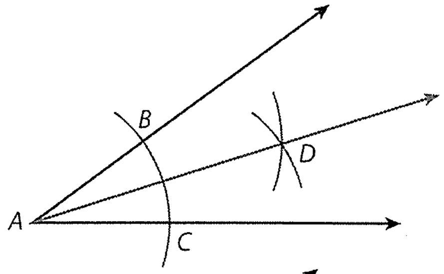


Explain 1 Proving the Angle Bisector and Perpendicular Bisector Constructions

You have constructed angle bisectors and perpendicular bisectors. You now have the tools you need to prove that these compass and straightedge constructions result in the intended figures.

Example 1 **Prove two bisector constructions.**

Ⓐ You have used the following steps to construct an angle bisector.

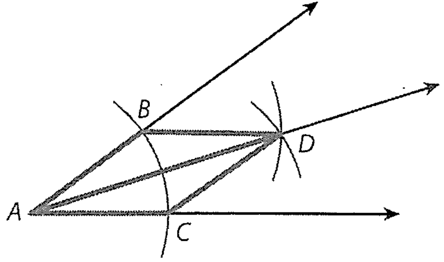


Step 1 Draw an arc intersecting the sides of the angle. Label the intersections B and C.

Step 2 Draw intersecting arcs from B and C. Label the intersection of the arcs as D.

Step 3 Use a straightedge to draw $\overrightarrow{AD}$.

Prove that the construction results in the angle bisector.

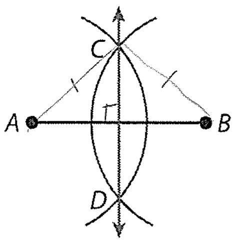


The construction results in the triangles ABD and ACD. Because the same compass setting was used to create them, $\overline{AB} \cong \overline{AC}$ and $\overline{BD} \cong \overline{CD}$. The segment $\overline{AD}$ is congruent to itself by the Reflexive Property of Congruence. So, by the SSS Triangle Congruence Theorem, $\triangle ABD \cong \triangle ACD$.

Corresponding parts of congruent figures are congruent, so $\angle BAD \cong \angle DAC$.

By the definition of angle bisector, $\overrightarrow{AD}$ is the angle bisector of $\angle A$.

Ⓑ You have used the following steps to construct a perpendicular bisector.



Step 1 Draw an arc centered at A.

Step 2 Draw an arc with the same diameter centered at B. Label the intersections C and D.

Step 3 Draw $\overline{CD}$.

Prove that the construction results in the perpendicular bisector. If a pt is equidistant from endpts of a seg, then it is on the ⊥ bisector of the seg.

The point C is equidistant from the endpoints of $\overline{AB}$, so by the ______ Theorem, it lies on the ⊥ bisector of $\overline{AB}$. The point D is also equidistant from the endpoints of $\overline{AB}$, so it also lies on the ⊥ bisector of $\overline{AB}$. Two points determine a line, so $\overline{CD}$ is the ⊥ bisector.

Reflect

6. In Part B, what can you conclude about the measures of the angles made by the intersection of $\overline{AB}$ and $\overline{CD}$?

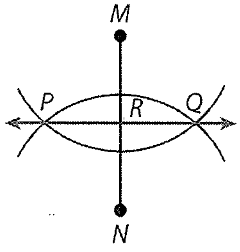
They are rt ∡s.

7. **Discussion** A classmate claims that in the construction shown in Part B, $\overline{AB}$ is the perpendicular bisector of $\overline{CD}$. Is this true? Justify your answer.

No. $\overline{CD}$ can be a line in which case, there is no ⊥ bisector.

Your Turn

8. The construction in Part B is also used to construct the midpoint R of $\overline{MN}$. How is the proof of this construction different from the proof of the perpendicular bisector construction in Part B?

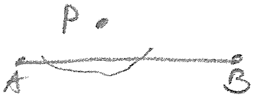


You need 1 extra step. After showing that $\overleftrightarrow{PQ}$ is ⊥ bisector, state that R is the midpt by the def. of ⊥ bisector.

9. How could you combine the constructions in Example 1 to construct a 45° angle?

Choose 1 of the angles.
Construct its angle bisector.
Since the angle was a rt. ∡ and rt ∡s = 90°, the two bisected ∡s are each 45°.

