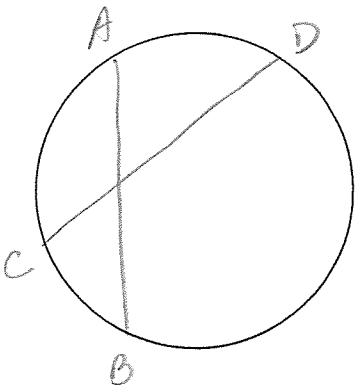
NOTES

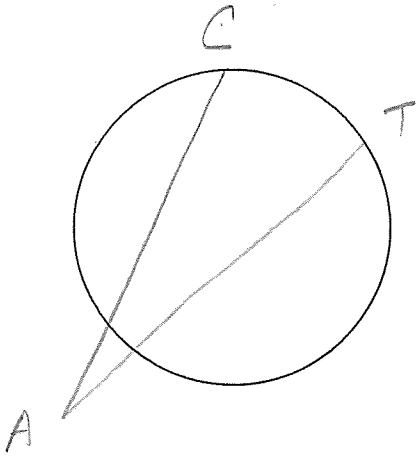
Geometry (Honors)
Section 9.6 – Angles of Chords, Secants and Tangents

So far we have studied, central angles, inscribed angles and tangent-chord angles. Today we will be introducing other types of angles that are formed by chords, secants and tangent. Here the vertex will either fall on the interior or exterior of the circle.

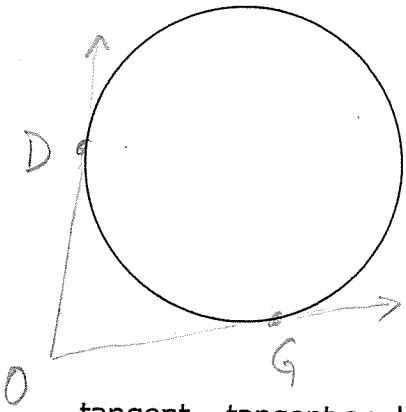
Let's look at an example of each new type of angle!



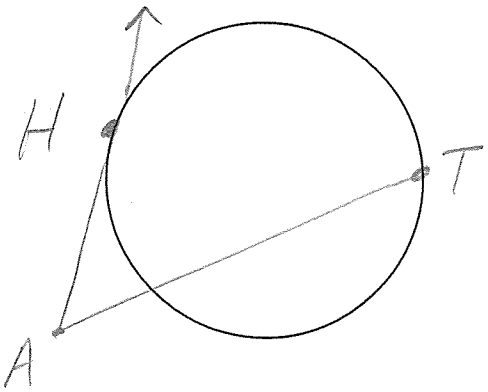
chord – chord angle



secant – secant angle



tangent – tangent angle



secant – tangent angle

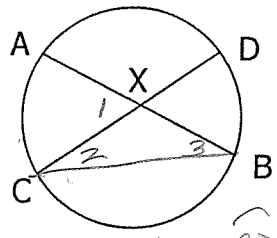
Notice that only the chord – chord angle has a vertex inside the circle.

Each angle has a special relationship to the arc(s) they intercept.

Thm 9.9 – The measure of an angle formed by two chords intersecting inside a circle is one half the <u>sum</u> of the intercepted arc.

Given: chords $\overline{AB}$ and $\overline{CD}$ intersecting at point X.

Prove: $m\angle AXC = \frac{1}{2}(mAC + mBD)$



① Draw $\overline{BC}$.

② $m\measuredangle 1 = m\measuredangle 2 + m\measuredangle 3$ } → ④ $m\measuredangle 1 = \frac{1}{2}m\overset{\frown}{BD} + m\measuredangle 3$

③ $m\measuredangle 2 = \frac{1}{2}m\overset{\frown}{BD}$ ⑤ $m\measuredangle 3 = \frac{1}{2}m\overset{\frown}{AC}$ } → ⑥ $m\measuredangle 1 = \frac{1}{2}m\overset{\frown}{BD} + \frac{1}{2}m\overset{\frown}{AC}$

↓

⑦ $m\measuredangle AXC = \frac{1}{2}(m\overset{\frown}{BD} + m\overset{\frown}{AC})$

① 2 pts determine a line.

② Exterior ∠ Thm

③ Inscribed ∠ = ½ intercept arc

④ Substitution

⑤ Inscribed ∠ = ½ intercept arc

⑥ Substitution

⑦ distributive prop.

Example 1: Find $m\angle 1$

$m\measuredangle 1 = \frac{1}{2}(m\overset{\frown}{DE} + 40)$

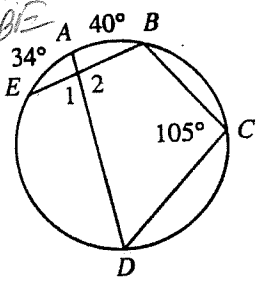
$= \frac{1}{2}(136 + 40)$

$m\measuredangle 1 = 88$

$m\overset{\frown}{DE} = m\overset{\frown}{BD} - m\overset{\frown}{BE}$

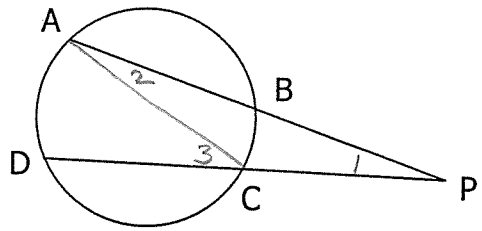
$= 210 - 74$

$= 136$



Thm 9.10 – The measure of an angle formed by two secants, two tangents or a secant and a tangent drawn from a point in the exterior of a circle is equal to half the <u>difference</u> of the measures of the intercepted arc.

Let's prove the case of two secant lines.



Given: $\overline{PA}$ and $\overline{PD}$ secants

Prove: $m\angle P = \frac{1}{2}(mAD - mBC)$

how? → $m\measuredangle P = \frac{1}{2}m\overset{\frown}{AD} - \frac{1}{2}m\overset{\frown}{BC}$

$m\measuredangle P + \frac{1}{2}m\overset{\frown}{BC} = \frac{1}{2}\overset{\frown}{AD}$

① Draw $\overline{AC}$.

② $m\measuredangle 3 = m\measuredangle 1 + m\measuredangle 2$ } → ④ $\frac{1}{2}m\overset{\frown}{AD} = m\measuredangle 1 + m\measuredangle 2$ } → ⑥ $\frac{1}{2}m\overset{\frown}{AD} = m\measuredangle 1 + \frac{1}{2}m\overset{\frown}{BC}$

③ $m\measuredangle 3 = \frac{1}{2}m\overset{\frown}{AD}$ ⑤ $m\measuredangle 2 = \frac{1}{2}m\overset{\frown}{BC}$

⑦ $\frac{1}{2}m\overset{\frown}{AD} - \frac{1}{2}m\overset{\frown}{BC} = m\measuredangle 1$

⑧ $\frac{1}{2}(m\overset{\frown}{AD} - m\overset{\frown}{BC}) = m\measuredangle P$

① 2 pts determine a line

② Ext. ∠ Thm

③ Inscribed ∠ = ½ intercepted arc

④ Substitution

⑤ Inscribe ∠ = ½ int. arc

⑥ Substitution

⑦ Subtraction prop

⑧ distributive prop