Elaborate

Note: If given


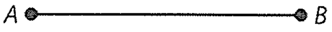
from P, draw an arc. Then draw ⊥ bisector.

10. Describe how you can construct a line that is parallel to a given line using the construction of a perpendicular to a line.

Use steps to construct ⊥ bisector.
Then choose a point on ⊥ and construct the ⊥ to that.

11. Use a straightedge and a piece of string to construct an equilateral triangle that has *AB* as one of its sides. Then explain how you know your construction works. (*Hint*: Consider an arc centered at *A* with radius *AB* and an arc centered at *B* with radius *AB*.)

Use compass.



12. **Essential Question Check-In** Is a construction something that must be proven? Explain.

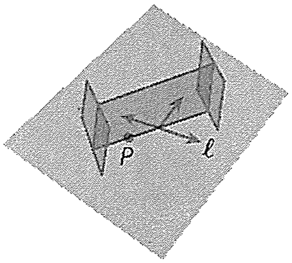
Yes. To know that a construction is valid, you must show that the resulting figure has the properties that are named in the construction.

For ex: for const. of ≅ ∡s, must prove ∡s ≅.

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

1. Julia is given a line ℓ and a point *P* not on line ℓ. She is asked to use a reflective device to construct a line through *P* that is perpendicular to line ℓ. She places the device as shown in the figure. What should she do next to draw the required line?

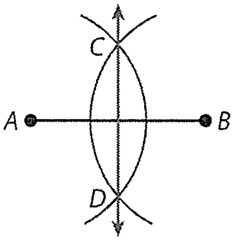


2. Describe how to construct a copy of a segment. Explain how you know that the segments are congruent.

Complete the proof of the construction of a segment bisector.

3. **Given:** the construction of the segment bisector of $\overline{AB}$

Prove: $\overline{CD}$ bisects $\overline{AB}$

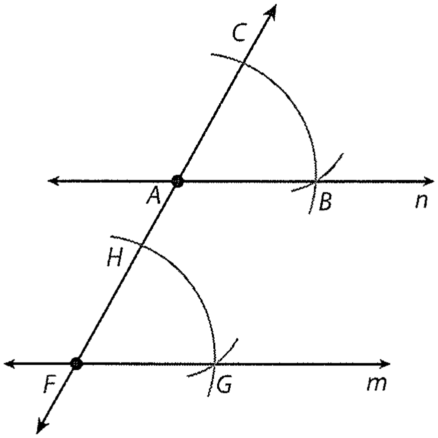


Statements	Reasons
1. $AC =$ BC and $AD =$ BD.	1. Same compass setting used
2. C is on the perpendicular bisector of $\overline{AB}$.	2. If a pt is equidistant from 2 endpts of a segmt, it is on the ⊥ bisector.
3. D is on the perpendicular bisector of $\overline{AB}$.	3. Same as #2
4. $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$.	4. Through any two points, there is exactly one line.
5. $\overline{CD}$ bisects $\overline{AB}$	5. Definition of ⊥ bisector

4. Complete the proof of the construction of a congruent angle.

Given: the construction of $\angle CAB$ given $\angle HFG$

Prove: $\angle CAB \cong \angle HFG$



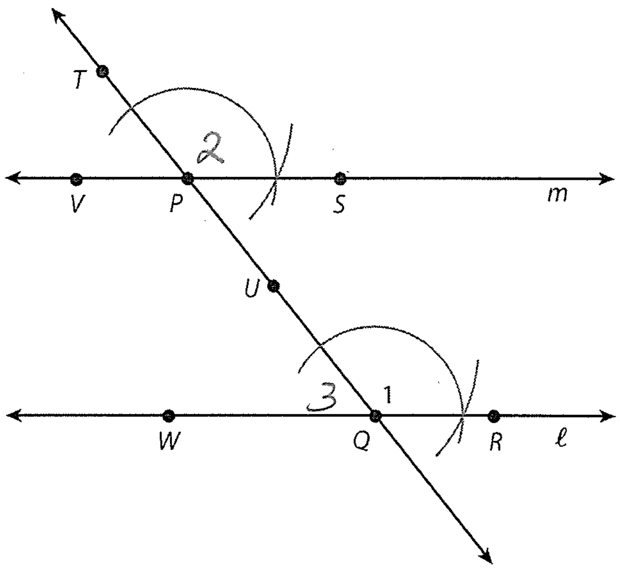
Statements	Reasons
1. $FG = FH =$ AB $= AC$	1. same compass setting
2. $GH = CB$	2. Same compass Setting
3. $\triangle FGH \cong \triangle ABC$	3. SSS △ congruence thm
4. $\angle CAB \cong \angle HFG$	4. CPCTC

To construct a line through the given point *P*, parallel to line ℓ, you use the following steps.

Step 1 Choose a point *Q* on line ℓ and draw $\overline{QP}$.

Step 2 Construct an angle congruent to ∠1 at *P*.

Step 3 Construct the line through the given point, parallel to the line shown.



Describe the relationship between the given angles or segments. Justify your answer.

5. ∠*TPS* and ∠*UQR* are ≅.
Proof of Construct of the copy of an ∡.

6. ∠*SPU* and ∠*RQU* are supplementary; Same-side interior ∡s of ∥ lines are supplem.

7. ∠*VPU* and ∠*UQR* are ≅.
Alternate int. ∡s of ∥ lines are ≅.

8. ∠*TPS* and ∠*WQU* are supplementary.
∡1 and ∡3 are a linear pair and thus, are supplementary. ∡2 ≅ ∡1 b/c they are corresponding ∡s. Replace ∡1 with ∡2 and we get ∡2 and ∡3 supplementary.

9. $\overline{QU}$ and ~~$\overline{PS}$~~

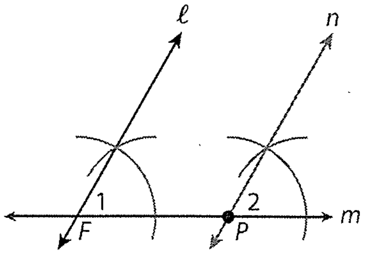
10. $\overline{QU}$ ~~and $\overline{PT}$~~

11. To construct a line through the given point *P*, parallel to line ℓ, you use the
following steps.

Step 1 Draw line *m* through *P* and intersecting line ℓ.

Step 2 Construct an angle congruent to ∠1 at *P*.

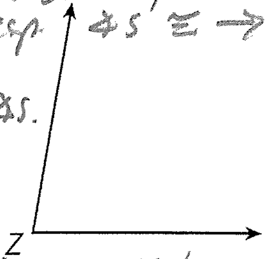
Step 3 Construct the line through the given point, parallel to the line shown.



How do you know that lines ℓ and *n* are parallel? Explain.

Line m is the transversal of lines l and n. By the justification for the construction of a copy of an ∡, we have ∡1 ≅ ∡2. ∡1 & ∡2 are corresponding ∡s. Thus, if 2 lines cut by trans. & Corresp. ∡s ≅ → ∥ lines.

12. Construct an angle whose measure is $\frac{1}{4}$ the measure of ∠*Z*. Justify the construction.



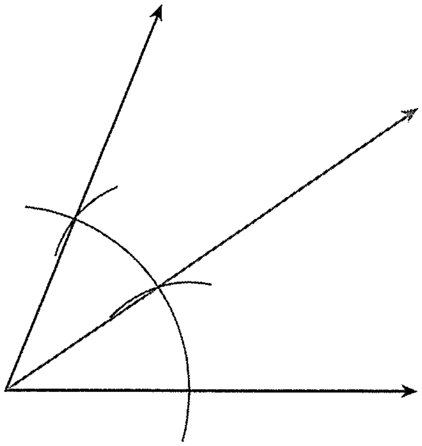
Bisect ∡Z; then bisect on of the smaller ∡s.

$\frac{1}{2} \cdot \left(\frac{1}{2} \cdot m\measuredangle Z\right) = \frac{1}{4} \cdot m\measuredangle Z$

Justification - Apply the proof for construction of ∡ bisector twice.

In Exercises 13 and 14, use the diagram shown. The diagram shows the result of constructing a copy of an angle adjacent to one of the rays of the original angle. Assume the pattern continues.

13. If it takes 10 more copies of the angle for the last angle to overlap the first ray (the horizontal ray), what is the measure of each angle?



14. If it takes 8 more copies of the angle for the last angle to overlap the first ray (the horizontal ray), what is the measure of each angle?

15. Sonia draws a segment on a piece of paper. She wants to find three points that are equidistant from the endpoints of the segment. Explain how she can use paper folding to help her locate the three points.

In Exercises 16–18, a polygon is inscribed in a circle if all of the polygon's vertices lie on the circle.

16. Follow the given steps to construct a square inscribed in a circle.

Use your compass to draw a circle. Mark the center.

Draw a diameter, $\overline{AB}$, using a straightedge.

Construct the perpendicular bisector of $\overline{AB}$. Label the points where the perpendicular bisector intersects the circle as C and D.

Use the straightedge to draw $\overline{AC}$, $\overline{CB}$, $\overline{BD}$, and $\overline{DA}$.

17. Suppose you are given a piece of tracing paper with a circle on it and you do not have a compass. How can you use paper folding to inscribe a square in the circle?

18. Follow the given steps to construct a regular hexagon inscribed in a circle.

Tie a pencil to one end of the string.

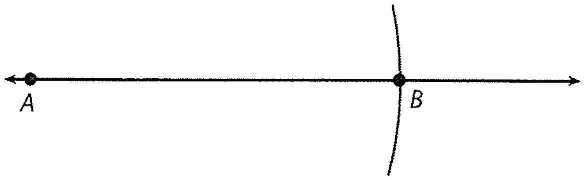
Mark a point *O* on your paper. Place the string on point *O* and hold it down with your finger. Pull the string taut and draw a circle. Mark and label a point *A*.

Hold the point on the string that you placed on point *O*, and move it to point *A*. Pull the string taut and draw an arc that intersects the circle. Label the point as *B*.

Hold the point on the string that you placed on point *A*, and move it to point *B*. Draw an arc to locate point *C* on the circle. Repeat to locate points *D*, *E*, and *F*. Use your straightedge to draw *ABCDEF*.

H.O.T. Focus on Higher Order Thinking

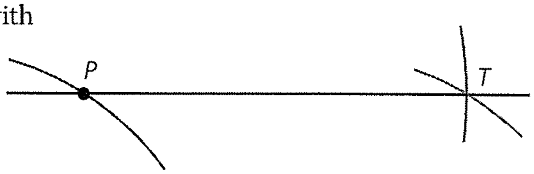
19. Your teacher constructed the figure shown. It shows the construction of line *PT* through point *P* and parallel to line *AB*.



a. Compass settings of length *AB* and *AP* were used in the construction. Complete the statements:

With the compass set to length *AP*, an arc was drawn with the compass point at point ____.

With the compass set to length ____, an arc was drawn with the compass point at point ____.

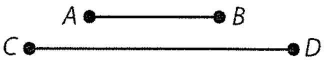


The two arcs intersect at point ____.

b. Write two congruence statements involving segments in the construction.

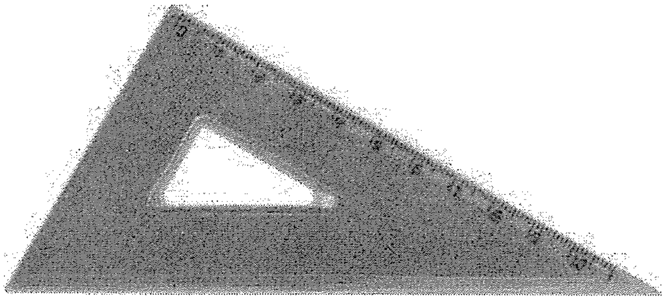
c. Write a proof that the construction is true. That is, given the construction, prove $\overline{PT} \| \overline{AB}$. (*Hint*: Draw segments to create two congruent triangles.)

20. Use the segments shown. Construct and label a segment, $\overline{XY}$, whose length is the average of the lengths of $\overline{AB}$ and $\overline{CD}$. Justify the method you used.



Lesson Performance Task

A plastic "mold" for copying a 30° angle is shown here.



a. If you drew a 30°—60°—90° triangle using the mold, how would you know that your triangle and the mold were congruent?

b. Explain how you know that any angle you would draw using the lower right corner of the mold would measure 30°.

c. Explain the meaning of "tolerance" in the context of drawing an angle using the mold.

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